# Package 'CNLTtsa' 

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Title Complex-Valued Wavelet Lifting for Univariate and Bivariate TimeSeries Analysis
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CNLTtsa-package Complex-Valued Wavelet Lifting for Univariate and Bivariate Time Series Analysis

## Description

Implementations of recent complex-valued wavelet spectral procedures for analysis of irregularly sampled signals, see Hamilton et al (2018) [doi:10.1080/00401706.2017.1281846](doi:10.1080/00401706.2017.1281846).

## Details

The DESCRIPTION file:

| Package: | CNLTtsa |
| :--- | :--- |
| Type: | Package |
| Title: | Complex-Valued Wavelet Lifting for Univariate and Bivariate Time Series Analysis |
| Version: | $0.1-2$ |
| Date: | $2018-07-18$ |
| Author: | Jean Hamilton [aut], Matt Nunes [aut, cre], Marina Knight [ctb], Piotr Fryzlewicz [ctb] |
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| Maintainer: | Matt Nunes <nunesrpackages@ gmail.com> |
| Description: | Implementations of recent complex-valued wavelet spectral procedures for analysis of irregularly sampled sign |
| License: | GPL-2 |
| Depends: | R(>=3.1), adlift, nlt, CNLTreg, fields |

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| :---: | :---: |
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| Google | End of second returns for Google from 1st March 2011 |
| cnlt.biv | Performs 'nondecimated' complex-valued wavelet lifting for bivariate time series analysis |
| cnlt.spec | A function to compute CNLT spectral quantities for univariate and bivariate series |
| cnlt.univ | Performs 'nondecimated' complex-valued wavelet lifting for univariate time series analysis |
| cnltspec.plot | A function to plot CNLT spectral quantities of interest |
| pre.per | Functions to form periodogram objects with a common time and scale bins for bivariate series with different sampling grids for each component |
| smooth.over.scale | Function to perform smoothing over scale of |

```
spectral quantities
smooth.over.time Function to perform smoothing over time of
spectral quantities
```

The main routines of the package are cnlt. univ and cnlt.biv which perform the nondecimated complex-valued lifting transform for univariate and bivariate time series, respectively; cnlt.spec computes spectral quantities of interest, for example the coherence and phase between two components of a bivariate series.

## Author(s)

Jean Hamilton [aut], Matt Nunes [aut, cre], Marina Knight [ctb], Piotr Fryzlewicz [ctb]
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## References

Hamilton, J., Nunes, M. A, Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

For related literature on the lifting methodology adopted in the technique, see
Knight, M. I. and Nason, G. P. (2009) A 'nondecimated' wavelet transform. Stat. Comput. 19 (1), 1-16.

Knight, M. I., Nunes, M. A. and Nason, G. P. (2012) Spectral Estimation for Locally Stationary Time Series with Missing Observations. Stat. Comput. 22 (4), 877-895.

```
See Also
```

```
cnlt.reg
```

```
cnlt.reg
```

Baidu End of second returns for Google from 1st March 2011

## Description

Often several trades per second of a stock occur; this dataset consists of the last quoted value for each second for 1st March 2011. Thus the finest sampling interval is one second, but as there are seconds with no trades, the data have an unequally spaced sampling regime.

## Usage

data("Baidu")

## Format

A data frame with 7984 observations on the following 3 variables.
Time A variable with the time of the trade.
Seconds.index An index representing the time (in seconds) from the start of the data, representing the sampling regime of the series.
Return The return price of the stock.

## References

Hamilton, J., Nunes, M. A, Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

## Examples

```
data(Baidu)
```

plot(Baidu\$Seconds.index, Baidu\$Return, type="l")
cnlt.biv Performs 'nondecimated' complex-valued wavelet lifting for bivariate time series analysis

## Description

The forward complex-valued lifting transform for decomposing a signal of interest is dependent on the trajectory (lifting order) used in the forward lifting transform. This procedure uses trajectory bootstrapping to provide (complex-valued) time-scale information at all times and scales for bivariate series

## Usage

cnlt.biv(x1, x2 = NULL, f1, f2, P = 100, nkeep = 2, use. same.trajectories = FALSE, verbose = TRUE, ...)

## Arguments

$x 1 \quad$ A vector of grid values. Can be of any length, not necessarily equally spaced.
x2 An optional vector of grid values corresponding to f2. Can be of any length, not necessarily equally spaced. If not specified (NULL), then the same grid is used for $f 2$ as $f 1$, i.e. $x 1$.
f1 A vector of function values of the first component of a bivariate series, corresponding to $x$. Must be of the same length as $x$.
f2 A vector of function values of the second component of a bivariate series, corresponding to $x$. Must be of the same length as $x$.
P
Number of trajectories to be used in the nondecimated lifting transform.

```
nkeep Number of scaling points we want at the end of the transform. The usual choice
    is nkeep=2.
use.same.trajectories
A boolean variable indicating whether the same set of trajectories should be used for both components of the bivariate signal.
verbose Indicates whether useful messages should be printed to the console during the procedure.
... Any other arguments to be passed to fwtnppermC, see the function documentation for more details.
```


## Details

Essentially, this function applies the forward complex wavelet lifting transform fwtnppermC $P$ times on both $(x, f 1)$ and $(x, f 2)$, each with a different random lifting trajectory. This results in $P$ sets of complex-valued detail coefficients, along with their associated scales. This information is stored in order to compute the cross-periodograms for the bivariate series ( $x, f 1, f 2$ ). The "degree of asymmetry" in the prediction is also recorded. This is the ratio between the maximum distance to the removed point to one of its neighbours and the minimum distance from the removed point to one of its neighbours, see Chapter 5.3 in Sanderson (2010) for more details.

## Value

An object of class cnlt (subclasses biv and either SG or DG).
If both components have the same grid (subclass SG), a list with components:

| x1 | The sampling grid corresponding to f1 used for the decomposition. |
| :---: | :---: |
| x2 | The sampling grid corresponding to f 2 used for the decomposition. If the object is of subclass SG, $x 1$ is the same as $\times 2$. |
| det1 | A list, entry i corresponding to detail coefficients associated to point x_i and f1. |
| det2 | A list, entry i corresponding to detail coefficients associated to point $x \_i$ and f2. |
| $1 r e$ | A list, entry i corresponding to the scales (integrals) when lifting point $x \_i$ and $f 1$. |
| lreA | A list, entry i corresponding to the degree of asymmetry of the neighbourhood used in the prediction step of point $x_{-} i$, see description above. |

If both components have different sampling grids, the additional following list components are returned:
lre2 A list, entry i corresponding to the scales (integrals) when lifting point x_i2 and f 2 .
lreA2 A list, entry i corresponding to the degree of asymmetry of the neighbourhood used in the prediction step of point $x_{-} i 2$ with f2, see description above.

## Warning

Using a large number of trajectories for long datasets could take a long time!

## Author(s)

Matt Nunes, Jean Hamilton

## References

Hamilton, J., Nunes, M. A., Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

Sanderson, J. (2010) Wavelet methods for time series with bivariate observations and irregular sampling grids. PhD Thesis, University of Bristol, UK.

For the real-valued equivalent procedure, see also
Knight, M. I., Nunes, M. A. and Nason, G. P. (2012) Spectral Estimation for Locally Stationary Time Series with Missing Observations. Stat. Comput. 22 (4), 877-895.

## See Also

fwtnppermC, link\{cnlt.univ\}

## Examples

```
# a bivariate series example with same grids
# simulate data, e.g. two sinusoids
dat <- seq(from=1, to=3, by=0.1)
x1 <- cumsum(sample(dat, 200, TRUE))
y1<-sin(2*pi*(1/25)*x1) + sin(2*pi*(1/50)*x1)+ 1*sin(2*pi*(1/10)*x1)+ rnorm(length(x1), 0,0.2)
y3 <- c(sin(2*pi*(1/25)*x1[1:100]),sin(2*pi*(1/25)*x1[97:196]))+ rnorm(length(x1), 0,0.1)
## Not run:
y1y3.dec<-cnlt.biv(x1, f1=y1, f2=y3, P = 500)
# the complex detail coefficients corresponding to the first timepoint are:
y1y3.dec$det1[[1]]
## End(Not run)
# a bivariate series example with different grids
# load some data in
data(Baidu)
data(Google)
```

```
## Not run:
BaiGoo<-cnlt.biv(Baidu$Seconds[1:100], Google$Seconds[1:100], Baidu$Return[1:100],
Google$Return[1:100], P = 500)
# now look at some of the coefficients from the decomposition
# (the complex detail coefficients corresponding to the first timepoint:
BaiGoo$det1[[1]]
BaiGoo$det2[[1]]
## End(Not run)
```

| $\mathrm{cnl} . \mathrm{spec}$ | A function to compute CNLT spectral quantities for univariate and <br> bivariate series |
| :--- | :--- |

## Description

The function takes a nondecimated complex lifting decomposition of a univariate or bivariate series, and uses smoothing before computing spectral quantities such as the complex periodograms, coherence and phase

## Usage

cnlt.spec (x, ...)
\#\# S3 method for class 'SG'
cnlt.spec (x, M = 50, fact $=1, \ldots$ )
\#\# S3 method for class 'DG'
cnlt.spec (x, M = 50, fact $=1, \ldots$ )

## Arguments

x

M
fact If length $(M)==1$, a factor indicating how the smoothing parameter (binwidth) in the time-domain kernel smoothing method should increase from one scale to the next, see smooth. over. time.

Any other parameters to be passed to the scale smoothing function, see the documentation for smooth. over. scale for univariate cnlt objects, or pre. per for bivariate cnlt objects.

## Details

For univariate series, the nondecimated complex lifting object can be used to form a spectral object by smoothing the squared details over scale (with smooth.over.scale), and then smoothing over time (using smooth.over.time). Smoothing over scale is done via smooth.spline; smoothing over time is done with a kernel smoother (e.g. a "box" kernel for a moving average). See Hamilton et al. (2018) for more details.

## Value

An object of class cnlt.spec (subclasses: DG, SG, univ, biv).

For subclass univ, a list with components:

S1 A spectral object (matrix) of dimension length(mscale) $x$ length(mtime), corresponding to the spectrum of the univariate series.
mscale A vector of scales corresponding to the rows of the spectrum S1 (after smoothing the periodogram), see smooth. over. scale.
mtime $\quad$ The vector cnltobj $\$ x$, the vector of times corresponding to the columns of the spectrum S1.

For subclass biv, a list with components:
coh A matrix of dimension length(mscale) x length(mtime), corresponding to the coherence between the two components of the bivariate series.
phase A matrix of dimension length(mscale) $x$ length (mtime), corresponding to the phase between the two components of the bivariate series.
C A matrix of dimension length(mscale) $x$ length(mtime), corresponding to the co-periodogram of the bivariate series.

Q A matrix of dimension length(mscale) $x$ length(mtime), corresponding to the quadrature periodogram of the bivariate series.
S1 A matrix of dimension length(mscale) $x$ length (mtime), corresponding to the spectrum of the first component of the bivariate series.

S2 A matrix of dimension length(mscale) x length (mtime), corresponding to the spectrum of the second component of the bivariate series.
mscale A vector of scales corresponding to the rows of the spectrum S1 (after smoothing the periodogram), see smooth.over.scale.
mtime A vector of times corresponding to the columns of the spectrum S1. If the class of cnlt.obj is SG, this is cnlt.obj\$x1, else this is a vector formed by binning detail coefficients within equal intervals of time, see pre. per for more details.

## Author(s)

Matt Nunes, Jean Hamilton

## References

Hamilton, J., Nunes, M. A., Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

## See Also

cnlt.biv, cnlt.univ, cnltspec.plot

## Examples

```
# read some data in (a bivariate series)
## Not run:
data(Baidu)
data(Google)
BaiGoo<-cnlt.biv(Baidu$Seconds[1:100], Google$Seconds[1:100], Baidu$Return[1:100],
Google$Return[1:100], P = 500)
specobj<-cnlt.spec(BaiGoo,M=10,fact=1.05, Tstar=20)
## End(Not run)
```

cnlt.univ | Performs 'nondecimated' complex-valued wavelet lifting for univari- |
| :--- |
| ate time series analysis |

## Description

The forward complex-valued lifting transform for decomposing a signal of interest is dependent on the trajectory (lifting order) used in the forward lifting transform. This procedure uses trajectory bootstrapping to provide (complex-valued) time-scale information at all times and scales

## Usage

cnlt.univ(x, f, $P=100$, nkeep $=2$, verbose $=$ TRUE, ...)

## Arguments

$x \quad$ A vector of grid values. Can be of any length, not necessarily equally spaced.
$f \quad$ A vector of function values corresponding to $x$. Must be of the same length as x .
P Number of trajectories to be used in the nondecimated lifting transform.
nkeep Number of scaling points we want at the end of the transform. The usual choice is nkeep $=2$.
verbose Indicates whether useful messages should be printed to the console during the procedure.
Any other arguments to be passed to fwtnppermC, see the function documentation for more details.

## Details

Essentially, this function applies the forward complex wavelet lifting transform fwtnppermC $P$ times, each with a different random lifting trajectory. This results in $P$ sets of complex-valued detail coefficients, along with their associated scales. This information is stored in order to compute the periodogram for $(x, f)$. The "degree of asymmetry" in the prediction is also recorded. This is the ratio between the maximum distance to the removed point to one of its neighbours and the minimum distance from the removed point to one of its neighbours, see Chapter 5.3 in Sanderson (2010) for more details.

## Value

An object of class cnlt (subclasses: univ, SG). A list with components:
$x \quad$ The sampling grid corresponding to $f$ used for the decomposition.
det1 A list, entry i corresponding to detail coefficients associated to point x_i.
lre A list, entry i corresponding to the scales (integrals) when lifting point x_i.
lreA A list, entry i corresponding to the degree of asymmetry of the neighbourhood used in the prediction step of point $x_{-}$i, see description above.

## Warning

Using a large number of trajectories for long datasets could take a long time!

## Author(s)

Jean Hamilton, Matt Nunes

## References

Hamilton, J., Nunes, M. A., Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

Sanderson, J. (2010) Wavelet methods for time series with bivariate observations and irregular sampling grids. PhD Thesis, University of Bristol, UK.

For the real-valued equivalent procedure, see also
Knight, M. I., Nunes, M. A. and Nason, G. P. (2012) Spectral Estimation for Locally Stationary Time Series with Missing Observations. Stat. Comput. 22 (4), 877-895.

## See Also

fwtnppermC, link\{get. percoeffsC.biv\}

## Examples

```
x<-sort(runif(100))
y<-sin(2*pi*(1/25)*x) + sin(2*pi*(1/50)*x)
## Not run:
xy.dec<-cnlt.univ(x,y,P=300)
xy.dec$det[[1]][1:10]
## End(Not run)
```

cnltspec.plot A function to plot CNLT spectral quantities of interest

## Description

The function takes a spectral quantity and plots it according to user inputted graphical options

## Usage

cnltspec.plot(spec, timevec, scalevec, zrange = NULL, xtitle = "Time", ytitle = "Scale", col.scale $=$ tim.colors(64)[1:45], SFratio $=2$, $d t=1$, parsw $=3$, axis $4=$ FALSE, frequencies = NULL)

## Arguments

spec
timevec A vector corresponding to the $x$-axis of the spectral object, often time or a sampling grid.
scalevec A vector of scales corresponding to the $y$-axis of the spectral object.
zrange An optional length two vector specifying the range of the z -axis of the plot.
xtitle A label for the x -axis of the plot.
ytitle A label for the $y$-axis of the plot.
col.scale a color palette to use for the spectral plot.
SFratio A number specifying the relationship between scale and Fourier frequency, see frequencies argument, and Sanderson (2010), chapter 6.2.
$\mathrm{dt} \quad$ A sampling interval, used to compute the relationship between scale and Fourier frequency, see Sanderson (2010), chapter 6.2.
parsw A number from 1 to 3, specifying different spacings between the plot and the legend. This is useful if you want to do call cnltspec. plot multiple times for e.g. multi-panel plots.

$$
\begin{array}{ll}
\text { axis4 } & \begin{array}{l}
\text { An optional boolean variable indicating whether a 4th (right) axis should be } \\
\text { added to the plot. }
\end{array} \\
\text { frequencies } & \text { If axis4 = TRUE, an optional vector for the ticks on the 4th axis. If these are } \\
\text { not specified, then a vector of Fourier frequencies are plotted, with the relation- } \\
\text { ship between scale and frequency specified by SFratio, see Sanderson (2010), } \\
\text { chapter 6.2. }
\end{array}
$$

## Value

A spectral quantity is plotted.

## Author(s)

Jean Hamilton, Matt Nunes

## References

Hamilton, J., Nunes, M. A., Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

Sanderson, J. (2010) Wavelet methods for time series with bivariate observations and irregular sampling grids. PhD Thesis, University of Bristol, UK.

## See Also

cnlt.spec

## Examples

```
# simulate data, e.g. two sinusoids
dat <- seq(from=1, to=3, by=0.1)
x1 <- cumsum(sample(dat, 200, TRUE))
y1 <-sin(2*pi*(1/25)*x1) + sin(2*pi*(1/50)*x1)+ 1*sin(2*pi*(1/10)*x1)+ rnorm(length(x1), 0,0.2)
y3 <- c(sin(2*pi*(1/25)*x1[1:100]),sin(2*pi*(1/25)*x1[97:196]))+ rnorm(length(x1), 0,0.1)
## Not run:
y1y3.dec<-cnlt.biv(x1, f1=y1, f2=y3, P = 500)
y1y3spec<-cnlt.spec(y1y3.dec)
cnltspec.plot(y1y3spec$coh,y1y3spec$mtime,y1y3spec$mscale)
## End(Not run)
```


## Description

Often several trades per second of a stock occur; this dataset consists of the last quoted value for each second for 1st March 2011. Thus the finest sampling interval is one second, but as there are seconds with no trades, the data have an unequally spaced sampling regime.

## Usage

data("Google")

## Format

A data frame with 6526 observations on the following 3 variables.
Time A variable with the time of the trade.
Seconds.index An index representing the time (in seconds) from the start of the data, representing the sampling regime of the series.

Return The return price of the stock.

## References

Hamilton, J., Nunes, M. A, Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

## Examples

```
data(Google)
plot(Google$Seconds.index,Google$Return,type="l")
```

```
pre.per
```

Functions to form periodogram objects with a common time and scale bins for bivariate series with different sampling grids for each component

## Description

The CNLT forms detail coefficients for each component of a bivariate series. Due to the two components having different sampling grids, the details (and associated scales) won't have a common association for both series. Hence the details are sampled and mapped to a common timescale and a common set of scales via binning and averaging. These functions compute spectral objects on these common times / scales

## Usage

pre.per(x, det, lre, lreA, scale.range $=$ NULL, time. range $=$ NULL, Arange $=$ NULL,
Jstar $=20$, Tstar = 50)
pre.per.comb(spec1, spec2)
pre.per.sample(spec1, spec2)

## Arguments

$x \quad$ A vector corrsponding to the sampling grid of a component of a series.
det A list of (real or imaginary parts of) the detail coefficients from a CNLT decomposition, such as from the output of cnlt . biv.
lre A list of scales (removed integral lengths) corresponding to det from a CNLT decomposition, such as from the output of cnlt.biv.

IreA A list of asymmetry values from a CNLT decomposition, such as from the output of cnlt.biv.
scale.range An optional two-vector specifying the range of scales to be considered in the resulting output spectrum.
time.range An optional two-vector specifying the range of times to be considered in the resulting output spectrum.
Arange An optional two-vector specifying whether the values used in forming the output spectrum should be limited to those from a specific range of asymmetry values, see Sanderson (2010), chapter 6.2.
Jstar The number of artificial scales in the output spectrum.
Tstar The number of artificial times in the output spectrum.
spec1 A periodogram corresponding to the first component of a bivariate series.
spec2 A periodogram corresponding to the second component of a bivariate series.

## Details

For a bivariate series where the two components have different sampling grids, the co- /quadrature periodogram values are first formed using pre.per, using a vector of Tstar equal time intervals, specified by setting Tstar and optionally time. range; they are also binned into Jstar equal artificial levels by setting Jstar and optionally scale. range. The details are sampled using a common sampling vector with pre. per . sample, and then combined using pre. per. comb. The periodogram is then smoothed over time. See Hamilton et al (2018), section 2.3 for more details.

## Value

For pre. per, a list with components:
spec A matrix of dimension Jstar $x$ Tstar corrsponding to a quadrature periodogram / co-periodogram of a bivariate series.
mscale A vector of scales (of length Jstar) corresponding to the rows of the spectrum spec.
mtime A vector of times (of length Tstar) corresponding to the columns of the spectrum spec.

For pre.per.sample, a list with components:
spec1 A matrix of dimension Jstar $x$ Tstar corrsponding to a periodogram of the first component of a bivariate series.
spec2 A matrix of dimension Jstar $x$ Tstar corrsponding to a periodogram of the second component of a bivariate series.

For pre.per.comb, a list with components:
spec A matrix of dimension Jstar $x$ Tstar corrsponding to a periodogram / quadrature periodogram / co-periodogram of a bivariate series.

## Note

Note that these functions aren't intended to be used directly, but are called internally from the function cnlt.spec.DG.
Note also that the argument Tstar should be chosen small enough so that the range of the sampling grid $x$ can be divided into equally spaced intervals, with *at least one* gridpoint in an interval.

## Author(s)

Jean Hamilton, Matt Nunes

## References

Hamilton, J., Nunes, M. A., Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

## See Also

cnlt.spec.DG, smooth.over.time

## Examples

```
# simulate data, e.g. two sinusoids
dat <- seq(from=1, to=3, by=0.1)
x1 <- cumsum(sample(dat, 200, TRUE))
x2 <- cumsum(sample(dat, 200, TRUE))
y1<-sin(2*pi*(1/25)*x1) + sin(2*pi*(1/50)*x1)+ 1*sin(2*pi*(1/10)*x1)+ rnorm(length(x1), 0,0.2)
y3 <- sin(2*pi*(1/25)*x2[97:196]) + rnorm(length(x2), 0,0.1)
## Not run:
y1y3.dec<-cnlt.biv(x1, f1=y1, f2=y3, P = 500)
# compute the co-periodogram for the first component...
```

```
C1 <- pre.per(x1, sapply(y1y3.dec$det1,Re), y1y3.dec$lre1, y1y3.dec$lreA1, Jstar = 10)
# . . and for the second component
C2 <- pre.per(x1, sapply(y1y3.dec$det2,Re), y1y3.dec$1re2, y1y3.dec$1reA2, Jstar = 10)
## End(Not run)
```

smooth.over.scale Function to perform smoothing over scale of spectral quantities

## Description

This function uses simple averaging or smoothing splines to smooth spectra over scale

## Usage

smooth.over.scale(x, det1, det2, lre, lreA, scale. range $=$ NULL, Arange $=$ NULL,
Jstar = 20, splines = FALSE, positive = FALSE, dfS = 10, interpolate = FALSE)

## Arguments

x
det1 A list of (real or imaginary parts of) the component 1 detail coefficients from a CNLT decomposition, such as from the output of cnlt.biv.
det2 A list of (real or imaginary parts of) the component 2 detail coefficients from a CNLT decomposition, such as from the output of cnlt.biv.
lre A list of scales (removed integral lengths) corresponding to det from a CNLT decomposition, such as from the output of cnlt.biv.
IreA A list of asymmetry values from a CNLT decomposition, such as from the output of cnlt.biv.
scale.range An optional two-vector specifying the range of scales to be considered in the resulting output spectrum.
Arange An optional two-vector specifying whether the values used in forming the output spectrum should be limited to those from a specific range of asymmetry values, see Sanderson (2010), chapter 6.2.
Jstar The number of artificial scales in the output spectrum.
splines An indicator variable whether smoothing splines should be used for the scalebased smoothing, or simple averaging (splines = FALSE).
positive An indicator variable whether the smoothing should ensure that the resulting output is positive or not (e.g. for spectra).
$\mathrm{dfS} \quad$ An argument, if splines = TRUE, specifying the number of degrees of freedom for the smoothing spline.
interpolate An indicator variable for whether interpolation should be used in the smoothing spline method for predicting values outside the range of the data.

## Details

For a univariate series or a bivariate series where the two components have the same sampling grids, the co- /quadrature periodogram values are first formed. They are then smoothed over scale (per timepoint), to give spectral values corresponding to equal artificial levels by setting Jstar and optionally scale.range.

## Value

A list with the following components:
spec A matrix of dimension Jstar $x$ length $(x)$ corrsponding to a periodogram / co-periodogram / quadrature periodogram.
mscale A vector of scales (of length Jstar) corresponding to the rows of the spectrum spec.

## Author(s)

Jean Hamilton, Matt Nunes

## References

Hamilton, J., Nunes, M. A., Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

## See Also

cnlt.spec.SG

## Examples

```
x<-sort(runif(100))
y <-sin(2*pi*(1/25)*x) + sin(2*pi*(1/50)*x)
## Not run:
xy.dec<-cnlt.univ(x,y,P=300)
# compute the real part of the spectrum (real details^2) and smooth over scale
ReS <- smooth.over.scale(x, sapply(xy.dec$det1,Re), sapply(xy.dec$det1,Re), xy.dec$lre,
xy.dec$lreA, positive = TRUE)
## End(Not run)
```


## Description

This function uses a running mean (box kernel) smoother to smooth spectra over time, with potentially different smoothing parameters used for each scale of the spectra

## Usage

smooth.over.time(x, spec, M, fact = 1)

## Arguments

x
spec
M The smoothing parameter (binwidth) or vector of smoothing parameters (one for each scale) for the smoothing method.
fact If length $(M)==1$, a factor indicating how the smoothing parameter (binwidth) in the time-domain kernel smoothing method should increase from one scale to the next.

## Details

The function takes in a matrix and performs a kernel smoother on row $i$ of the matrix, using a bandwidth of $M[i]$ if length $(M)==n r o w(s p e c)$, and $M *$ fact^\{i-1\} if length $(M)==1$. Thus if the scaling factor, fact, is chosen to be greater than one, a wider kernel is used for the smoothing for later scales.

## Value

smooth.spec A matrix of same dimension as spec, containing smoothed spectral values.

## Author(s)

Jean Hamilton

## References

Hamilton, J., Nunes, M. A., Knight, M. I. and Fryzlewicz, P. (2018) Complex-valued wavelet lifting and applications. Technometrics, 60 (1), 48-60, DOI 10.1080/00401706.2017.1281846.

## See Also

cnlt.spec, pre.per

## Examples

```
x<-sort(runif(100))
y<-sin(2*pi*(1/25)*x) + sin(2*pi*(1/50)*x)
## Not run:
xy.dec<-cnlt.univ(x,y,P=300)
# compute the real part of the spectrum (real details^2) and smooth over scale, then over time
ReS <- smooth.over.scale(x, sapply(xy.dec$det1,Re), sapply(xy.dec$det1,Re), xy.dec$lre,
xy.dec$lreA, positive = TRUE)
ReS.smooth <- smooth.over.time(x,ReS$spec,5,1.05)
## End(Not run)
```


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