Package 'WLinfer'

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Type Package

Title Statistical Inference in Weighted Lindley Distribution

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Description Estimate the parameters of weighted Lindley distribution, and provide some useful results like goodness of fit or confidence interval etc.
Ghitany, M., Alqallaf, F., Al-
Mutairi, D., Husain, H. (2011) <doi:10.1016/j.matcom.2010.11.005>.
Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020) submitted.
Wang, M., Wang, W. (2017) <doi:10.1080/03610918.2014.970696>.

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`CoxSnell_bias` *Cox & Snell bias correction methods for estimators*

Description

`CoxSnell_bias` and `CoxSnell_bias_log` provide a vector of MLE bias, and it can be used for bias correction of both MLE and MLEc. The estimation method was suggested by by Cox and Snell(1968).

Usage

```
CoxSnell_bias(n, lambda, phi)

CoxSnell_bias_log(n, lambda, phi)
```

Arguments

`n`	a numeric value of data length.
`lambda`	a numeric value of estimated lambda.
`phi`	a numeric value of estimated phi.

Details

`CoxSnell_bias` provides the bias of original lambda and phi. However, `CoxSnell_bias_log` provides the bias of log lambda and log phi. In some cases, estimators are smaller than bias, and it means that the bias corrected estimators are out of parameter space. To solve this problem, `CoxSnell_bias_log` is useful. Correction formula is based on Fisher information and some cumulants, which are given in Kim and Jang (2020).

Value

A numeric vector of Cox & Snell biases of lambda and phi.

Background

These functions implement formulas given in Hyoung-Moon Kim. et al. (2020).

References

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

Examples

```
data <- fail_fiber
mlec <- MLEc_WL(data)
n <- length(data)
CoxSnell_bias(n, mlec[1], mlec[2])
CoxSnell_bias_log(n, mlec[1], mlec[2])
```

`fail_fiber` *The failure stresses (in GPA) of single carbon fibers*

Description

The data represents the strength measured in GPA for single carbon fibers and impregnated 1000-carbon fiber tows. Single fibers were tested under tension at gauge lengths of 50 mm, with sample sizes n = 65 (Bader et al., 1982).

Usage

```
data(fail_fiber, package = "WL")
```

Format

A dataframe with 65 observations.

References

Bader, M. G., Priest, A. M. (1982). Statistical aspects of fibre and bundle strength in hybrid composites. In: Hayashi, T., Kawata, K., Umekawa, S., eds. Progress in Science and Engineering Composites: Proceedings of the Fourth International Conference on Composite Materials, ICCM-IV. Tokyo: Japan Society for Composite Materials.

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

`fail_time` *The lifetime failure data of an electronic device*

Description

It represents the lifetime failure data of an electronic device (Wang, 2000).

Usage

```
data(fail_time, package = "WL")
```

Format

A dataframe with 18 observations.

References

Wang, F. (2000). A new model with bathtub-shaped failure rate using an additive Burr XII distribution. *Reliability Engineering and System Safety*, 70:305-312.

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

`Firth_WL` *Firth's method for bias correction of estimators*

Description

Firth's method is bias correction option included in `WL`. `Firth_WL` and `Firth_WL_log` provides the bias corrected lambda and phi based on Firth's method.

Usage

```
Firth_WL(y, init)

Firth_WL_log(y, init)
```

Arguments

`y`	a numeric vector.
`init`	a vector of initial values for iterative algorithm designed to solve the modified likelihood equations.

Details

`Firth_WL` and `Firth_WL_log` returns a vector of estimates of parameters corrected by Firth's method which uses the modified likelihood equations. In case of weighted Lindley distribution, two non-linear equations should be solved since the solutions are not in closed form. To this end, R package `nleqslv` is used. To avoid poor local maxima, other estimators like MMEm or MLEc are recommended to be used as initial values.

Value

A vector of corrected estimators lambda and phi.

Background

Non-linear equations to be solved are derived in Kim and Jang (2020).

References

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

Examples

```
data <- fail_fiber
Firth_WL(data,MMEm_WL(data))
Firth_WL_log(data,MMEm_WL(data))
```

`lifetime_alum` *The lifetimes of aluminum specimens*

Description

This data set represents the lifetimes of 102 aluminum specimens exposed to a maximum stress/cycle of 26000 PSI (PSI is a unit of pressure expressed in pounds of force per square inch of area; it stands for Pounds per Square Inch. 1 PSI = 6894 Pascals = 0.070 atmospheres = 51.715 torr) (Owen et al., 2000).

Usage

```
data(lifetime_alum, package = "WL")
```

Format

A dataframe with 102 observations.

References

Owen W.J., Padget W.J. (2000). A Birnbaum-Saunders accelerated life model. *IEEE Trans Reliability*, 49(2):224-229.

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

MLE_WL *MLE in weighed Lindley distribution*

Description

MLE_WL returns maximum likelihood estimates of parameters. First, the estimate of phi is obtained from one dimensional non linear equation. Then, by plugging in the estimated phi, the estimate of lambda is easily obtained.

Usage

```
MLE_WL(y, init)
```

Arguments

y	a numeric vector of observations.
init	a initial value for estimate phi.

Details

These functions implement formulas given in Hyoung-Moon Kim. et al. (2020).

Value

A numeric vector of MLE for lambda and phi.

References

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

Examples

```
data <- fail_fiber
mme <- MME_WL(data)
modified_mme <- MMEm_WL(data)
mle <- MLE_WL(data, mme[2])
mlec <- MLEc_WL(data)

rbind(mme, modified_mme, mle, mlec)
```

MME_var *Asymptotic covariance matrix of estimators*

Description

These four functions: `MME_var`, `MMEm_var`, `MLE_var` and `MLEc_var` provide asymptotic covariance matrixs for MME, modified MME, MLE, MLEc, repectively. All of these can be calculated to a closed form value.

Usage

```
MME_var(l, p, n)

MMEm_var(l, p, n)

MLE_var(l, p, n)

MLEc_var(l, p, n)
```

Arguments

`l`	a numeric value.
`p`	a numeric value.
`n`	a numeric value.

Details

These functions implement formulas given in Hyoung-Moon Kim. et al. (2020).

Value

A matrix of asymptotic covariance of lambda and phi.

References

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

Examples

```
data <- fail_fiber
n <- length(data)
mme <- MME_WL(data)
modified_mme <- MMEm_WL(data)
mle <- MLE_WL(data, mme[2])
mlec <- MLEc_WL(data)
MME_var(mme[1],mme[2],n)
MMEm_var(modified_mme[1],modified_mme[2],n)
```

```
MLE_var(mle[1],mle[2],n)
MLEc_var(mlec[1],mlec[2],n)
```

MME_WL *Estimating parameters in weighed Lindley distribution*

Description

Parameter estimate functions: MME_WL, MMEm_WL, MLE_WL and MLEc_WL can be used to estimate parameters in weighed Lindley distribution. MME_WL, MMEm_WL and MLEc_WL have closed form values for parameters, lambda and phi. On the other hands, parameter lambda in MLE_WL is based on numerical solution. It use the function nleqslv for solving one variable equation.

Usage

```
MME_WL(x)

MMEm_WL(x)

MLEc_WL(x)
```

Arguments

x	a numeric vector.

Details

These functions implement formulas given in Hyoung-Moon Kim. et al. (2020).

Value

A numeric vector of lambda and phi estimated by each method.

References

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

Examples

```
data <- fail_fiber
mme <- MME_WL(data)
modified_mme <- MMEm_WL(data)
mle <- MLE_WL(data, mme[2])
mlec <- MLEc_WL(data)

rbind(mme, modified_mme, mle, mlec)
```

plot.WL *Some plots for* WL

Description

plot method for a class "WL".

Usage

```
## S3 method for class 'WL'
plot(
  x,
  which = c(1, 2, 3, 4),
  ask = prod(par("mfcol")) < length(which) && dev.interactive(),
  ...
)
```

Arguments

x	an object of class "WL" made by the function WL.
which	if a subset of the plots is required, specify a subset of 1:4; see 'Details' for a description of the plots.
ask	logical; if TRUE, the user is asked before each plot.
...	other parameters to be passed through to plotting functions.

Details

The first plot (which=1) is histogram of input values and pdf from estimated lambda and phi. This is a good plot to judge how well the estimated values match the actual data.

The second plot (which=2) is the boxplot of the input data. It shows the simple information about median and outlier points. Additionally, you can easily see which the "Outlier Points" are with box on the right side.

The third plot (which=3) is Q-Q plot. This provides the same information as the normal Q-Q plot.

The last plot (which=4) is contour plot of lambda and phi. You can observe the location of estimator, and also compare estimator to Cox&Snell bias corrected MLE and MLEc.

Plots are basically displayed one by one in the order mentioned above, and it is also possible to view only one plot or four at a time. How to do this is shown in the example below.

See Also

See also WL and plot.

Examples

```
example <- lifetime_alum
result <- WL(example)

plot(result)
plot(result, which=1)
plot(result, which=3)
par(mfrow = c(2, 2)) ; plot(result)
par(mfrow = c(1, 1))
```

---

summary.WL *Summarizing WL function*

Description

summary method for a class "WL".

Usage

```
## S3 method for class 'WL'
summary(object, ...)

## S3 method for class 'summary.WL'
print(x, digits = max(3, getOption("digits") - 3), ...)
```

Arguments

object	an object of class "WL" made by the function WL.
...	not used, but exists because of the compatibility.
x	an object of class "summary.WL".
digits	a numeric number of significant digits.

Examples

```
example <- fail_fiber
result <- WL(example, est_method="MLEc")
summary(result)
```

`wilks.test` *Estimator test based on Wilks' theorem*

Description

This is a test based on Wilks' theorem, to determine which parameter space the estimated parameter is included in.

Usage

```
wilks.test(x, estimator, side = "two")
```

Arguments

`x`	a numeric vector or data frame.
`estimator`	a numeric vector with estimated lambda and phi.
`side`	a character string which selects the direction of wilks' theorem test ("two", "less" or "greater").

Details

By Wilks' theorem, we can test the k-dimensional parameter with chi-square distribution. The Wilks' theorem test can be performed by assuming the parameter space of null hypothesis and setting the part not included in it as the parameter space of the alternative hypothesis.

Value

`wilks.test` returns a list with these components:

`side`	a character string of one of "two", "less" or "greater".
`stat`	a numeric value the statistics from Wilks' theorem.
`pvalue`	a numeric value the p-value of the statistics.

Examples

```
data <- fail_fiber
mme <- MME_WL(data)
wilks.test(data, mme, side="two")
wilks.test(data, mme, side="less")
wilks.test(data, mme, side="greater")
```

WL *Statistical inference in weighted Lindley distribution*

Description

Weighted Lindley distribution is suggested by Ghitany et al. (2011). WL provides four types of estimator, which are MME, modified MME, MLE, and MLEc from weighted Lindley distribution. And there are four sub-options, which are bias-correction, goodness of fit test, confidence interval, and Wilks' theorem test.

Usage

```
WL(
  x,
  est_method = "MLEc",
  bias_cor = "None",
  dist_test = "ks",
  gof_alpha = 0.05,
  ks_side = "two",
  CI_method = "asymp",
  CI_scale = "normal",
  CI_side = "two",
  CI_alpha = 0.05,
  boots_iter = 10^3,
  wilks_test = TRUE,
  wilks_alpha = 0.05,
  wilks_side = "two"
)
```

Arguments

x	a numeric vector or data frame.
est_method	a character string which selects the estimation method ("MME", "MMEm", "MLE", "MLEc"), default is "MLEc".
bias_cor	an optional character character string which selects the bias correction method ("coxsnell", "boots" or "firth").
dist_test	a character string or character vector which choose the test of goodness of fit ("all","ks","ad","cvm").
gof_alpha	a numeric value between 0 and 1 for controlling the significance level of goodness of fit test; default value is 0.05.
ks_side	a character string which selects the alternative hypothesis ("two", "less" or "greater") for Kolmogorov-Smirnov Test, default is "two".
CI_method	a character string which selects the method for calculating confidence intervals ("asymp" or "boots"), default is "asymp". Since the "asymp" option is not available with bias correction, only the "boots" is available with bias correction.

CI_scale	a character string which selects the scale of confidence intervals ("exp" or "normal")
CI_side	a character string which selects the direction of confidence intervals ("two", "less" or "greater").
CI_alpha	a numeric value between 0 and 1 for controlling the significance level of confidence intervals; default value is 0.05.
boots_iter	a numeric value for iteration number of bootstrap method.
wilks_test	logical. If TRUE, wilks' theorem test is performed.
wilks_alpha	a numeric value between 0 and 1 for controlling the significance level of wilks' theorem test; default value is 0.05.
wilks_side	a character string which selects the direction of wilks' theorem test ("two", "less" or "greater").

Details

First, the user can determine the type of estimator from MME, modified MME, MLE, and MLEc. The closed form formulas for MME, modified MME, and MLEc are given in Hyoung-Moon Kim. et al. (2020). And MLE is obtained numerically. Additionally MLE and MLEc have bias correction options. MLE has Cox&Snell method and Firth's method, however MLEc has Cox&Snell method and bootstrap method.

Second, it provides a goodness of fit test. There are three kinds of tests, Kolmogorov-Smirnov test, Anderson Darling test, and Cramer-von Mises test. They provide statistics and also p-values. If the input value gof_alpha is selected, it determines whether or not to reject the null hypothesis.

Third, it provides information on the confidence interval. There are two kinds of confidence intervals, one is based on bootstrap method, and the other is asymptotic variance based method. Asymptotic variance based method is only available without bias correction, however bootstrap method is always available. Sometimes the confidence interval is outside the parameter space. If it occers, confidence interval will be calculated with log scale and show the exponential confidence interval of log scaled estimators. This option can also be used separately with selecting "exp" in CI_scale.

Lastly, through wilks.test, WL test the parameter space of estimators lambda and phi. There is an option for Wilks' theorem test, and that option provides options for the side of Wilks' theorem test.

Value

WL returns a list with these components:

data	a numeric vector the input values.
dataname	a character string the name of input values.
stat_summary	a numeric vector with min, 1st quantile, median, 3rd quantile, and max.
mean	a numeric value mean of input values.
var	a numeric value variance of input values.
est	a numeric vector with estimated lambda and phi.
lambda_var	a numeric value variance of estimated lambda.

phi_var	a numeric value variance of estimationed lambda.
bias_cor	a character string from bias correction method ("coxsnell","firth" or "boots").
est_method	a character string from estimation method ("MME", "MMEm", "MLE" or "MLEc").
boots_iter	a numeric value of bootstrap iteration.
test_list	a list with results of goodness of fit test.
CI_list	a list with confidence interval related outputs.
wilks_list	a list with results of wilks' test.

References

Ghitany, M., Alqallaf, F., Al-Mutairi, D., Husain, H. (2011). A two-parameter weighted Lindley distribution and its applications to survival data. *Mathematics and Computers in Simulation* 81: 1190-1201.

Hyoung-Moon Kim. and Yu-Hyeong Jang. (2020). New Closed-Form Estimators for Weighted Lindley Distribution. , submitted.

Wang, M., Wang, W. (2017). Bias-Corrected maximum likelihood estimation of the parameters of the weighted Lindley distribution. *Communications in Statistics Simulation and Computation* 46: 530-545.

Examples

```
example <- lifetime_alum
result <- WL(example)
print(result)
```

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