Package 'EMSNM'

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Type Package

Title EM Algorithm for Sigmoid Normal Model

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Description It provides a method based on EM algorithm to estimate the parameter of a mixture model, Sigmoid-Normal Model, where the samples come from several normal distributions (also call them subgroups) whose mean is determined by co-variable Z and coefficient alpha while the variance are homogeneous. Meanwhile, the subgroup each item belongs to is determined by co-variables X and coefficient eta through Sigmoid link function which is the extension of Logistic Link function. It uses bootstrap to estimate the standard error of parameters. When sample is indeed separable, removing estimation with abnormal sigma, the estimation of alpha is quite well. I used this method to explore the subgroup structure of HIV patients and it can be used in other domains where exists subgroup structure.

License GPL (>= 2)

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R topics documented:

`EMSNM-package` *EM Algorithm for Sigmoid Normal Model*

Description

It provides a method based on EM algorithm to estimate the parameter of a mixture model, Sigmoid-Normal Model, where the samples come from several normal distributions (also call them subgroups) whose mean is determined by co-variable Z and coefficient alpha while the variance are homogeneous. Meanwhile, the subgroup each item belongs to is determined by co-variables X and coefficient eta through Sigmoid link function which is the extension of Logistic Link function. It uses bootstrap to estimate the standard error of parameters. When sample is indeed separable, removing estimation with abnormal sigma, the estimation of alpha is quite well. I used this method to explore the subgroup structure of HIV patients and it can be used in other domains where exists subgroup structure.

Details

The DESCRIPTION file:

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License:	GPL(>=2)

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`EMalgorithm`	`Parameter Estimation`
`EMbootstrap`	`Bootstrap Method`
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`Ggenerate`	`Subgroup Determination`

Wgenerate	Sigmoid Logistic Data Generation
fnorm	Density Value
softmax	Softmax Value
standard	Data Standardlization
update_eta	Updata Eta
update_gamma	Updata Alpha and Sigma
weight_matrix	Weighted Inner Product

The EMalgorithm is used to estimate the parameters, EMbootstrap is used to estimate the parameters with bootstrap method. In EMsimulation we can simulate the situation with given parameters, so parameter estimation can be verified.

Author(s)

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Examples

```
#parameter initialization
etasize <- 2
classsize <- 2
alphasize <- 3
samplesize <- 100
expriments <- 30

etatest <- matrix(c(1,1,
                    0,0),etasize,classsize)

alphatest <- matrix(c(1,0,2,
                      4,3,5),alphasize,classsize)
sigmatest <- 0.5

#test of EMsimulation
EMsimulation_result <- EMsimulation(eta=etatest,alpha=alphatest,sigma=sigmatest,
                            samplesize=samplesize,expriments=expriments,
                            compact_flag=TRUE,C0=5,C1=0.5,C2=5)
index <- which(EMsimulation_result$sigma<0.8)
EMsimulation_result_sort <- EM_result_sort(EMsimulation_result$alpha[index,,],
                                           EMsimulation_result$eta[index,,])
EM_parameter <- EM_parameter_sd(EMsimulation_result_sort$alpha,
                                EMsimulation_result_sort$eta,
                                EMsimulation_result$sigma[index])

#test of EMbootstrap
samplesize <- 1000
X <- matrix(c(matrix(1,samplesize),
                  rnorm(samplesize*(etasize-1))+1),samplesize,etasize)
Z <- matrix(c(matrix(1,samplesize),rbinom(prob=1/2,size=1,n=samplesize),
                 rnorm(samplesize*(alphasize-2))+1),samplesize,alphasize)
```

```
Wtest <- Wgenerate(alpha=alphatest,eta=etatest,sigma=sigmatest,X=X,Z=Z)

boots_samplesize <- 100
boots_expriments <- 30
samplesize <- dim(Wtest$X)[1]
EMbootstrap_theta <- EMbootstrap(Wtest$X,Wtest$Y,Wtest$Z,samplesize,
                                 boots_samplesize,boots_expriments,
                                 classsize=2,compact_flag=TRUE,C0=5,C1=0.2,C2=5)
index <- which(EMbootstrap_theta$sigma<0.8)
EMsimulation_result_sort <- EM_result_sort(EMbootstrap_theta$alpha[index,,],
                                           EMbootstrap_theta$eta[index,,])
EM_parameter <- EM_parameter_sd(EMsimulation_result_sort$alpha,
                                EMsimulation_result_sort$eta,
                                 EMbootstrap_theta$sigma[index])
```

`Ccompute` *Ccompute*

Description

Compute the probability of Y in given parameters alphat, sigmat, etat and variables X, Z by the Bayesian Formula under the assumption of Sigmoid-Normal Model.

Usage

```
Ccompute(alphat, sigmat, etat, X, Z, Y)
```

Arguments

`alphat`	the coeffients of the mean of each subgroup
`sigmat`	the variance of Y
`etat`	the coeffients determining subgroup
`X`	the covariables of the mean of each subgroup
`Z`	the covaraibles determining subgroup
`Y`	the respond variable

Value

the probability of Y in given parameters alphat, sigmat, etat and variables X, Z under the assumption of Sigmoid-Normal Model.

Author(s)

Linsui Deng

Examples

```
#some variables
samplesize <- 1000
classsize <- 6
etasize <- 3
alphasize <- 2

Xtest <- data.frame(matrix(rnorm(samplesize*etasize),samplesize,etasize))
Ztest <- matrix(rnorm(samplesize*alphasize),samplesize,alphasize)

etatest <- matrix(seq(1.15,1,length=etasize*classsize),etasize,classsize)
alphatest <- matrix(seq(1.15,1,length=alphasize*classsize),alphasize,classsize)

Wtest <- Wgenerate(alpha=alphatest,eta=etatest,X=Xtest,Z=Ztest)

#test of Ccompute
sigmatest <- 1
Ctest <-
  Ccompute(alphat=alphatest,sigmat=sigmatest,
           etat=etatest,X=Wtest$X,Z=Wtest$Z,Y=Wtest$Y)
```

`EMalgorithm` *Parameter Estimation*

Description

Estimate paramters value by EM algorithm under the assumption of Sigmoid-Normal Model.

Usage

```
EMalgorithm(X, Y, Z, etat, alphat, sigmat, classsize = 2, learning_rate = 0.1,
            regular_parameter_eta = 0.001, max_iteration = 10000,
          max_iteration_eta = 10000, compact_flag = FALSE, C0 = 5, C1 = 2, C2 = 9)
```

Arguments

`X`	the covariables of the mean of each subgroup
`Y`	the respond variable
`Z`	the covaraibles determining subgroup
`etat`	the coeffients determining subgroup
`alphat`	the coeffients of the mean of each subgroup
`sigmat`	the variance of Y
`classsize`	the number of subgroup types in your model assumption
`learning_rate`	learning rate of updating eta
`regular_parameter_eta`	regular value of updating eta by gradiant descending methond.

max_iteration	maximum steps of interation to avoid unlimited looping.
max_iteration_eta	maximal steps of eta interation to avoid unlimited looping.
compact_flag	if the value of eta is limited in a compact set, set it TRUE
C0	the maximum of intercept of eta.
C1	the minimum of the norm of slope of eta
C2	the maximum of the norm of slope of eta

Value

alpha	alpha estimation
eta	eta estimation
sigma	sigma estimation

Author(s)

Linsui Deng

Examples

```
#data generation
samplesize <- 1000
classsize <- 2
etasize <- 3
alphasize <- 3

set.seed(1)
Xtest <- data.frame(matrix(rnorm(samplesize*etasize),samplesize,etasize))
etatest <- matrix(c(1,2,-1,
                    0,0,0),etasize,classsize)

Ztest <- matrix(rnorm(samplesize*alphasize),samplesize,alphasize)
alphatest <- matrix(c(1,0,2,
                      5,0,7),alphasize,classsize)
sigmatest <- 5

Wtest <- Wgenerate(alpha=alphatest,eta=etatest,X=Xtest,Z=Ztest,sigma=sigmatest)

eta_initial <- matrix(c(rnorm(3),0,0,0),etasize,classsize)
alpha_initial<- matrix(rnorm(alphasize*classsize)*3,alphasize,classsize)
sigma_initial <- 1

EMtheta <- EMalgorithm(X=Wtest$X,Z=Wtest$Z,Y=Wtest$Y,classsize=2,
                       etat=eta_initial,alphat=alpha_initial,sigmat=sigma_initial,
                       learning_rate=0.01,regular_parameter_eta=0.001,
                       max_iteration=1000,max_iteration_eta=10000,
                       compact_flag = TRUE, C0 = 5, C1 = 2, C2 = 9)
```

EMbootstrap *Bootstrap Method*

Description

Estimate the value of parameters several times by bootstrap method where the parameters are estimated by EM algorithm. In this way, we can observe the distribution of parameter.

Usage

```
EMbootstrap(X, Y, Z, samplesize, boots_samplesize, boots_expriments, classsize = 2,
          learning_rate = 0.1, regular_parameter_eta = 0.001, max_iteration = 10000,
          max_iteration_eta = 10000, compact_flag = FALSE, C0 = 5, C1 = 2, C2 = 9)
```

Arguments

X	the co-variables of the mean of each subgroup
Y	the respond variable
Z	the co-variables determining subgroup
samplesize	the size of this sample.
boots_samplesize	the size of chosen sample in one step of bootstrap
boots_expriments	the steps of bootstrap
classsize	the number of subgroup types in your model assumption
learning_rate	learning rate of updating eta
regular_parameter_eta	regular value of updating eta by gradient descending method.
max_iteration	maximum steps of iteration to avoid unlimited looping.
max_iteration_eta	maximal steps of eta iteration to avoid unlimited looping.
compact_flag	if the value of eta is limited in a compact set, set it TRUE
C0	the maximum of intercept of eta.
C1	the minimum of the norm of slope of eta
C2	the maximum of the norm of slope of eta

Details

Actually, the method can be extended to other parameter estimation where the standard error of parameter can't be calculated in a simple way.

Value

alpha	alpha estimated by bootstrap method.
eta	eta estimated by bootstrap method.
sigma	sigma estimated by bootstrap method.

Author(s)

Linsui Deng

Examples

```
#parameter initialization
etasize <- 2
classsize <- 2
alphasize <- 3
samplesize <- 1000

etatest <- matrix(c(1,1,
                    0,0),etasize,classsize)

alphatest <- matrix(c(1,0,2,
                      4,3,5),alphasize,classsize)
sigmatest <- 0.5

#test of EMbootstrap
X <- matrix(c(matrix(1,samplesize),
                  rnorm(samplesize*(etasize-1))+1),samplesize,etasize)
Z <- matrix(c(matrix(1,samplesize),rbinom(prob=1/2,size=1,n=samplesize),
                 rnorm(samplesize*(alphasize-2))+1),samplesize,alphasize)

Wtest <- Wgenerate(alpha=alphatest,eta=etatest,sigma=sigmatest,X=X,Z=Z)

boots_samplesize <- 100
boots_expriments <- 30
samplesize <- dim(Wtest$X)[1]
EMbootstrap_theta <- EMbootstrap(Wtest$X,Wtest$Y,Wtest$Z,samplesize,
                                 boots_samplesize,boots_expriments,
                                 classsize=2,compact_flag=TRUE,C0=5,C1=0.2,C2=5)
index <- which(EMbootstrap_theta$sigma<0.8)
EMsimulation_result_sort <- EM_result_sort(EMbootstrap_theta$alpha[index,,],
                                           EMbootstrap_theta$eta[index,,])
EM_parameter <- EM_parameter_sd(EMsimulation_result_sort$alpha,
                                EMsimulation_result_sort$eta,
                                 EMbootstrap_theta$sigma[index])
```

EMsimulation *Simulation For Estimation*

Description

It simulates the experiments with given alpha, eta and sigma to verify the EMalgorithm

Usage

```
EMsimulation(eta, alpha, sigma, samplesize, expriments,
             compact_flag = FALSE, C0 = 5, C1 = 2, C2 = 9)
```

Arguments

eta	the true value of eta
alpha	the true value of alpha
sigma	the true value of sigma
samplesize	the size of sample
expriments	the times of experiments
compact_flag	if the value of eta is limited in a compact set, set it TRUE
C0	the maximum of intercept of eta.
C1	the minimum of the norm of slope of eta
C2	the maximum of the norm of slope of eta

Value

alpha	alpha estimated in simulation.
eta	eta estimated in simulation.
sigma	sigma estimated in simulation.

Author(s)

Linsui Deng

Examples

```
#parameter initialization
etasize <- 2
classsize <- 2
alphasize <- 3
samplesize <- 100
expriments <- 30

etatest <- matrix(c(1,1,
                    0,0),etasize,classsize)

alphatest <- matrix(c(1,0,2,
                      4,3,5),alphasize,classsize)
sigmatest <- 0.5

#test of EMsimulation
```

```
EMsimulation_result <- EMsimulation(eta=etatest,alpha=alphatest,sigma=sigmatest,
                            samplesize=samplesize,expriments=expriments,
                            compact_flag=TRUE,C0=5,C1=0.5,C2=5)
```

`EM_parameter_sd` *Bootstrap Parameter Inference*

Description

Estimating the parameters and their stand error through the sorted parameters estimated by bootstrap method.

Usage

```
EM_parameter_sd(EMsimulation_sort_alpha, EMsimulation_sort_eta, EMsimulation_sort_sigma)
```

Arguments

`EMsimulation_sort_alpha`
: sorted alpha estimated by bootstrap method.

`EMsimulation_sort_eta`
: sorted eta estimated by bootstrap method.

`EMsimulation_sort_sigma`
: sorted sigma estimated by bootstrap method.

Value

`sigma`	the estimation of sigma
`sigma_sd`	the estimation of standard error of sigma
`alpha`	the estimation of alpha
`alpha_sd`	the estimation of standard error of alpha
`eta`	the estimation of eta
`eta_sd`	the estimation of standard error of eta

Author(s)

Linsui Deng

Examples

```
#parameter initialization
etasize <- 2
classsize <- 2
alphasize <- 3
samplesize <- 100
expriments <- 30
```

```
etatest <- matrix(c(1,1,
                    0,0),etasize,classsize)

alphatest <- matrix(c(1,0,2,
                      4,3,5),alphasize,classsize)
sigmatest <- 0.5

EMsimulation_result <- EMsimulation(eta=etatest,alpha=alphatest,sigma=sigmatest,
                           samplesize=samplesize,expriments=expriments,
                           compact_flag=TRUE,C0=5,C1=0.5,C2=5)
index <- which(EMsimulation_result$sigma<0.8)
EMsimulation_result_sort <- EM_result_sort(EMsimulation_result$alpha[index,,],
                                           EMsimulation_result$eta[index,,])
#test of EM_parameter_sd
EM_parameter <- EM_parameter_sd(EMsimulation_result_sort$alpha,
                                EMsimulation_result_sort$eta,
                                EMsimulation_result$sigma[index])
```

`EM_result_sort` *Sort Parameter*

Description

Since the number of subgroup types is always beyond 1, the order of subgroup may be different, we can sort them in this function.

Usage

```
EM_result_sort(EMsimulation_result_alpha, EMsimulation_result_eta)
```

Arguments

`EMsimulation_result_alpha`	alpha estimated by bootstrap method.
`EMsimulation_result_eta`	eta estimated by bootstrap method.

Value

`alpha`	sorted alpha estimated by bootstrap method.
`eta`	sorted eta estimated by bootstrap method.

Author(s)

Linsui Deng

Examples

```
#parameter initialization
etasize <- 2
classsize <- 2
alphasize <- 3
samplesize <- 100
expriments <- 30

etatest <- matrix(c(1,1,
                    0,0),etasize,classsize)

alphatest <- matrix(c(1,0,2,
                      4,3,5),alphasize,classsize)
sigmatest <- 0.5

#test of EMsimulation
EMsimulation_result <- EMsimulation(eta=etatest,alpha=alphatest,sigma=sigmatest,
                            samplesize=samplesize,expriments=expriments,
                            compact_flag=TRUE,C0=5,C1=0.5,C2=5)

#test of EM_result_sort
EMsimulation_result_sort <- EM_result_sort(EMsimulation_result$alpha,
                                           EMsimulation_result$eta)
```

`fnorm` *Density Value*

Description

Calculate the density value of respond value Y under each mean and homogeneous variance.

Usage

```
fnorm(Y, mu, sigma)
```

Arguments

`Y`	the respond variable
`mu`	different mean of each subgroup
`sigma`	standard error

Value

the density value of Y under different mu and common sigma.

Author(s)

Linsui Deng

Examples

```
fnormtest <- fnorm(matrix(1:6,3,2),matrix(seq(1,3,length=6),3,2),1)
```

`Ggenerate` *Subgroup Determination*

Description

In data genaration, determining the subgroup of each item belonging to through random number and Sigmoid Link function.

Usage

```
Ggenerate(eta, X, seed = 0)
```

Arguments

`eta`	the coeffients determining subgroup
`X`	the covariables determining subgroup
`seed`	random seed

Value

the classes items belonging to, it's a vector. If X1 belongs to class 3, then the 1st row 3rd colmn is 1 and the rest of 1st row are 0.

Author(s)

Linsui Deng

Examples

```
#some variables
samplesize <- 1000
classsize <- 6
etasize <- 3
alphasize <- 2

#test of Ggenerate
Xtest <- data.frame(matrix(rnorm(samplesize*etasize),samplesize,etasize))
etatest <- matrix(seq(1.15,1,length=etasize*classsize),etasize,classsize)

Gtest1 <- Ggenerate(etatest,Xtest)
Gtest2 <- Ggenerate(etatest,Xtest,1)
```

softmax *Softmax Value*

Description

Calculate the Softmax Value of each subgroup to represent the probability of items belonging to specific class.

Usage

```
softmax(eta, X)
```

Arguments

eta	the coeffients determining subgroup
X	the covariables determining subgroup

Value

Softmax Value of each subgroup

Author(s)

Linsui Deng

Examples

```
#some variables
samplesize <- 1000
classsize <- 6
etasize <- 3
alphasize <- 2

#test of softmax
Xtest <- data.frame(matrix(rnorm(samplesize*etasize),samplesize,etasize))
etatest <- matrix(seq(1.15,1,length=etasize*classsize),etasize,classsize)
softmax_value <- softmax(etatest,Xtest)
```

standard *Data Standardlization*

Description

Standardlize the data to elimilate the effect of scale

Usage

```
standard(X)
```

Arguments

X	the original data

Value

standarlized data

Author(s)

Linsui Deng

Examples

```
#data generata

Y <- rnorm(100)*2+5

Y_sta <- standard(Y)
```

update_eta *Updata Eta*

Description

Updata eta in step t+1 with given data and coeffients estimated in step t.

Usage

```
update_eta(fun, alphat, sigmat, etat, X, Y, Z, learning_rate_eta = 0.001,
           regular_parameter_eta = 0.001, max_iteration_eta = 10000)
```

Arguments

fun	the function updata eta
alphat	the estimated coeffients of the mean of each subgroup in step t
sigmat	the estimated standard error of Y in step t
etat	the estimated coeffients determining subgroup in step t
X	the covariables of the mean of each subgroup
Z	the covaraibles determining subgroup
Y	the respond variable
learning_rate_eta	learning rate of updating eta
regular_parameter_eta	regular value of updating eta by gradiant descending methond.
max_iteration_eta	maximal steps of eta interation to avoid unlimited looping.

Value

alpha	alpha estimated in step t.
eta	eta estimated in step t+1.
sigma	sigma estimated in step t.

Author(s)

Linsui Deng

Examples

```
#some variables
samplesize <- 1000
classsize <- 6
etasize <- 3
alphasize <- 2

Xtest <- data.frame(matrix(rnorm(samplesize*etasize),samplesize,etasize))
Ztest <- matrix(rnorm(samplesize*alphasize),samplesize,alphasize)

etatest <- matrix(seq(1.15,1,length=etasize*classsize),etasize,classsize)
alphatest <- matrix(seq(1.15,1,length=alphasize*classsize),alphasize,classsize)
sigmatest <- 0.1

Wtest <- Wgenerate(alpha=alphatest,eta=etatest,X=Xtest,Z=Ztest)

#test of update_eta
thetaupdate_eta <- update_eta(fun=eta_gradient_fun,alphat=alphatest,sigmat=sigmatest,
           etat=etatest,X=Wtest$X,Z=Wtest$Z,Y=Wtest$Y,
           learning_rate=0.1,regular_parameter=0.001,max_iteration=10000)
```

update_gamma *Updata Alpha and Sigma*

Description

Updata alpha and sigma in t+1 step with given data and coeffients estimated in step t.

Usage

```
update_gamma(alphat, sigmat, etat, X, Z, Y)
```

Arguments

alphat	the estimated coeffients of the mean of each subgroup in step t
sigmat	the estimated standard error of Y in step t
etat	the estimated coeffients determining subgroup in step t
X	the covariables of the mean of each subgroup
Z	the covaraibles determining subgroup
Y	the respond variable

Value

alpha	alpha estimated in step t+1.
eta	eta estimated in step t.
sigma	sigma estimated in step t+1.

Author(s)

Linsui Deng

Examples

```
#some variables
samplesize <- 1000
classsize <- 6
etasize <- 3
alphasize <- 2

Xtest <- data.frame(matrix(rnorm(samplesize*etasize),samplesize,etasize))
Ztest <- matrix(rnorm(samplesize*alphasize),samplesize,alphasize)

etatest <- matrix(seq(1.15,1,length=etasize*classsize),etasize,classsize)
alphatest <- matrix(seq(1.15,1,length=alphasize*classsize),alphasize,classsize)
sigmatest <- 0.1

Wtest <- Wgenerate(alpha=alphatest,eta=etatest,X=Xtest,Z=Ztest)
```

```
#test of updata_gamma
thetaupdate_gamma <- update_gamma(alphat=alphatest,sigmat=sigmatest,
                            etat=etatest,X=Wtest$X,Z=Wtest$Z,Y=Wtest$Y)
```

`weight_matrix` *Weighted Inner Product*

Description

Calculate the weighted inner product of A and B with the weight W. This result is useful in Logistic Regression.

Usage

```
weight_matrix(A, W, B)
```

Arguments

A	the left matrix
W	the weight
B	the right matrix

Value

the wighted inner product, noticing it can be vector when A or B is 2 dimension.

Author(s)

Linsui Deng

Examples

```
#data generation
A <- matrix(rnorm(200),100,2)
W <- rnorm(100)
B <- matrix(rnorm(100),100,1)
weighted <- weight_matrix(A,W,B)
```

Wgenerate *Sigmoid Logistic Data Generation*

Description

Generate data satisties Sigmoid Logistic Model to check EMalgorithm.

Usage

```
Wgenerate(alpha, sigma = 1, eta, samplesize = 0, X, Z, seed1 = 0, seed2 = 0)
```

Arguments

alpha	the coeffients of the mean of each subgroup
sigma	the variance of Y
eta	the coeffients determining subgroup
samplesize	the size of sample you wanna generate
X	the covariables of the mean of each subgroup
Z	the covaraibles determining subgroup
seed1	random seed of generating Y
seed2	random seed of generating G

Value

X	the covariables of the mean of each subgroup
Z	the covaraibles determining subgroup
Y	the generated respond variable
G	the classes items belonging to

Author(s)

Linsui Deng

Examples

```
#some variables
samplesize <- 1000
classsize <- 6
etasize <- 3
alphasize <- 2

#test of Wgenerate
Xtest <- data.frame(matrix(rnorm(samplesize*etasize),samplesize,etasize))
Ztest <- matrix(rnorm(samplesize*alphasize),samplesize,alphasize)
```

```
etatest <- matrix(seq(1.15,1,length=etasize*classsize),etasize,classsize)
alphatest <- matrix(seq(1.15,1,length=alphasize*classsize),alphasize,classsize)

Wtest <- Wgenerate(alpha=alphatest,eta=etatest,X=Xtest,Z=Ztest)
```

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