# Package 'BRISC'

April 30, 2022

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# **Description**

The function BRISC\_bootstrap performs bootstrap to provide confidence intervals for parameters of univariate spatial regression models using outputs of BRISC\_estimation. The details of the bootstrap method can be found in BRISC (Saha & Datta, 2018). The optimization is performed with C library of limited-memory BFGS libLBFGS: a library of Limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS), http://www.chokkan.org/software/liblbfgs/ (Naoaki Okazaki). For user convenience the soure codes of the package libLBFGS are provided in the package. Some code blocks are borrowed from the R package: spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes

https://CRAN.R-project.org/package=spNNGP.

# Usage

#### **Arguments**

BRISC_Out	an object of class BRISC_Out, obtained as an output of BRISC_estimation.
n_boot	number of bootstrap samples. Default value is 100.
h	number of core to be used in parallel computing setup for bootstrap samples. If $h = 1$ , there is no parallelization. Default value is 1.
n_omp	number of threads to be used, value can be more than 1 if source code is compiled with OpenMP support. Default is 1.
init	keyword that specifies initialization scheme to be used. Supported keywords are: "Initial" and "Estimate" for initialization of parameter values for bootstrap samples with initial values used in BRISC_estimate and estimated values of parameters in BRISC_estimate respectively.
verbose	if TRUE, model specifications along with information regarding OpenMP support and progress of the algorithm is printed to the screen. Otherwise, nothing is printed to the screen. Default value is TRUE.
nugget_status	if $nugget_status = 0$ , tau.sq is fixed to 0, if $nugget_status = 1$ tau.sq is estimated. Default value is 1.

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#### Value

A list comprising of the following:

# Author(s)

```
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```

in R, reported using proc.time().

#### References

Saha, A., & Datta, A. (2018). BRISC: bootstrap for rapid inference on spatial covariances. Stat, e184, DOI: 10.1002/sta4.184.

Okazaki N. libLBFGS: a library of Limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS), http://www.chokkan.org/software/liblbfgs/ .

Andrew Finley, Abhirup Datta and Sudipto Banerjee (2017). spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes. R package version 0.1.1. https://CRAN.R-project.org/package=spNNGP

```
rmvn \leftarrow function(n, mu = 0, V = matrix(1)){
  p <- length(mu)</pre>
  if(any(is.na(match(dim(V),p))))
    stop("Dimension not right!")
  D <- chol(V)
  t(matrix(rnorm(n*p), ncol=p)%*%D + rep(mu,rep(n,p)))
}
set.seed(1)
n <- 300
coords <- cbind(runif(n,0,1), runif(n,0,1))
beta <- c(1,5)
x <- cbind(rnorm(n), rnorm(n))</pre>
sigma.sq = 1
phi = 5
tau.sq = 0.1
B <- as.matrix(beta)</pre>
D <- as.matrix(dist(coords))</pre>
```

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```
R <- exp(-phi*D)
w <- rmvn(1, rep(0,n), sigma.sq*R)

y <- rnorm(n, x%*%B + w, sqrt(tau.sq))

estimation_result <- BRISC_estimation(coords, y, x)
bootstrap_result <- BRISC_bootstrap(estimation_result, n_boot = 10)</pre>
```

BRISC\_correlation

Function for simulating correlated data with BRISC

# Description

The function BRISC\_correlation creates correlated data (known structure) using Nearest Neighbor Gaussian Processes (NNGP). BRISC\_correlation uses the sparse Cholesky representation of Vecchia's likelihood developed in Datta et al., 2016. Some code blocks are borrowed from the R package: spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes https://CRAN.R-project.org/package=spNNGP.

# Usage

# **Arguments**

coords	an $n \times 2$ matrix of the observation coordinates in $\mathbb{R}^2$ (e.g., easting and northing).
sim	an $n \times k$ matrix of the $k$ many $n \times 1$ vectors from which the correlated data are calculated (see Details below).
sigma.sq	value of sigma square. Default value is 1.
tau.sq	value of tau square. Default value is 0.1.
phi	value of phi. Default value is 1.
nu	value of nu, only required for matern covariance model. Default value is 1.5.
n.neighbors	number of neighbors used in the NNGP. Default value is $\max(100, n-1)$ . We suggest a high value of n.neighbors for lower value of phi.
n_omp	number of threads to be used, value can be more than 1 if source code is compiled with OpenMP support. Default is 1.
cov.model	keyword that specifies the covariance function to be used in modelling the spatial dependence structure among the observations. Supported keywords are: "exponential", "matern", "spherical", and "gaussian" for exponential, Matern, spherical and Gaussian covariance function respectively. Default value is "exponential".

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search.type

keyword that specifies type of nearest neighbor search algorithm to be used. Supported keywords are: "brute", "tree" and "cb".

"brute" and "tree" provide the same result, though "tree" should be faster. "cb" implements fast code book search described in Ra and Kim (1993) modified for NNGP. If locations do not have identical coordinate values on the axis used for the nearest neighbor determination, then "cb" and "brute" should produce identical neighbor sets. However, if there are identical coordinate values on the axis used for nearest neighbor determination, then "cb" and "brute" might produce different, but equally valid neighbor sets, e.g., if data are on a grid. Default value is "tree".

stabilization

when we use a very smooth covarince model (lower values of phi for spherical and Gaussian covariance and low phi and high nu for Matern covarinace) in absence of a non-negligble nugget, the correlation process may fail due to computational instability. If stabilization = TRUE, performs stabilization by setting tau.sq = maxtau.sq, sigma.sq \* 1e-06. Default value is TRUE for cov.model = "expoenential" and FALSE otherwise.

verbose

if TRUE, model specifications along with information regarding OpenMP support and progress of the algorithm is printed to the screen. Otherwise, nothing is printed to the screen. Default value is TRUE.

tol

the input observation coordinates are rounded to this many places after the decimal. The default value is 12.

# **Details**

Denote g be the input sim. Let  $\Sigma$  be the precision matrix associated with the covariance model determined by the cov.model and model parameters. Then BRISC\_correlation calculates h, where h is given as follows:

$$S^{-0.5}h = a$$

where,  $S^{-0.5}$  is a sparse approximation of the cholesky factor  $\Sigma^{-0.5}$  of the precision matrix  $\Sigma^{-1}$ , obtained from NNGP.

# Value

A list comprising of the following:

coords the matrix coords.

n.neighbors the used value of n.neighbors.

cov.model the used covariance model.

Theta parameters of covarinace model; accounts for stabilization.

input.data the matrix sim.

output.data the output matrix h in Details.

time (in seconds) required after preprocessing data in R,

reported using, proc.time().

BRISC\_decorrelation

#### Author(s)

```
Arkajyoti Saha <arkajyotisaha93@gmail.com>,
Abhirup Datta <abhidatta@jhu.edu>
```

#### References

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. Journal of the American Statistical Association, 111:800-812.

Andrew Finley, Abhirup Datta and Sudipto Banerjee (2017). spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes. R package version 0.1.1. https://CRAN.R-project.org/package=spNNGP

#### **Examples**

BRISC\_decorrelation Function for decorrelating data with BRISC

#### **Description**

The function BRISC\_decorrelation is used to decorrelate data (known structure) using Nearest Neighbor Gaussian Processes (NNGP). BRISC\_decorrelation uses the sparse Cholesky representation of Vecchia's likelihood developed in Datta et al., 2016. Some code blocks are borrowed from the R package: spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes https://CRAN.R-project.org/package=spNNGP.

# Usage

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#### **Arguments**

coords an  $n \times 2$  matrix of the observation coordinates in  $\mathbb{R}^2$  (e.g., easting and northing).

sim an  $n \times k$  matrix of the k many  $n \times 1$  vectors from which the decorrelated data

are calculated (see Details below).

sigma.sq value of sigma square. Default value is 1. tau.sq value of tau square. Default value is 0.1.

phi value of phi. Default value is 1.

nu value of nu, only required for Matern covariance model. Default value is 1.5.

n.neighbors number of neighbors used in the NNGP. Default value is max(100, n-1). We

suggest a high value of n.neighbors for lower value of phi.

n\_omp number of threads to be used, value can be more than 1 if source code is com-

piled with OpenMP support. Default is 1.

cov.model keyword that specifies the covariance function to be used in modelling the spa-

tial dependence structure among the observations. Supported keywords are: "exponential", "matern", "spherical", and "gaussian" for exponential, Matern, spherical and Gaussian covariance function respectively. Default value

is "exponential".

search.type keyword that specifies type of nearest neighbor search algorithm to be used.

Supported keywords are: "brute", "tree" and "cb".

"brute" and "tree" provide the same result, though "tree" should be faster. "cb" implements fast code book search described in Ra and Kim (1993) modified for NNGP. If locations do not have identical coordinate values on the axis used for the nearest neighbor determination, then "cb" and "brute" should produce identical neighbor sets. However, if there are identical coordinate values on the axis used for nearest neighbor determination, then "cb" and "brute" might produce different, but equally valid neighbor sets, e.g., if data are on a

grid. Default value is "tree".

stabilization when the correlated data are generated from a very smooth covarince model

(lower values of phi for spherical and Gaussian covariance and low phi and high nu for Matern covarinace), the decorrelation process may fail due to computational instability. If stabilization = TRUE, performs stabilization by adding a white noise to the data with nugget tau.sq = sigma.sq \* 1e-06. Default value

is TRUE for cov.model = "expoenential" and FALSE otherwise.

verbose if TRUE, model specifications along with information regarding OpenMP support

and progress of the algorithm is printed to the screen. Otherwise, nothing is

printed to the screen. Default value is TRUE.

tol the input observation coordinates are rounded to this many places after the dec-

imal. The default value is 12.

#### **Details**

Denote h be the input sim. Let  $\Sigma$  be the covariance matrix associated with the covariance model determined by the cov.model and model parameters. Then BRISC\_decorrelation calculates g, where g is given as follows:

 $S^{-0.5}h = a$ 

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where,  $S^{-0.5}$  is a sparse approximation of the cholesky factor  $\Sigma^{-0.5}$  of the precision matrix  $\Sigma^{-1}$ , obtained from NNGP.

#### Value

A list comprising of the following:

coords the matrix coords.

n.neighbors the used value of n.neighbors.

cov.model the used covariance model.

Theta parameters of covarinace model; accounts for stabilization.

input.data if stabilization = FALSE, return the matrix sim. If stabilization = TRUE,

returns sim + used white noise in stabilization process.

output . data the output matrix g in Details.

time (in seconds) required after preprocessing data in R,

reported using, proc.time().

#### Author(s)

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#### References

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. Journal of the American Statistical Association, 111:800-812.

Andrew Finley, Abhirup Datta and Sudipto Banerjee (2017). spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes. R package version 0.1.1. https://CRAN.R-project.org/package=spNNGP

```
rmvn <- function(n, mu = 0, V = matrix(1)){
  p <- length(mu)
  if(any(is.na(match(dim(V),p))))
    stop("Dimension not right!")
  D <- chol(V)
  t(matrix(rnorm(n*p), ncol=p)%*%D + rep(mu,rep(n,p)))
}
set.seed(1)
n <- 1000
coords <- cbind(runif(n,0,1), runif(n,0,1))
sigma.sq = 1
phi = 1
set.seed(1)</pre>
```

```
D <- as.matrix(dist(coords))
R <- exp(-phi*D)
sim <- rmvn(3, rep(0,n), sigma.sq*R)
decorrelation_result <- BRISC_decorrelation(coords, sim = sim)</pre>
```

BRISC\_estimation

Function for estimation with BRISC

#### **Description**

The function BRISC\_estimation fits univariate spatial regression models for large spatial data using Vecchia's approximate likelihood (Vecchia, 1988). BRISC\_estimation uses the sparse Cholesky representation of Vecchia's likelihood developed in Datta et al., 2016. The Maximum Likelihood Estimates (MLE) of the parameters are used later for calculating the confidence interval via the BRISC\_bootstrap (BRISC, Saha & Datta, 2018).

We recommend using  $BRISC\_estimation$  followed by  $BRISC\_bootstrap$  to obtain the confidence intervals for the model parameters.

The optimization is performed with C library of limited-memory BFGS libLBFGS: a library of Limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS),

http://www.chokkan.org/software/liblbfgs/ (Naoaki Okazaki). For user convenience the source codes of the package libLBFGS are provided in the package. The code for the coordinate ordering method, approximate Maximum Minimum Distance (Guinness, 2018) is available in

https://github.com/joeguinness/gp\_reorder/tree/master/R and is adopted with minor modification. Some code blocks are borrowed from the R package: spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes https://CRAN.R-project.org/package=spNNGP

# Usage

#### **Arguments**

coords an  $n \times 2$  matrix of the observation coordinates in  $R^2$  (e.g., easting and northing). y an n length vector of response at the observed coordinates. x an  $n \times p$  matrix of the covariates in the observation coordinates. Default value is  $n \times 1$  matrix of 1 to adjust for the mean(intercept).

sigma.sq starting value of sigma square. Default value is 1. tau.sq starting value of tau square. Default value is 0.1.

phi starting value of phi. Default value is 1.

nu starting value of nu, only required for matern covariance model. Default value

is 1.5.

n.neighbors number of neighbors used in the NNGP. Default value is 15.

n\_omp number of threads to be used, value can be more than 1 if source code is com-

piled with OpenMP support. Default is 1.

order keyword that specifies the ordering scheme to be used in ordering the obser-

vations. Supported keywords are: "AMMD" and "Sum\_coords" for approximate Maximum Minimum Distance and sum of coordinate based ordering, respectively. Default value is "Sum\_coords". n>65 is required for "AMMD". Ignored,

if "ordering" in not NULL.

cov.model keyword that specifies the covariance function to be used in modelling the spa-

tial dependence structure among the observations. Supported keywords are: "exponential", "matern", "spherical", and "gaussian" for exponential, Matern, spherical and Gaussian covariance function respectively. Default value

is "exponential".

search.type keyword that specifies type of nearest neighbor search algorithm to be used.

Supported keywords are: "brute", "tree" and "cb".

"brute" and "tree" provide the same result, though "tree" should be faster. "cb" implements fast code book search described in Ra and Kim (1993) modified for NNGP. If locations do not have identical coordinate values on the axis used for the nearest neighbor ordering (see order argument) then "cb" and "brute" should produce identical neighbor sets. However, if there are identical coordinate values on the axis used for nearest neighbor ordering, then "cb" and "brute" might produce different, but equally valid, neighbor sets, e.g., if data are on a grid. Default value is "tree". Ignored, if "neighbor" in not NULL.

stabilization

when the spatial errors are generated from a very smooth covarince model (lower values of phi for spherical and Gaussian covariance and low phi and high nu for Matern covarinace), the estimation process may fail due to computational instability. If stabilization = TRUE, performs stabilization by adding a white noise to the reordered data with nugget tau.sq = sigma.sq \* 1e-06. Estimation is performed on this new data with nugget\_status = 1 (see nugget\_status argument below). Default value is TRUE for cov.model = "expoenential" and

FALSE otherwise.

pred.stabilization

if not NULL, will truncate the estimated tau square to pred.stabilization \*

estimated sigma square. This provides additional stability in

BRISC\_prediction. Default value is 1e - 8.

verbose if TRUE, model specifications along with information regarding OpenMP support

and progress of the algorithm is printed to the screen. Otherwise, nothing is

printed to the screen. Default value is TRUE.

eps tolerance to be used in centred finite difference approximation of derivatives.

Default value is 2e-05.

nugget\_status if nugget\_status = 0, tau.sq is fixed to 0, if nugget\_status = 1 tau.sq is

estimated. Default value is 1.

ordering if NULL, the observed locations will be ordered following the scheme in "order".

If not NULL, the ordering step is bypassed and the input must denote the n length integer vector of ordering of the input coordinates that is to be used as the ordering of the coordinates for determination of the set of nearest neighbors. Output

from BRISC\_order can be used here.

neighbor if NULL, neighbor set and corresponding information are created using the search

type specified in "search.type". If not NULL, the step of searching the neighbors is bypassed and the input must be an output from BRISC\_neighbor with

identical input in "order", "ordering" and "search.type".

tol the input observation coordinates, response and the covariates are rounded to

this many places after the decimal. The default value is 12.

#### Value

An object of class BRISC\_Out, which is a list comprising:

ord the vector of indices used to order data necessary for fitting the NNGP model.

coords the matrix coords[ord,].

y If stabilization = FALSE, returns the vector y[ord].

If stabilization = TRUE, returns y[ord] + used white noise in stabilization

process.

X the matrix x[ord,,drop=FALSE].

n.neighbors the used value of n.neighbors.

cov.model the used covariance model.

eps value of used eps for approximate derivation. Default value is 2e-05.

init initial values of the parameters of the covariance model;

accounts for stabilization.

Beta estimate of beta.

Theta estimate of parameters of covarinace model.

log\_likelihood value of Vecchia's approximate log likelihood with parameter estimates.

estimation.time

time (in seconds) required to perform the model fitting after ordering and pre-

processing data in R, reported using proc.time().

BRISC\_Object object required for bootstrap and prediction.

#### Author(s)

Arkajyoti Saha <arkajyotisaha93@gmail.com>, Abhirup Datta <abhidatta@jhu.edu>

#### References

Saha, A., & Datta, A. (2018). BRISC: bootstrap for rapid inference on spatial covariances. Stat, e184, DOI: 10.1002/sta4.184.

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. Journal of the American Statistical Association, 111:800-812.

Guinness, J. (2018) Permutation and Grouping Methods for Sharpening Gaussian Process Approximations, Technometrics, DOI: 10.1080/00401706.2018.1437476, https://github.com/joeguinness/gp\_reorder/tree/master/R.

Vecchia, A. V. (1988) Estimation and model identification for continuous spatial processes. Journal of the Royal Statistical Society. Series B (Methodological), 297-312.

Okazaki N. libLBFGS: a library of Limited-memory Broyden-Fletcher-Goldfarb-Shanno (L-BFGS), http://www.chokkan.org/software/liblbfgs/ .

Andrew Finley, Abhirup Datta and Sudipto Banerjee (2020). spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes. R package version 0.1.4. https://CRAN.R-project.org/package=spNNGP

Ra, S. W., & Kim, J. K. (1993). A fast mean-distance-ordered partial codebook search algorithm for image vector quantization. IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing, 40(9), 576-579.

```
rmvn \leftarrow function(n, mu = 0, V = matrix(1))
  p <- length(mu)</pre>
  if(any(is.na(match(dim(V),p))))
    stop("Dimension not right!")
  D <- chol(V)
  t(matrix(rnorm(n*p), ncol=p)%*%D + rep(mu,rep(n,p)))
set.seed(1)
n <- 1000
coords <- cbind(runif(n,0,1), runif(n,0,1))
beta <- c(1,5)
x <- cbind(rnorm(n), rnorm(n))</pre>
sigma.sq = 1
phi = 1
tau.sq = 0.1
B <- as.matrix(beta)</pre>
D <- as.matrix(dist(coords))</pre>
R <- exp(-phi*D)</pre>
w <- rmvn(1, rep(0,n), sigma.sq*R)</pre>
y \leftarrow rnorm(n, x%*B + w, sqrt(tau.sq))
```

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```
estimation_result <- BRISC_estimation(coords, y, x)
estimation_result$Theta ##Estimates of covariance model parameters.
estimation_result$Beta ##Estimates of Beta</pre>
```

BRISC\_neighbor

Function for finding set of nearest neighbors for BRISC

#### **Description**

The function BRISC\_neighbor creates the set of nearest neighbors for a given set of coordinates, which can be used as an input for "neighbor" argument in BRISC\_estimation. This is especially useful for avoiding often computationally intensive nearest neighbor finding scheme in case of multiple application of BRISC\_estimation on a fixed set of coordinates.

#### Usage

# Arguments

coords an  $n \times 2$  matrix of the observation coordinates in  $R^2$  (e.g., easting and northing).

n.neighbors number of neighbors used in the NNGP. Default value is 15.

n\_omp number of threads to be used, value can be more than 1 if source code is com-

piled with OpenMP support. Default is 1.

order keyword that specifies the ordering scheme to be used in ordering the obser-

vations. Supported keywords are: "AMMD" and "Sum\_coords" for approximate Maximum Minimum Distance and sum of coordinate based ordering, respectively. Default value is "Sum\_coords". n>65 is required for "AMMD". Ignored,

if ordering in not NULL.

search.type keyword that specifies type of nearest neighbor search algorithm to be used.

Supported keywords are: "brute", "tree" and "cb".

"brute" and "tree" provide the same result, though "tree" should be faster. "cb" implements fast code book search described in Ra and Kim (1993) modified for NNGP. If locations do not have identical coordinate values on the axis used for the nearest neighbor ordering (see order argument) then "cb" and "brute" should produce identical neighbor sets. However, if there are identical coordinate values on the axis used for nearest neighbor ordering, then "cb" and "brute" might produce different, but equally valid, neighbor sets, e.g., if

data are on a grid. Default value is "tree".

verbose if TRUE, information regarding OpenMP support and progress of the algorithm is

printed to the screen. Otherwise, nothing is printed to the screen. Default value

is TRUE.

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ordering if not NULL, denotes the n length integer vector of ordering of the input coordi-

nates and is used as the ordering of the coordinates for determination of the set

of nearest neighbors.

tol the input observation coordinates, response and the covariates are rounded to

this many places after the decimal. The default value is 12.

#### Value

A list containing information regarding nearest neighbors which can be used as an input for "neighbor" argument in BRISC\_estimation.

#### Author(s)

```
Arkajyoti Saha <arkajyotisaha93@gmail.com>,
Abhirup Datta <abhidatta@jhu.edu>
```

#### References

Saha, A., & Datta, A. (2018). BRISC: bootstrap for rapid inference on spatial covariances. Stat, e184, DOI: 10.1002/sta4.184.

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. Journal of the American Statistical Association, 111:800-812.

Guinness, J. (2018) Permutation and Grouping Methods for Sharpening Gaussian Process Approximations, Technometrics, DOI: 10.1080/00401706.2018.1437476, https://github.com/joeguinness/gp\_reorder/tree/master/R.

#### **Examples**

```
set.seed(1)
n <- 1000
coords <- cbind(runif(n,0,1), runif(n,0,1))
ordering_result <- BRISC_order(coords)</pre>
```

BRISC\_order

Function for ordering coordinates with BRISC

# **Description**

The function BRISC\_order outputs the ordering for a set of coordinates, which can be used as an input for "ordering" argument in BRISC\_estimation. This is especially useful for avoiding often computationally intensive location ordering scheme in case of multiple application of BRISC\_estimation on a fixed set of coordinates.

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#### Usage

```
BRISC_order(coords, order = "Sum_coords", verbose = TRUE)
```

# Arguments

coords an  $n \times 2$  matrix of the observation coordinates in  $R^2$  (e.g., easting and northing).

order keyword that specifies the ordering scheme to be used in ordering the obser-

vations. Supported keywords are: "AMMD" and "Sum\_coords" for approximate Maximum Minimum Distance and sum of coordinate based ordering, respec-

tively. Default value is "Sum\_coords". n>65 is required for "AMMD".

verbose if TRUE, progress of the algorithm is printed to the screen. Otherwise, nothing is

printed to the screen. Default value is TRUE.

#### Value

An integer vector of ordering of the input coordinates which can be used as an input for "ordering" argument in BRISC\_estimation.

#### Author(s)

```
Arkajyoti Saha <arkajyotisaha93@gmail.com>,
Abhirup Datta <abhidatta@jhu.edu>
```

#### References

Saha, A., & Datta, A. (2018). BRISC: bootstrap for rapid inference on spatial covariances. Stat, e184, DOI: 10.1002/sta4.184.

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. Journal of the American Statistical Association, 111:800-812.

Guinness, J. (2018) Permutation and Grouping Methods for Sharpening Gaussian Process Approximations, Technometrics, DOI: 10.1080/00401706.2018.1437476, https://github.com/joeguinness/gp\_reorder/tree/master/R.

```
set.seed(1)
n <- 1000
coords <- cbind(runif(n,0,1), runif(n,0,1))
ordering_result <- BRISC_order(coords)</pre>
```

BRISC\_prediction

BRISC_prediction	Function for	narformina	prediction w	ith RRISC
prisc_breaterion	Function for	perjorming	prediction w	un brisc

# Description

The function BRISC\_prediction performs fast prediction on a set of new locations with univariate spatial regression models using Nearest Neighbor Gaussian Processes (NNGP) (Datta et al., 2016). BRISC\_prediction uses the parameter estimates from BRISC\_estimation for the prediction. Some code blocks are borrowed from the R package: spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes https://CRAN.R-project.org/package=spNNGP.

# Usage

# **Arguments**

BRISC_Out	an object of class BRISC_Out, obtained as an output of BRISC_estimation.
coords.0	the spatial coordinates corresponding to prediction locations. Its structure should be same as that of coords in BRISC_estimation. Default value is a column of 1 to adjust for the mean (intercept).
X.0	the covariates for prediction locations. Its Structure should be identical (including intercept) with that of covariates provided for estimation purpose in BRISC_estimation.
n_omp	number of threads to be used, value can be more than 1 if source code is compiled with OpenMP support. Default is 1.
verbose	if TRUE, model specifications along with information regarding OpenMP support and progress of the algorithm is printed to the screen. Otherwise, nothing is printed to the screen. Default value is TRUE.
tol	the coordinates and the covariates corresponding to the prediction locations are rounded to this many places after the decimal. The default value is 12.

#### Value

A list comprising of the following:

```
prediction predicted response corresponding to X.0 and coords.0.

prediction.ci confidence intervals corresponding to the predictions.

prediction.time
```

time (in seconds) required to perform the prediction after preprocessing data in R, reported using proc.time().

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#### Author(s)

```
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Abhirup Datta <abhidatta@jhu.edu>
```

#### References

Datta, A., S. Banerjee, A.O. Finley, and A.E. Gelfand. (2016) Hierarchical Nearest-Neighbor Gaussian process models for large geostatistical datasets. Journal of the American Statistical Association, 111:800-812.

Andrew Finley, Abhirup Datta and Sudipto Banerjee (2017). spNNGP: Spatial Regression Models for Large Datasets using Nearest Neighbor Gaussian Processes. R package version 0.1.1. https://CRAN.R-project.org/package=spNNGP

```
rmvn <- function(n, mu = 0, V = matrix(1)){
  p <- length(mu)</pre>
  if(any(is.na(match(dim(V),p))))
    stop("Dimension not right!")
  D <- chol(V)
  t(matrix(rnorm(n*p), ncol=p)%*%D + rep(mu,rep(n,p)))
}
set.seed(1)
n <- 500
coords <- cbind(runif(n,0,1), runif(n,0,1))</pre>
beta <- c(1,5)
x <- cbind(rnorm(n), rnorm(n))</pre>
sigma.sq = 1
phi = 1
tau.sq = 0.1
B <- as.matrix(beta)</pre>
D <- as.matrix(dist(coords))</pre>
R <- exp(-phi*D)</pre>
w \leftarrow rmvn(1, rep(0,n), sigma.sq*R)
y \leftarrow rnorm(n, x%*%B + w, sqrt(tau.sq))
estimation_result <- BRISC_estimation(coords[1:400,], y[1:400], x[1:400,])</pre>
prediction_result <- BRISC_prediction(estimation_result,</pre>
                                          coords[401:500,], x[401:500,])
```

18 BRISC\_simulation

BRISC\_simulation

Function to simulate data with BRISC

# **Description**

The function BRISC\_simulation simulates correlated data (known structure) using Nearest Neighbor Gaussian Processes (NNGP). BRISC\_simulation uses the sparse Cholesky representation of Vecchia's likelihood developed in Datta et al., 2016. BRISC\_simulation uses BRISC\_correlation for this purpose.

# Usage

# Arguments

coords	an $n \times 2$ matrix of the observation coordinates in $\mathbb{R}^2$ (e.g., easting and northing).
sim_number	number of simulations. Default value is 1.
seeds	seeds which are used in generation of the initial independent data. Default value is NULL. If non-null, the number of seeds must be equal to sim_number.
sigma.sq	value of sigma square. Default value is 1.
tau.sq	value of tau square. Default value is 0.1.
phi	value of phi. Default value is 1.
nu	starting value of nu, only required for matern covariance model. Default value is 1.5.
n.neighbors	number of neighbors used in the NNGP. Default value is 15.
n_omp	number of threads to be used, value can be more than 1 if source code is compiled with OpenMP support. Default is 1.
cov.model	keyword that specifies the covariance function to be used in modelling the spatial dependence structure among the observations. Supported keywords are: "exponential", "matern", "spherical", and "gaussian" for exponential, Matern, spherical and Gaussian covariance function respectively. Default value is "exponential".
search.type	keyword that specifies type of nearest neighbor search algorithm to be used. Supported keywords are: "brute", "tree" and "cb".

"brute" and "tree" provide the same result, though "tree" should be faster.

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"cb" implements fast code book search described in Ra and Kim (1993) modified for NNGP. If locations do not have identical coordinate values on the axis used for the nearest neighbor ordering (see order argument) then "cb" and "brute" should produce identical neighbor sets. However, if there are identical coordinate values on the axis used for nearest neighbor ordering, then "cb" and "brute" might produce different, but equally valid, neighbor sets, e.g., if data are on a grid. Default value is "tree".

stabilization

when we use a very smooth covarince model (lower values of phi for spherical and Gaussian covariance and low phi and high nu for Matern covarinace) in absence of a non-negligble nugget, the correlation process may fail due to computational instability. If stabilization = TRUE, performs stabilization by setting tau.sq = maxtau.sq, sigma.sq \* 1e-06. Default value is TRUE for cov.model = "expoenential" and FALSE otherwise.

verbose

if TRUE, model specifications along with information regarding OpenMP support and progress of the algorithm is printed to the screen. Otherwise, nothing is printed to the screen. Default value is TRUE.

tol

the input observation coordinates are rounded to this many places after the decimal. The default value is 12.

#### Value

A list comprising of the following:

coords the matrix coords.

n.neighbors the used value of n.neighbors.

cov.model the used covariance model.

Theta parameters of covarinace model; accounts for stabilization.

input.data the  $n \times sim_n umber$  matrix of generated independent data. Here  $i^{th}$  column

denotes the data corresponding to the  $i^{th}$  simulation.

output.data the  $n \times sim_n umber$  matrix of generated correlated data. Here  $i^{th}$  column de-

notes the data corresponding to the  $i^{th}$  simulation.

time (in seconds) required after preprocessing data in R,

reported using, proc.time().

# Author(s)

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```
set.seed(1)
n <- 1000
coords <- cbind(runif(n,0,1), runif(n,0,1))
sigma.sq = 1</pre>
```

BRISC\_variogram.ci

```
phi = 1
simulation_result <- BRISC_simulation(coords, sim_number = 3)</pre>
```

BRISC\_variogram.ci

Function for plotting estimated Variogram and confidence region

# Description

The function BRISC\_variogram.ci plots estimated Variogram and associated confience region. BRISC\_variogram.ci uses the parameter estimates from BRISC\_estimation and associated confidence interval from BRISC\_bootstrap.

# Usage

#### **Arguments**

BRISC\_Out an object of class BRISC\_Out, obtained as an output of

 ${\tt BRISC\_estimation}.$ 

confidence\_est bootstrp sample of the Theta parameters, obtained from BRISC\_bootstrap.

plot.variogram if TRUE, plots the variogram and the associated confidence region. Default is

FALSE.

# Value

A list comprising of the following:

variogram variogram and associated confidence region corresponding to lag ranging from

0 to 20, evaluated at 0.01 frequency.

Plot plots the Variogram and associated confidence region with legends.

#### Author(s)

```
Arkajyoti Saha <arkajyotisaha93@gmail.com>,
Abhirup Datta <abhidatta@jhu.edu>
```

BRISC\_variogram.ci

# **Examples**

```
rmvn <- function(n, mu = 0, V = matrix(1)){
  p <- length(mu)</pre>
  if(any(is.na(match(dim(V),p))))
    stop("Dimension not right!")
  D <- chol(V)
  t(matrix(rnorm(n*p), ncol=p)%*%D + rep(mu,rep(n,p)))
}
set.seed(1)
n <- 300
coords <- cbind(runif(n,0,1), runif(n,0,1))</pre>
beta <- c(1,5)
x <- cbind(rnorm(n), rnorm(n))</pre>
sigma.sq = 1
phi = 5
tau.sq = 0.1
B <- as.matrix(beta)</pre>
D <- as.matrix(dist(coords))</pre>
R <- exp(-phi*D)</pre>
w \leftarrow rmvn(1, rep(0,n), sigma.sq*R)
y \leftarrow rnorm(n, x%*%B + w, sqrt(tau.sq))
estimation_result <- BRISC_estimation(coords, y, x)</pre>
bootstrap_result <- BRISC_bootstrap(estimation_result, n_boot = 10)</pre>
varg <- BRISC_variogram.ci(estimation_result,</pre>
                              bootstrap\_result\$boot.Theta,
                              plot.variogram = TRUE)
```

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```