

Package ‘FastHCS’

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Type Package

Title Robust Algorithm for Principal Component Analysis

Version 0.0.7

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Depends R (>= 3.1.1), matrixStats

Imports methods

LinkingTo Rcpp, RcppEigen

SystemRequirements C++11

Description The FastHCS algorithm of Schmitt and Vakili (2015) for high-dimensional, robust PCA modelling and associated outlier detection and diagnostic tools.

License GPL (>= 2)

LazyLoad yes

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NeedsCompilation yes

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FastHCS-package	<i>Package implementing the FastHCS robust PCA algorithm.</i>
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Description

Uses the FastHCS algorithm to compute a robust PCA model.

Details

```

Package:    FastHCS
Type:      Package
Version:   0.1
Date:     2013-01-13
Suggests:  mvtnorm
License:   GPL (>= 2)
LazyLoad:  yes

```

Index:

compPcaParams	Internal function used to compute the FastHCS PCA model parameters.
DnaAlteration	Cytosine methylation beta values for a sample of 198 non-pathological human tissue specimens.
FastHCS	Function to compute the FastHCS outlyingness index for high-dimensional data-sets.
FHCSnumStarts	Computes the number of starting subsets for the FastHCS algorithm.
FHCSkernelEVD	Reduces the data space to the affine subspace spanned by the n observations.
FHCSpsdo	Computes the pseudo Stahel Donoho based PCA estimates.
MultipleFeatures	Fourier coefficients describing the shape of many hand written replications of the numerals '0' and '1'.
plot.FastHCS	PCA diagnostic plot for object of class FastHCS.
quanf	Internal function used to compute the size of the h -subsets used in FastHCS based on the input parameter α .
signFlip	Performs the sign flip operation on a matrix of loadings.
Tablets	Near-infrared (NIR) spectroscopy of a sample of 310 tablets.

Author(s)

Kaveh Vakili (primary programmer), Eric Schmitt Maintainer: Kaveh Vakili <vakili.kaveh.email@gmail.com>

compPcaParams	<i>Computes the center vector, eigenvalues and loading matrix corresponding to a PCA model of a data matrix with respect to a subset of observations in a data set</i>
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Description

This function is used in FastHCS to compute the parameter estimates of the PCA models used at different steps of the algorithm. It is an internal function not intended to be called by the user.

Usage

```
compPcaParams(x, fitd, q=NULL, z0=NULL, seed=1)
```

Arguments

x	A data matrix x.
fitd	The (internal) result of a call to FastHCS.
q	Desired rank of the SVD decomposition.
z0	Optional. Result of a call to FHCSkernelEVD.
seed	Seed used to initialize the RNG. Defaults to 1.

Value

A list with the following components:

center	The multivariate mean of the observations with indexes in best.
loadings	The (rank q) loadings matrix of the observations with indexes in best.
eigenvalues	The eigenvalues of the observations with indexes in best multiplied by a consistency factor.
scores	The value of the projected on the space of the principal components data (the centred data multiplied by the loadings matrix) is returned. Hence, cov(scores) is the diagonal matrix diag(eigenvalues).

Author(s)

Kaveh Vakili, Eric Schmitt

DnaAlteration	<i>Cytosine methylation beta values for a sample of 198 non-pathological human tissue specimens.</i>
---------------	--

Description

A data frame with the subset of the 'Dna Alteration' data set corresponding to the sample of 'blood' and 'non-blood, non placenta' tissues.

Usage

```
data(DnaAlteration)
```

Format

Labels Observations with label "0" correspond to the subset of 'blood' tissues.

Column 2–1414 Cytosine methylation beta values collected at 1413 autosomal CpG loci.

Source

Christensen, B.C Houseman, E.A. Marsit, C.J. Zheng, S. Wrench, M.R. Wiemels, J.L. Nelson, H.H. Karagas, M.R. Padbury, J.F. Bueno, R. Sugarbaker, D.J Yeh, R., Wiencke, J.K. Kelsey, K.T. (2009). Aging and Evironmental Exposure Alter Tissue-Specific DNA Methylation Dependent upon CpG Island Context. PLoS Genet 5(8), e1000602.

Examples

```
data(DnaAlteration)
alpha<-0.5
Q<-15
p<-ncol(DnaAlteration[,-1])
ns<-FHCSnumStarts(q=Q,eps=(1-alpha)*4/5)
RunExample<-FALSE
if(RunExample){
Fit<-FastHCS(x=DnaAlteration[,-1],q=Q,nSamp=ns,seed=0)
colvec<-rep("orange",nrow(DnaAlteration))
colvec[DnaAlteration[,1]==1]<- "blue"
plot(Fit,col=colvec,pch=16)
}
```

FastHCS

*Performs the FastHCS algorithm for robust PCA.***Description**

Computes a robust PCA model with q components for an n by p matrix of multivariate data using the FastHCS algorithm.

Usage

```
FastHCS(x, nSamp=NULL, alpha=0.5, q=10, seed=1)
```

Arguments

<code>x</code>	A numeric n ($n > 5 * q$) by p ($p > 1$) matrix or data frame.
<code>nSamp</code>	A positive integer giving the number of resamples required; "nsamp" may not be reached if too many of the q -subsamples, chosen out of the observed vectors, are in a hyperplane. If "nSamp" is omitted, it is calculated to provide a breakdown point of "alpha" with probability 0.99.
<code>alpha</code>	Numeric parameter controlling the size of the active subsets i.e., " $h = \text{quantf}(\text{alpha}, n, q)$ ". Allowed values are between 0.5 and 1 and the default is 0.5.
<code>q</code>	Number of principal components to compute. Note that $p > q > 1$, $1 < q < n$. Default is $q = 10$.
<code>seed</code>	Starting value for random generator. Default is <code>seed = 1</code> .

Value

A list with components:

<code>rawBest</code> :	The indexes of the h members of H^* , the raw FastHCS optimal subset.
<code>obj</code> :	The FastHCS objective function corresponding to H^* , the selected subset of h observations.
<code>rawDist</code> :	Outlyingness index of the data on the raw q -dimensional subset that initialized H^* .
<code>best</code> :	the indexes of the members of the H_+ , the FastHSC subset after the C-steps.
<code>center</code> :	the p -vector of column means of the observations with indexes in <code>best</code> .
<code>loadings</code> :	the (rank q) loadings matrix of the observations with indexes in <code>best</code> .
<code>eigenvalues</code> :	the first q) eigenvalues of the observations with indexes in <code>best</code> .
<code>od</code> :	the orthogonal distances of the centered data wrt to the subspace spanned by the <code>loadings</code> matrix.
<code>sd</code> :	the score distances of the data projected on the subspace spanned by the <code>loadings</code> matrix with respect to the estimated center.
<code>cutoff.od</code> :	the cutoff for the vector of orthogonal distances.

cutoff.sd: the cutoff for the vector of score distances.

scores The value of the projected on the space of the principal components data (the centred data multiplied by the loadings matrix) is returned. Hence, cov(scores) is the diagonal matrix diag(eigenvalues).

Author(s)

Kaveh Vakili, Eric Schmitt

References

Schmitt E. and Vakili K. and (2015). Robust PCA with FastHCS. (<http://arxiv.org/abs/1402.3514>)

Examples

```
## testing outlier detection
n<-100
p<-30
Q<-5
set.seed(123)
x0<-matrix(rnorm(n*p),nc=p)
x0[1:30,]<-matrix(rnorm(30*p,4.5,1/100),nc=p)
z<-c(rep(0,30),rep(1,70))
nStarts<-FHCSnumStarts(q=Q,eps=0.4)
Fit<-FastHCS(x=x0,nSamp=nStarts,q=Q)
z[Fit$best]
plot(Fit,col=(!z)+1,pch=16)

## testing outlier detection, different value of alpha
n<-100
p<-30
Q<-5
set.seed(123)
x0<-matrix(rnorm(n*p),nc=p)
x0[1:20,]<-matrix(rnorm(20*p,4.5,1/100),nc=p)
z<-c(rep(0,20),rep(1,80))
nStarts<-FHCSnumStarts(q=Q,eps=0.25)
Fit<-FastHCS(x=x0,nSamp=nStarts,q=Q,alpha=0.75)
z[Fit$best]

#testing exact fit
n<-100
p<-5
Q<-4
set.seed(123)
x0<-matrix(rnorm(n*p),nc=p)
x0[1:30,]<-matrix(rnorm(30*p,4.5,1/100),nc=p)
x0[31:100,4:5]<-x0[31:100,2]
z<-c(rep(0,30),rep(1,70))
nStart<-FHCSnumStarts(q=Q,eps=0.4)
results<-FastHCS(x=x0,nSamp=nStart,q=Q)
```

```

z[results$best]
results$obj

#testing rotation equivariance
n<-100
p<-10
Q<-3
set.seed(123)
x0<-scale(matrix(rnorm(n*p),nc=p))
A<-diag(rep(1,p))
A[1:2,1:2]<-c(0,1,-1,0)
x1<-x0%%A
nStart<-FHCSnumStarts(q=Q,eps=0.4)
r0<-FastHCS(x=x0,nSamp=nStart,q=Q,seed=0)
r1<-FastHCS(x=x1,nSamp=nStart,q=Q,seed=0)
max(abs(log(r1$eigenvalues[1:Q]/r0$eigenvalues[1:Q])))

```

FHCSkernelEVD

Carries out the kernelEVD algorithm for data reduction

Description

This step reduces the data space to the affine subspace spanned by the n observations.

Usage

```
FHCSkernelEVD(x,best=NULL,q=NULL)
```

Arguments

<code>x</code>	A data matrix.
<code>best</code>	An optional subset of $1:n$.
<code>q</code>	Desired rank of the SVD decomposition. Optional.

Value

A reduced data set with full rank.

Author(s)

Small modification of the code from the `classPC` from `rrcov`.

References

Wu, W., Massart, D. L., and de Jong, S. (1997), 'The Kernel PCA Algorithms for Wide Data. Part I: Theory and Algorithms,' *Chemometrics and Intelligent Laboratory Systems*,36,165–172

Examples

```
n<-50
p<-200
x<-matrix(rnorm(n*p),nc=p)
W<-FHCSkerne1EVD(x)
```

FHCSnumStarts*Computes the number of starting q-subsets*

Description

Computes the number of starting q-subsets to take so that there is a 99 This is an internal function not intended to be called by the user.

Usage

```
FHCSnumStarts(q,gamma=0.99,eps=0.5)
```

Arguments

q	Number of desired components for the PCA model.
gamma	Desired probability of having at least one clean starting q-subset.
eps	suspected contamination rate of the sample.

Value

An integer number of starting q-subsets.

Author(s)

Kaveh Vakili

Examples

```
FHCSnumStarts(q=3,gamma=0.99,eps=0.4)
```


FHCSpsdo

*Computes the univariate MCD estimator of scatter***Description**

Pseudo Stahel Donoho Outlyingness based estimates of PCA.

Usage

```
FHCSpsdo(z0, h=NULL, seed=1, q=NULL, ndir=1000)
```

Arguments

<code>z0</code>	Either a data matrix or the result of a call to <code>FHCSkerne1EVD</code> .
<code>h</code>	Number of observation used to compute the univairate outlyingness. Defaults to $\lfloor (n+q+1)/2 \rfloor + 1$.
<code>seed</code>	Seed used to initialize the RNG. Defaults to 1.
<code>q</code>	Number of components. Defaults to <code>ncol(z0)</code> .
<code>ndir</code>	Number of projection used to compute the PP outlyngness.

Value

A list with components:

<code>rawDist:</code>	Outlyingness index of the data on the raw q-dimensional subset that initialized H^* .
<code>best:</code>	the indexes of the members of the H^+ , the FastHSC subset after the C-steps.
<code>center:</code>	the p-vector of column means of the observations with indexes in <code>best</code> .
<code>loadings:</code>	the (rank q) loadings matrix of the observations with indexes in <code>best</code> .
<code>eigenvalues:</code>	the first $\min(q)$ eigenvalues of the observations with indexes in <code>best</code> .

Author(s)

Vakili Kaveh.

References

Rousseeuw, P. J. (1984), Least Median of Squares Regression, Journal of the American Statistical Association, 79, 871–880.

Examples

```
n<-50
p<-10
x<-matrix(rnorm(n*p),nc=p)
FHCSpsdo(x)
```

MultipleFeatures	<i>Fourier coefficients describing the shape of many hand written replications of the numerals '0' and '1'.</i>
------------------	---

Description

A data frame with the subset of the 'Multiple Features' dataset corresponding to the sample of '0' and '1' numerals.

Usage

```
data(MultipleFeatures)
```

Format

Labels Numerals.

Column 2–77 Fourier coefficients describing the shape of each observation.

Source

Van Breukelen, M. Duin, R.P.W. Tax, D.M.J. and Den Hartog, J.E. (1998). Handwritten digit recognition by combined classifiers, *Kybernetika*, vol. 34, 381–386.

Examples

```
data(MultipleFeatures)
alpha<-0.5
Q<-15
p<-ncol(MultipleFeatures[, -1])
ns<-FHCSnumStarts(q=Q, eps=(1-alpha)*4/5)
RunExample<-FALSE
if(RunExample){
  Fit<-FastHCS(x=MultipleFeatures[, -1], q=Q, nSamp=ns, seed=1)
  colvec<-rep("orange", nrow(MultipleFeatures))
  colvec[MultipleFeatures[, 1]==1]<- "blue"
  plot(Fit, col=colvec, pch=16)
}
```

`plot.FastHCS`*Robust diagnostic plots for FastHCS*

Description

Creates a diagnostic plot of the robust SD and OD values from a FastHCS model fit, and their parametric cutoffs.

Usage

```
## S3 method for class 'FastHCS'  
plot(x,col="black",pch=16,...)
```

Arguments

<code>x</code>	For the <code>plot()</code> method, a FastHCS object, typically resulting as output from FastHCS .
<code>col</code>	A specification for the default plotting color. Vectors of values are recycled.
<code>pch</code>	Either an integer specifying a symbol, or a single character to be used as the default in plotting points. Note that only integers and single-character strings can be set as graphics parameters. Vectors of values are recycled.
<code>...</code>	Further arguments passed to the plot function.

Details

This function produces the PCA diagnostic plot of Hubert et al. (2005). Score distances are the n -vector of distances of each observation to the robust estimate of location on the robust PCA subspace. Likewise, orthogonal distances are the n -vector of distances of each observations to the robust PCA subspace. The observations whose score distance is larger than `cutoff.sd` or whose orthogonal distance is larger than `cutoff.od` are considered outliers and receive a flag equal to zero. The orthogonal distances are displayed along the vertical axis and the score distances along the horizontal axis, with the dotted lines indicating their respective cut-offs.

Author(s)

Kaveh Vakili

References

M. Hubert, P. J. Rousseeuw, K. Vanden Branden (2005), ROBPCA: a new approach to robust principal components analysis, *Technometrics*, **47**, 64–79.

See Also

[FastHCS](#)

Examples

```
data(Tablets)
alpha<-0.5
Q<-15
p<-ncol(Tablets[, -1])
ns<-FHCSnumStarts(q=Q, eps=(1-alpha)*4/5)
RunExample<-FALSE
if(RunExample){
Fit<-FastHCS(x=Tablets[, -1], q=Q, nSamp=ns, seed=1, alpha=0.5)
colvec<-rep("orange", nrow(Tablets))
colvec[Tablets[, 1]==1]<-"blue"
plot(Fit, col=colvec, pch=16)
}
```

signFlip*Carries out the signflip adjustment of a loadings matrix*

Description

This function solves the sign indeterminacy of the loadings by setting the maximum element in a singular vector to be positive.

Usage

```
signFlip(loadings)
```

Arguments

loadings A matrix of loadings.

Value

An (eventually sign flipped) loadings matrix.

Author(s)

Kaveh Vakili

Examples

```
x<-diag(10)
x[1,1]<--2
W<-signFlip(x)
W[1,1]
```

Tablets

Near-infrared (NIR) spectroscopy of a sample of 310 tablets.

Description

The original data set contains near-infrared (NIR) spectroscopy data for 310 tablets of four different dosages from pilot, laboratory and full scale production settings are included in the study. In this subset, we combine all 80 samples of 80mg tablets with the first 50 samples of 250mg tablets.

Usage

```
data(Tablets)
```

Format

Labels The observations with label '1' correspond to the 80mg Tablets samples and the '0' to the 250mg ones.

Column 2–405 Near Infrared Transmittance; 404 variables; 7400 to 10507 cm-1.

Source

M. Dyrby, S.B. Engelsen, L. Norgaard, M. Bruhn and L. Lundsberg Nielsen (2002). Chemometric Quantitation of the Active Substance in a Pharmaceutical Tablet Using Near Infrared (NIR) Transmittance and NIR FT Raman Spectra *Applied Spectroscopy* 56(5): 579–585.

Examples

```
data(Tablets)
alpha<-0.5
Q<-15
p<-ncol(Tablets[, -1])
ns<-FHCSnumStarts(q=Q, eps=(1-alpha)*4/5)
RunExample<-FALSE
if(RunExample){
Fit<-FastHCS(x=Tablets[, -1], q=Q, nSamp=ns, seed=1, alpha=0.5)
colvec<-rep("orange", nrow(Tablets))
colvec[Tablets[, 1]==1]<-"blue"
plot(Fit, col=colvec, pch=16)
}
```

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