# Package 'accSDA' 

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Description Implementation of sparse linear discriminant analysis, which is a supervised classification method for multiple classes. Various novel optimization approaches to this problem are implemented including alternating direction method of multipliers (ADMM), proximal gradient (PG) and accelerated proximal gradient (APG) (See Atkins et al. [arXiv:1705.07194](arXiv:1705.07194)).
Functions for performing cross validation are also supplied along with basic prediction and plotting functions.
Sparse zero variance discriminant analysis (SZVD) is also included in the package (See Ames and Hong, [arXiv:1401.5492](arXiv:1401.5492)). See the github wiki for a more extended description.

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URL https://github.com/gumeo/accSDA/wiki

BugReports https://github.com/gumeo/accSDA/issues
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accSDA

accSDA: A package for performing sparse discriminant analysis in various ways.

## Description

The accSDA package provides functions to perform sparse discriminant analysis using a selection of three optimization methods, proximal gradient (PG), accelerated proximal gradient (APG) and alternating direction method of multipliers (ADMM). The package is intended to extend the available tools to perform sparse discriminant analysis in R. The three methods can be called from the function ASDA. Cross validation is also implemented for the L 1 regularization parameter. Functions for doing predictions, summary, printing and simple plotting are also provided. The sparse discriminant functions perform lda on the projected data by default, using the lda function in the MASS package. The functions return an object of the same class as the name of the function and provide the lda solution, along with the projected data, thus other kinds of classification algorithms can be employed on the projected data.

## Description

Applies accelerated proximal gradient algorithm, proximal gradient algorithm or alternating direction methods of multipliers algorithm to the optimal scoring formulation of sparse discriminant analysis proposed by Clemmensen et al. 2011.

$$
\operatorname{argmin}\left|\left(Y_{t} \theta-X_{t} \beta\right)\right|_{2}^{2}+t|\beta|_{1}+\lambda|\beta|_{2}^{2}
$$

## Usage

```
ASDA(Xt, ...)
    ## Default S3 method:
    ASDA(
    Xt,
    Yt,
    Om = diag(p),
    gam = 0.001,
    lam = 1e-06,
    q = K - 1,
    method = "SDAAP",
    control = list(),
)
```


## Arguments

Xt
...
Yt $\quad n$ by $K$ matrix of indicator variables ( $\mathrm{Y} \mathrm{ij}=1$ if i in class j ). This will later be changed to handle factor variables as well. Each observation belongs in a single class, so for a given row/observation, only one element is 1 and the rest is 0 .
Om p by p parameter matrix Omega in generalized elastic net penalty.
gam Regularization parameter for elastic net penalty.
lam Regularization parameter for 11 penalty, must be greater than zero. If crossvalidation is used ( $C V=$ TRUE) then this must be a vector of length greater than one.
q Desired number of discriminant vectors.
method This parameter selects which optimization method to use. It is specified as a character vector which can be one of the three values
SDAP Proximal gradient algorithm.
SDAAP Accelerated proximal gradient algorithm.
SDAD Alternating directions method of multipliers algorithm.
Note that further parameters are passed to the function in the argument control, which is a list with named components.
control List of control arguments. See Details.

## Details

The control list contains the following entries to further tune the algorithms.
PGsteps Maximum number if inner proximal gradient/ADMM algorithm for finding beta. Default value is 1000 .

PGtol Stopping tolerance for inner method. If the method is SDAD, then this must be a vector of two values, absolute (first element) and relative tolerance (second element). Default value is $1 \mathrm{e}-5$ for both absolute and relative tolerances.
maxits Number of iterations to run. Default value is 250 .
tol Stopping tolerance. Default value is $1 \mathrm{e}-3$.
mu Penalty parameter for augmented Lagrangian term, must be greater than zero and only needs to be specified when using method SDAD. Default value is 1 .
CV Logical value which is TRUE if cross validation is supposed to be performed. If cross-validation is performed, then lam should be specified as a vector containing the regularization values to be tested. Default value is FALSE.
folds Integer determining the number of folds in cross-validation. Not needed if CV is not specified. Default value is 5 .
feat Maximum fraction of nonzero features desired in validation scheme. Not needed if CV is not specified. Default value is 0.15 .
quiet Set to FALSE if status updates are supposed to be printed to the R console. Default value is TRUE. Note that this triggers a lot of printing to the console.
ordinal Set to TRUE if the labels are ordinal. Only available for methods SDAAP and SDAD.
initTheta Option to set the initial theta vector, by default it is a vector of all ones for the first theta.
bt Logical indicating whether backtracking should be used, only applies to the Proximal Gradient based methods. By default, backtracking is not used.
L Initial estimate for Lipshitz constant used for backtracking. Default value is 0.25.
eta Scalar for Lipshitz constant. Default value is 1.25 .
rankRed Boolean indicating whether Om is factorized, such that $\mathrm{R}^{\wedge} \mathrm{t}^{*} \mathrm{R}=\mathrm{Om}$, currently only applicable for accelerated proximal gradient.

## Value

ASDA returns an object of class "ASDA" including a list with the following named components:
call The matched call.
B p by q matrix of discriminant vectors, i.e. sparse loadings.
Q K by q matrix of scoring vectors, i.e. optimal scores.
varNames Names of the predictors used, i.e. column names of Xt.
origP Number of variables in Xt.
fit Output from function lda on projected data. This is NULL the trivial solution is found, i.e. B is all zeroes. Use lower values of lam if that is the case.
classes The classes in Yt.
lambda The lambda/lam used, best value found by cross- validation if CV is TRUE.

## Note

The input matrix Xt should be normalized, i.e. each column corresponding to a variable should have its mean subtracted and scaled to unit length. The functions normalize and normalizetest are supplied for this purpose in the package.

## See Also

SDAAP, SDAP and SDAD

## Examples

```
set.seed(123)
# Prepare training and test set
train <- c(1:40,51:90,101:140)
Xtrain <- iris[train,1:4]
nX <- normalize(Xtrain)
Xtrain <- nX$Xc
Ytrain <- iris[train,5]
Xtest <- iris[-train,1:4]
Xtest <- normalizetest(Xtest,nX)
Ytest <- iris[-train,5]
# Define parameters for Alternating Direction Method of Multipliers (SDAD)
Om <- diag(4)+0.1*matrix(1,4,4) #elNet coef mat
gam <- 0.0001
lam <- 0.0001
method <- "SDAD"
q <- 2
control <- list(PGsteps = 100,
    PGtol = c(1e-5,1e-5),
    mu = 1,
    maxits = 100,
    tol = 1e-3,
    quiet = FALSE)
    # Run the algorithm
    res <- ASDA(Xt = Xtrain,
        Yt = Ytrain,
        Om = Om,
        gam = gam ,
        lam = lam,
        q = q,
        method = method,
        control = control)
    # Can also just use the defaults, which is Accelerated Proximal Gradient (SDAAP):
    resDef <- ASDA(Xtrain,Ytrain)
    # Some example on simulated data
    # Generate Gaussian data on three classes with plenty of redundant variables
    # This example shows the basic steps on how to apply this to data, i.e.:
```

```
# 1) Setup training data
# 2) Normalize
# 3) Train
# 4) Predict
# 5) Plot projected data
# 6) Accuracy on test set
P <- 300 # Number of variables
N <- 50 # Number of samples per class
# Mean for classes, they are zero everywhere except the first 3 coordinates
m1 <- rep(0,P)
m1[1] <- 3
m2 <- rep(0,P)
m2[2] <- 3
m3 <- rep(0,P)
m3[3] <- 3
# Sample dummy data
Xtrain <- rbind(MASS::mvrnorm(n=N,mu = m1, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m2, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m3, Sigma = diag(P)))
Xtest <- rbind(MASS::mvrnorm(n=N,mu = m1, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m2, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m3, Sigma = diag(P)))
# Generate the labels
Ytrain <- factor(rep(1:3,each=N))
Ytest <- Ytrain
# Normalize the data
Xt <- accSDA::normalize(Xtrain)
Xtrain <- Xt$Xc # Use the centered and scaled data
Xtest <- accSDA::normalizetest(Xtest,Xt)
# Train the classifier and increase the sparsity parameter from the default
# so we penalize more for non-sparse solutions.
res <- accSDA::ASDA(Xtrain,Ytrain,lam=0.01)
# Plot the projected training data, it is projected to
# 2-dimension because we have 3 classes. The number of discriminant
# vectors is maximum number of classes minus 1.
XtrainProjected <- Xtrain%*%res$beta
plot(XtrainProjected[,1],XtrainProjected[,2],col=Ytrain)
# Predict on the test data
preds <- predict(res, newdata = Xtest)
# Plot projected test data with predicted and correct labels
```

```
XtestProjected <- Xtest%*%res$beta
plot(XtestProjected[,1],XtestProjected[, 2],col=Ytest,
    main="Projected test data with original labels")
plot(XtestProjected[,1],XtestProjected[,2],col=preds$class,
    main="Projected test data with predicted labels")
# Calculate accuracy
sum(preds$class == Ytest)/(3*N) # We have N samples per class, so total 3*N
```


## Description

This is a function to visualize the discriminant vector from the ASDA method. The plot is constructed as a ggplot barplot and the main purpose of it is to visually inspect the sparsity of the discriminant vectors. The main things to look for are how many parameters are non-zero and if there is any structure in the ones that are non-zero, but the structure is dependent on the order you specify your variables. For time-series data, this could mean that a chunk of variables are non-zero that are close in time, meaning that there is some particular event that is best for discriminating between the classes that you have.

## Usage

ASDABarPlot(asdaObj, numDVs = 1, xlabel, ylabel, getList = FALSE, main, ...)

## Arguments

| asdaObj |  |
| :--- | :--- |
| numDVs | Object from the ASDA function. <br> Number of discriminant vectors (DVs) to plot. This is limited by the number <br> of DVs outputted from the ASDA function or k-1 DVs where $k$ is the number of <br> classes. The first 1 to numDVs are plotted. |
| xlabel | Label to put under every plot |
| ylabel | Vector of y-axis labels for each plot, e.g. if there are three DVs, then ylab = <br> c('Discriminant Vector 1 ' , 'Discriminant Vector $2^{\prime}, ~, ~ D i s c r i m i n a n t ~ V e c t o r ~$ |
| 3') is a valid option. |  |
| getList | Logical value indicating whether the output should be a list of the plots or the <br> plots stacked in one plot using the gridExtra package. By default the function <br> produces a single plot combining all plots of the DVs. |
| main | Main title for the plots, this is not used if getList is set to TRUE. |
| $\ldots$ | Extra arguments to grid.arrange. |

## Value

barplot. ASDA returns either a single combined plot or a list of individual ggplot objects.

## Note

This function is used as a quick diagnostics tool for the output from the ASDA function. Feel free to look at the code to customize the plots in any way you like.

## See Also

ASDA

## Examples

```
# Generate and ASDA object with your data, e.g.
# Prepare training and test set
# This is a very small data set, I advise you to try it on something with more
# variables, e.g. something from this source: http://www.cs.ucr.edu/~eamonn/time_series_data/
    # or possibly run this on the Gaussian data example from the ASDA function
    train <- c(1:40,51:90,101:140)
    Xtrain <- iris[train,1:4]
    nX <- normalize(Xtrain)
    Xtrain <- nX$Xc
    Ytrain <- iris[train,5]
    Xtest <- iris[-train,1:4]
    Xtest <- normalizetest(Xtest,nX)
    Ytest <- iris[-train,5]
    # Run the method
    resIris <- ASDA(Xtrain,Ytrain)
    # Look at the barplots of the DVs
    ASDABarPlot(resIris)
```

    genDat
    Generate data for ordinal examples in the package

## Description

Given the parameters, the function creates a dataset for testing the ordinal functionality of the package. The data is samples from multivariate Gaussians with different means, where the mean varies along a sinusoidal curve w.r.t. the class label.

## Usage

genDat(numClasses, numObsPerClass, mu, sigma)

## Arguments

numClasses Positive integer specifying the number of classes for the dataset.
numObsPerClass Number of observations sampled per class.

| mu | Mean of the first class. |
| :--- | :--- |
| sigma | 2 by 2 covariance matrix |

## Details

This function is used to demonstrate the usage of the ordinal classifier.

## Value

genDat Returns a list with the following attributes:
X A matrix with two columns and numObsPerClass*numClasses rows.
$Y$ Labels for the rows of $X$.

## Author(s)

Gudmundur Einarsson

## See Also

> ordASDA

## Examples

```
set.seed(123)
    # You can play around with these values to generate some 2D data to test one
    numClasses <- 15
    sigma <- matrix(c(1,-0.2,-0.2,1),2,2)
    mu <- c(0,0)
    numObsPerClass <- 5
    # Generate the data, can access with train$X and train$Y
    train <- accSDA::genDat(numClasses,numObsPerClass,mu,sigma)
    test <- accSDA::genDat(numClasses,numObsPerClass*2,mu,sigma)
    # Visualize it, only using the first variable gives very good separation
    plot(train$X[,1],train$X[,2],col = factor(train$Y),asp=1,main="Training Data")
```

    normalize Normalize training data
    
## Description

Normalize a vector or matrix to zero mean and unit length columns.

## Usage

normalize(X)

## Arguments

X
a matrix with the training data with observations down the rows and variables in the columns.

## Details

This function can e.g. be used for the training data in the ASDA function.

## Value

normalize Returns a list with the following attributes:

Xc The normalized data
$\mathbf{m x}$ Mean of columns of $X$.
$\mathbf{v x}$ Length of columns of $X$.
Id Logical vector indicating which variables are included in X. If some of the columns have zero length they are omitted

## Author(s)

Line Clemmensen

## References

Clemmensen, L., Hastie, T. and Ersboell, K. (2008) "Sparse discriminant analysis", Technical report, IMM, Technical University of Denmark

## See Also

```
normalizetest, predict.ASDA, ASDA
```


## Examples

```
## Data
X<-matrix(sample(seq(3),12,replace=TRUE),nrow=3)
## Normalize data
Nm<-normalize(X)
print(Nm$Xc)
## See if any variables have been removed
which(!Nm$Id)
```

```
normalizetest Normalize training data
```


## Description

Normalize test data using output from the normalize() of the training data

## Usage

normalizetest(Xtst, Xn)

## Arguments

Xtst a matrix with the test data with observations down the rows and variables in the columns.
Xn List with the output from normalize(Xtr) of the training data.

## Details

This function can e.g. be used for the test data in the predict. ASDA function.

## Value

normalizetest returns the normalized test data Xtst

## Author(s)

Line Clemmensen

## References

Clemmensen, L., Hastie, T. and Ersboell, K. (2008) "Sparse discriminant analysis", Technical report, IMM, Technical University of Denmark

## See Also

normalize, predict.ASDA, ASDA

## Examples

```
## Data
Xtr<-matrix(sample(seq(3),12,replace=TRUE),nrow=3)
Xtst<-matrix(sample(seq(3),12,replace=TRUE),nrow=3)
## Normalize training data
Nm<-normalize(Xtr)
## Normalize test data
Xtst<-normalizetest(Xtst,Nm)
```

ordASDA Ordinal Accelerated Sparse Discriminant Analysis

## Description

Applies accelerated proximal gradient algorithm to the optimal scoring formulation of sparse discriminant analysis proposed by Clemmensen et al. 2011. The problem is further casted to a binary classification problem as described in "Learning to Classify Ordinal Data: The Data Replication Method" by Cardoso and da Costa to handle the ordinal labels. This function serves as a wrapper for the ASDA function, where the appropriate data augmentation is performed. Since the problem is casted into a binary classication problem, only a single discriminant vector comes from the result. The first * $\mathrm{p} *$ entries correspond to the variables/coefficients for the predictors, while the following K-1 entries correspond to biases for the found hyperplane, to separate the classes. The resulting object is of class ordASDA and has an accompanying predict function. The paper by Cardoso and dat Costa can be found here: (http://www.jmlr.org/papers/volume8/cardoso07a/cardoso07a.pdf).

```
Usage
    ordASDA(Xt, ...)
    ## Default S3 method:
    ordASDA(
        Xt,
        Yt,
        s = 1,
        Om,
        gam = 0.001,
        lam = 1e-06,
        method = "SDAAP",
        control,
    )
```


## Arguments

$X t \quad n$ by $p$ data matrix, (can also be a data.frame that can be coerced to a matrix)
... Additional arguments for ASDA and lda function in package MASS.
Yt vector of length $n$, equal to the number of samples. The classes should be $1,2, \ldots, \mathrm{~K}$ where K is the number of classes. Yt needs to be a numeric vector.
s
We need to find a hyperplane that separates all classes with different biases. For each new bias we define a binary classification problem, where a maximum of $s$ ordinal classes or contained in each of the two classes. A higher value of $s$ means that more data will be copied in the data augmentation step. BY default s is 1 .
Om p by p parameter matrix Omega in generalized elastic net penalty, where p is the number of variables.

| gam | Regularization parameter for elastic net penalty, must be greater than zero. |
| :--- | :--- |
| lam | Regularization parameter for ll penalty, must be greater than zero. |
| method | String to select method, now either SDAD or SDAAP, see ?ASDA for more info. |
| control | List of control arguments further passed to ASDA. See ASDA. |

## Value

ordASDA returns an object of class "ordASDA" including a list with the same components as an ASDA objects and:
h Scalar value for biases.
K Number of classes.
NULL

## Note

Remember to normalize the data.

## See Also

ASDA.

## Examples

set.seed(123)
\# You can play around with these values to generate some 2D data to test one numClasses <- 5
sigma <- matrix (c(1,-0.2,-0.2,1), 2, 2)
mu <- c $(0,0)$
numObsPerClass <- 5
\# Generate the data, can access with train\$X and train\$Y
train <- accSDA::genDat(numClasses,numObsPerClass,mu,sigma)
test <- accSDA::genDat(numClasses, numObsPerClass*2,mu, sigma)
\# Visualize it, only using the first variable gives very good separation
plot(train\$X[,1],train\$X[,2],col = factor(train\$Y), asp=1, main="Training Data")
\# Train the ordinal based model
res <- accSDA::ordASDA(train\$X,train\$Y,s=2,h=1, gam=1e-6, lam=1e-3)
vals <- predict (object = res, newdata = test\$X) \# Takes a while to run ~ 10 seconds
sum(vals==test\$Y)/length(vals) \# Get accuracy on test set
\#plot(test\$X[,1],test\$X[,2],col = factor(test\$Y),asp=1,
\# main="Test Data with correct labels")
\#plot(test\$X[,1],test\$X[,2],col = factor(vals), asp=1,
\# main="Test Data with predictions from ordinal classifier")
predict.ASDA Predict method for sparse discriminant analysis

## Description

Predicted values based on fit from the function ASDA. This function is used to classify new observations based on their explanatory variables/features.

## Usage

\#\# S3 method for class 'ASDA'
predict(object, newdata $=$ NULL, ...)

## Arguments

object Object of class ASDA. This object is returned from the function ASDA.
newdata A matrix of new observations to classify.
... Arguments passed to predict.lda.
Value
A list with components:
class The classification (a factor)
posterior posterior probabilities for the classes
$x$ the scores

## Note

The input matrix newdata should be normalized w.r.t. the normalization of the training data

## See Also

SDAAP, SDAP and SDAD

## Examples

```
    # Prepare training and test set
    train <- c(1:40,51:90,101:140)
    Xtrain <- iris[train,1:4]
    nX <- normalize(Xtrain)
    Xtrain <- nX$Xc
    Ytrain <- iris[train,5]
    Xtest <- iris[-train,1:4]
    Xtest <- normalizetest(Xtest,nX)
    Ytest <- iris[-train,5]
    # Define parameters for SDAD
```

```
Om <- diag(4)+0.1*matrix(1,4,4) #elNet coef mat
gam <- 0.01
lam <- 0.01
method <- "SDAD"
q <- 2
control <- list(PGsteps = 100,
    PGtol = c(1e-5,1e-5),
    mu = 1,
    maxits = 100,
    tol = 1e-3,
    quiet = FALSE)
# Run the algorithm
res <- ASDA(Xt = Xtrain,
    Yt = Ytrain,
    Om = Om,
    gam = gam ,
    lam = lam,
    q = q,
    method = method,
    control = control)
# Do the predictions on the test set
preds <- predict(object = res, newdata = Xtest)
```

predict.ordASDA Predict method for ordinal sparse discriminant analysis

## Description

Predicted values based on fit from the function ordASDA. This function is used to classify new observations based on their explanatory variables/features. There is no need to normalize the data, the data is normalized based on the normalization data from the ordASDA object.

## Usage

\#\# S3 method for class 'ordASDA'
predict(object, newdata $=$ NULL, $\ldots$ )

## Arguments

object Object of class ordASDA. This object is returned from the function ordASDA.
newdata A matrix of new observations to classify.
... Arguments passed to predict.lda.

## Value

A vector of predictions.

## See Also

ordASDA

## Examples

```
    set.seed(123)
    # You can play around with these values to generate some 2D data to test one
    numClasses <- 5
    sigma <- matrix(c(1,-0.2,-0.2,1),2,2)
    mu <- c(0,0)
    numObsPerClass <- 5
    # Generate the data, can access with train$X and train$Y
    train <- accSDA::genDat(numClasses,numObsPerClass,mu, sigma)
    test <- accSDA::genDat(numClasses,numObsPerClass*2,mu,sigma)
    # Visualize it, only using the first variable gives very good separation
    plot(train$X[,1],train$X[,2],col = factor(train$Y),asp=1,main="Training Data")
    # Train the ordinal based model
    res <- accSDA::ordASDA(train$X, train$Y,s=2,h=1, gam=1e-6, lam=1e-3)
    vals <- predict(object = res,newdata = test$X) # Takes a while to run ~ 10 seconds
    sum(vals==test$Y)/length(vals) # Get accuracy on test set
    #plot(test$X[,1],test$X[,2],col = factor(test$Y),asp=1,
    # main="Test Data with correct labels")
    #plot(test$X[,1],test$X[,2],col = factor(vals),asp=1,
    # main="Test Data with predictions from ordinal classifier")
```

    print.ASDA Print method for ASDA object
    
## Description

Prints a summary of the output from the ASDA function. The output summarizes the discriminant analysis in human readable format.

## Usage

\#\# S3 method for class 'ASDA'
print(x, digits $=\max (3$, getOption("digits") - 3), numshow = 5, ...)

## Arguments

X
digits
numshow Number of best ranked variables w.r.t. to their absolute coefficients.
... arguments passed to or from other methods.

## Value

An invisible copy of $x$.

## See Also

ASDA, predict. ASDA and SDAD

## Examples

\# Prepare training and test set
train <- c (1:40,51:90,101:140)
Xtrain <- iris[train, 1:4]
nX <- normalize(Xtrain)
Xtrain <- nX\$Xc
Ytrain <- iris[train,5]
Xtest <- iris[-train, 1:4]
Xtest <- normalizetest(Xtest, nX)
Ytest <- iris[-train,5]
\# Run the algorithm
resDef <- ASDA(Xtrain, Ytrain)
\# Print
print(resDef)

SZVD Sparse Zero Variance Discriminant Analysis

## Description

Applies SZVD heuristic for sparse zero-variance discriminant analysis to given training set.

```
Usage
SZVD(train, ...)
## Default S3 method:
SZVD(
    train,
    gamma,
    D,
    penalty = TRUE,
    scaling = TRUE,
    tol = list(abs = 1e-04, rel = 1e-04),
    maxits = 2000,
    beta = 1,
    quiet = TRUE
)
```


## Arguments

| train | Data matrix where first column is the response class. |
| :--- | :--- |
| $\ldots$ | Parameters passed to SZVD.default. |
| gamma | Set of regularization parameters controlling l1-penalty. |
| D | dictionary/basis matrix. |
| penalty | Controls whether to apply reweighting of 11-penalty (using sigma = within-class <br> std devs). |
| scaling | Logical indicating whether to scale data such that each feature has variance 1. <br> tol |
| maxits | Stopping tolerances for ADMM algorithm, must include tol\$rel and tol\$abs. <br> beta |
| quiet | penalty term controlling the splitting constraint. |
|  | Print intermediate outpur or not. |

## Details

This function will currently solve as a standalone function in accSDA for time comparison. A wrapper function like ASDA will be created to use the functionality of plots and such. Maybe call it ASZDA. For that purpose the individual ZVD function will need to be implemented.

## Value

SZVD returns an object of class "SZVD" including a list with the following named components:
DVs Discriminant vectors.
its Number of iterations required to find DVs.
pen_scal Weights used in reweighted 11-penalty.
$N$ Basis for the null-space of the sample within-class covariance.
means Training class-means.
mus Training meand and variance scaling/centering terms.
w0 unpenalized zero-variance discriminants (initial solutions) plus B and W, etc.
NULL

## See Also

Used by: SZVDcv.

## Examples

```
    set.seed(123)
    P <- 300 # Number of variables
    N <- 50 # Number of samples per class
    # Mean for classes, they are zero everywhere except the first 3 coordinates
    m1 <- rep(0,P)
```

```
m1[1] <- 3
m2 <- rep(0,P)
m2[2] <- 3
m3 <- rep(0,P)
m3[3] <- 3
# Sample dummy data
Xtrain <- rbind(MASS::mvrnorm(n=N,mu = m1, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m2, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m3, Sigma = diag(P)))
# Generate the labels
Ytrain <- rep(1:3,each=N)
# Normalize the data
Xt <- accSDA::normalize(Xtrain)
Xtrain <- Xt$Xc
# Train the classifier and increase the sparsity parameter from the default
# so we penalize more for non-sparse solutions.
res <- accSDA::SZVD(cbind(Ytrain,Xtrain),beta=2.5,
    maxits=1000,tol = list(abs = 1e-04, rel = 1e-04))
```

    SZVDcv
    
## Description

Applies alternating direction methods of multipliers to solve sparse zero variance discriminant analysis.

## Usage

```
SZVDcv(Atrain, ...)
```

\#\# Default S3 method:
SZVDcv(
Atrain,
Aval,
k,
num_gammas,
g_mults,
D,
sparsity_pen,
scaling,
penalty,

```
    beta,
    tol,
    ztol,
    maxits,
    quiet
)
```


## Arguments

| Atrain | Training data set. |
| :---: | :---: |
|  | Parameters passed to SZVD.default. |
| Aval | Validation set. |
| k | Number of classes within training and validation sets. |
| num_gammas | Number of gammas to train on. |
| g_mults | Parameters defining range of gammas to train, $\mathrm{g}_{-} \max *\left(\mathrm{c} \_\min , \mathrm{c} \_\max \right)$. Note that it is an array/vector with two elements. |
| D <br> sparsity_pen | Penalty dictionary basis matrix. weight defining validation criteria as weighted sum of misclassification error and cardinality of discriminant vectors. |
| scaling | Whether to rescale data so each feature has variance 1. |
| penalty | Controls whether to apply reweighting of 11-penalty (using sigma $=$ within-class std devs) |
| beta | Parameter for augmented Lagrangian term in the ADMM algorithm. |
| tol | Stopping tolerances for the ADMM algorithm, must have tol\$rel and tol\$abs. |
| ztol | Threshold for truncating values in DVs to zero. |
| maxits | Maximum number of iterations used in the ADMM algorithm. |
| quiet | Controls display of intermediate results. |

## Details

This function might require a wrapper similar to ASDA.

## Value

SZVDcv returns an object of class "SZVDcv" including a list with the following named components:
DVs Discriminant vectors for the best choice of gamma.
all_DVs Discriminant vectors for all choices of gamma.
10_DVs Discriminant vectors for gamma minimizing cardinality.
mc_DVs Discriminant vector minimizing misclassification.
gamma Choice of gamma minimizing validation criterion.
gammas Set of all gammas trained on.
max_g Maximum value of gamma guaranteed to yield a nontrivial solution.
ind Index of best gamma.
w0 unpenalized zero-variance discriminants (initial solutions) plus B and W, etc. from ZVD
NULL

## See Also

Non CV version: SZVD.

## Examples

```
P <- 300 # Number of variables
N <- 50 # Number of samples per class
# Mean for classes, they are zero everywhere except the first 3 coordinates
m1 <- rep(0,P)
m1[1] <- 3
m2 <- rep(0,P)
m2[2] <- 3
m3 <- rep(0,P)
m3[3] <- 3
# Sample dummy data
Xtrain <- rbind(MASS::mvrnorm(n=N,mu = m1, Sigma = diag(P)),
                                    MASS::mvrnorm(n=N,mu = m2, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m3, Sigma = diag(P)))
Xval <- rbind(MASS::mvrnorm(n=N,mu = m1, Sigma = diag(P)),
                                    MASS::mvrnorm(n=N,mu = m2, Sigma = diag(P)),
                                    MASS::mvrnorm(n=N,mu = m3, Sigma = diag(P)))
# Generate the labels
Ytrain <- rep(1:3,each=N)
Yval <- rep(1:3,each=N)
# Train the classifier and increase the sparsity parameter from the default
# so we penalize more for non-sparse solutions.
res <- accSDA::SZVDcv(cbind(Ytrain,Xtrain),cbind(Yval,Xval),num_gammas=4,
    g_mults = c(0,1),beta=2.5,
    D=diag(P), maxits=100,tol=list(abs=1e-3,rel=1e-3), k = 3,
    ztol=1e-4,sparsity_pen=0.3,quiet=FALSE,penalty=TRUE,scaling=TRUE)
```

    SZVD_kFold_cv
    Cross-validation of sparse zero variance discriminant analysis
    
## Description

Applies alternating direction methods of multipliers to solve sparse zero variance discriminant analysis.

## Usage

```
SZVD_kFold_cv(X, ...)
    ## Default S3 method:
    SZVD_kFold_cv(
        X,
        Y,
        folds,
        gams,
        beta,
        D,
        q,
        maxits,
        tol,
        ztol,
        feat,
        penalty,
        quiet
    )
```


## Arguments

X
... Parameters passed to SZVD.default.
$Y \quad n$ by K indicator matrix.
folds number of folds to use in K-fold cross-validation.
gams Number of regularly spaced regularization parameters to try in [0,1]*max_gamma. See details for how max_gamma is computed in the function.
beta Augmented Lagrangian parameter. Must be greater than zero.
D Penalty dictionary basis matrix.
$\mathrm{q} \quad$ Desired number of discriminant vectors.
maxits Number of iterations to run ADMM algorithm.
tol Stopping tolerances for ADMM, must have tol\$rel and tol\$abs.
ztol Rounding tolerance for truncating entries to 0 .
feat Maximum fraction of nonzero features desired in validation scheme.
penalty $\quad$ Controls whether to apply reweighting of 11-penalty (using sigma $=$ within-class std devs)
quiet toggles between displaying intermediate statistics.

## Details

Add how max_gamma is calculated from the ZVD solution. This function might require a wrapper similar to ASDA.

## Value

SZVDcv returns an object of class "SZVDcv" including a list with the named components DVs and gambest. Where DVs are the discriminant vectors for the best 11 regularization parameter and gambest is the best regularization parameter found in the cross-validation.

NULL

## See Also

Used by: SZVDcv.

## Examples

```
P <- 150 # Number of variables
N <- 20 # Number of samples per class
# Mean for classes, they are zero everywhere except the first 3 coordinates
m1 <- rep(0,P)
m1[1] <- 3
m2 <- rep(0,P)
m2[2] <- 3
m3 <- rep(0,P)
m3[3] <- 3
# Sample dummy data
Xtrain <- rbind(MASS::mvrnorm(n=N,mu = m1, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m2, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m3, Sigma = diag(P)))
# Generate the labels
Ytrain <- cbind(c(rep (1,N),rep (0, 2*N)),
    c(rep(0,N),rep(1,N),rep (0,N)),
    c(rep(0,2*N),rep(1,N)))
# Normalize the data
Xt <- accSDA::normalize(Xtrain)
Xtrain <- Xt$Xc
# Train the classifier and increase the sparsity parameter from the default
# so we penalize more for non-sparse solutions.
res <- accSDA::SZVD_kFold_cv(Xtrain,Ytrain,folds=2,gams=2,beta=2.5,q=1, D=diag(P),
maxits=50,tol=list(abs=1e-2,rel=1e-2),
ztol=1e-3,feat=0.3,quiet=FALSE,penalty=TRUE)
```


## Description

Implements the ZVD algorithm to solve dicriminant vectors.

## Usage

ZVD (A, ...)
\#\# Default S3 method:
ZVD (A, scaling = FALSE, get_DVs = FALSE)

## Arguments

A
... Parameters passed to ZVD.default.
scaling Logical whether to rescale data so each feature has variance 1.
get_DVs Logical whether to obtain unpenalized zero-variance discriminant vectors.

## Details

This function should potentially be made internal for the release.

## Value

SZVDcv returns an object of class "ZVD" including a list with the following named components:
dvs discriminant vectors (optional).
B sample between-class covariance.
W sample within-class covariance.
$N$ basis for the null space of the sample within-class covariance.
mu training mean and variance scaling/centering terms
means vectors of sample class-means.
$k$ number of classes in given data set.
labels list of classes.
obs matrix of data observations.
class_obs Matrices of observations of each class.
NULL

See Also
Used by: SZVDcv.

## Examples

```
# Generate Gaussian data on three classes with bunch of redundant variables
P <- 300 # Number of variables
N <- 50 # Number of samples per class
# Mean for classes, they are zero everywhere except the first 3 coordinates
m1 <- rep(0,P)
m1[1] <- 3
m2 <- rep(0,P)
m2[2] <- 3
m3 <- rep(0,P)
m3[3] <- 3
# Sample dummy data
Xtrain <- rbind(MASS::mvrnorm(n=N,mu = m1, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m2, Sigma = diag(P)),
    MASS::mvrnorm(n=N,mu = m3, Sigma = diag(P)))
# Generate the labels
Ytrain <- rep(1:3,each=N)
# Normalize the data
Xt <- accSDA::normalize(Xtrain)
Xtrain <- Xt$Xc
# Train the classifier and increase the sparsity parameter from the default
# so we penalize more for non-sparse solutions.
res <- accSDA::ZVD(cbind(Ytrain,Xtrain))
```


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