Package 'alphashape3d'

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Title Implementation of the 3D Alpha-Shape for the Reconstruction of
3D Sets from a Point Cloud

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Depends geometry, rgl

Suggests alphahull

Description Implementation in R of the alpha-shape of a finite set of points in the three-dimensional space. The alpha-shape generalizes the convex hull and allows to recover the shape of non-convex and even non-connected sets in 3D, given a random sample of points taken into it. Besides the computation of the alpha-shape, this package provides users with functions to compute the volume of the alpha-shape, identify the connected components and facilitate the three-dimensional graphical visualization of the estimated set.

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R topics documented:

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ashape3d   *3D α-shape computation*

Description

This function calculates the 3D α-shape of a given sample of points in the three-dimensional space for $\alpha > 0$.

Usage

```
ashape3d(x, alpha, pert = FALSE, eps = 1e-09)
```

Arguments

x	A 3-column matrix with the coordinates of the input points. Alternatively, an object of class "ashape3d" can be provided, see Details.
alpha	A single value or vector of values for α.
pert	Logical. If the input points are not in general position and pert it set to TRUE the observations are perturbed by adding random noise, see Details.
eps	Scaling factor used for data perturbation when the input points are not in general position, see Details.

Details

If x is an object of class "ashape3d", then ashape3d does not recompute the 3D Delaunay triangulation (it reduces the computational cost).

If the input points x are not in general position and pert is set to TRUE, the function adds random noise to the data. The noise is generated from a normal distribution with mean zero and standard deviation eps*sd(x).

Value

An object of class "ashape3d" with the following components (see Edelsbrunner and Mucke (1994) for notation):

tetra	For each tetrahedron of the 3D Delaunay triangulation, the matrix tetra stores the indices of the sample points defining the tetrahedron (columns 1 to 4), a value that defines the intervals for which the tetrahedron belongs to the α-complex (column 5) and for each α a value (1 or 0) indicating whether the tetrahedron belongs to the α-shape (columns 6 to last).
triang	For each triangle of the 3D Delaunay triangulation, the matrix triang stores the indices of the sample points defining the triangle (columns 1 to 3), a value (1 or 0) indicating whether the triangle is on the convex hull (column 4), a value (1 or 0) indicating whether the triangle is attached or unattached (column 5), values that define the intervals for which the triangle belongs to the α-complex (columns 6 to 8) and for each α a value (0, 1, 2 or 3) indicating, respectively, that

the triangle is not in the α-shape or it is interior, regular or singular (columns 9 to last). As defined in Edelsbrunner and Mucke (1994), a simplex in the α-complex is interior if it does not belong to the boundary of the α-shape. A simplex in the α-complex is regular if it is part of the boundary of the α-shape and bounds some higher-dimensional simplex in the α-complex. A simplex in the α-complex is singular if it is part of the boundary of the α-shape but does not bounds any higher-dimensional simplex in the α-complex.

edge
For each edge of the 3D Delaunay triangulation, the matrix edge stores the indices of the sample points defining the edge (columns 1 and 2), a value (1 or 0) indicating whether the edge is on the convex hull (column 3), a value (1 or 0) indicating whether the edge is attached or unattached (column 4), values that define the intervals for which the edge belongs to the α-complex (columns 5 to 7) and for each α a value (0, 1, 2 or 3) indicating, respectively, that the edge is not in the α-shape or it is interior, regular or singular (columns 8 to last).

vertex
For each sample point, the matrix vertex stores the index of the point (column 1), a value (1 or 0) indicating whether the point is on the convex hull (column 2), values that define the intervals for which the point belongs to the α-complex (columns 3 and 4) and for each α a value (1, 2 or 3) indicating, respectively, if the point is interior, regular or singular (columns 5 to last).

x
A 3-column matrix with the coordinates of the original sample points.

alpha
A single value or vector of values of α.

xpert
A 3-column matrix with the coordinates of the perturbated sample points (only when the input points are not in general position and pert is set to TRUE).

References

Edelsbrunner, H., Mucke, E. P. (1994). Three-Dimensional Alpha Shapes. *ACM Transactions on Graphics*, 13(1), pp.43-72.

Examples

```
T1 <- rtorus(1000, 0.5, 2)
T2 <- rtorus(1000, 0.5, 2, ct = c(2, 0, 0), rotx = pi/2)
x <- rbind(T1, T2)
# Value of alpha
alpha <- 0.25
# 3D alpha-shape
ashape3d.obj <- ashape3d(x, alpha = alpha)
plot(ashape3d.obj)

# For new values of alpha, we can use ashape3d.obj as input (faster)
alpha <- c(0.15, 1)
ashape3d.obj <- ashape3d(ashape3d.obj, alpha = alpha)
plot(ashape3d.obj, indexAlpha = 2:3)
```

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components_ashape3d   *Connected subsets computation*

Description

This function calculates and clusters the different connected components of the α-shape of a given sample of points in the three-dimensional space.

Usage

```
components_ashape3d(as3d, indexAlpha = 1)
```

Arguments

as3d   An object of class "ashape3d" that represents the α-shape of a given sample of points in the three-dimensional space, see ashape3d.

indexAlpha   A single value or vector with the indexes of as3d$alpha that should be used for the computation, see Details.

Details

The function components_ashape3d computes the connected components of the α-shape for each value of α in as3d$alpha[indexAlpha] when indexAlpha is numeric.

If indexAlpha="all" or indexAlpha="ALL" then the function computes the connected components of the α-shape for all values of α in as3d$alpha.

Value

If indexAlpha is a single value then the function returns a vector v of length equal to the sample size. For each sample point i, v[i] represents the label of the connected component to which the point belongs (for isolated points, v[i]=-1). The labels of the connected components are ordered by size where the largest one (in number of vertices) gets the smallest label which is one.

Otherwise components_ashape3d returns a list of vectors describing the connected components of the α-shape for each selected value of α.

See Also

ashape3d, plot.ashape3d

Examples

```
T1 <- rtorus(1000, 0.5, 2)
T2 <- rtorus(1000, 0.5, 2, ct = c(2, 0, 0), rotx = pi/2)
x <- rbind(T1, T2)
alpha <- c(0.25, 2)
ashape3d.obj <- ashape3d(x, alpha = alpha)
```

```
plot(ashape3d.obj, indexAlpha = "all")

# Connected components of the alpha-shape for both values of alpha
comp <- components_ashape3d(ashape3d.obj, indexAlpha = "all")
class(comp)
# Number of components and points in each component for alpha=0.25
table(comp[[1]])
# Number of components and points in each component for alpha=2
table(comp[[2]])

# Plot the connected components for alpha=0.25
plot(ashape3d.obj, byComponents = TRUE, indexAlpha = 1)
```

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inashape3d   *Test of the inside of an α-shape*

Description

This function checks whether points are inside an α-shape.

Usage

```
inashape3d(as3d, indexAlpha = 1, points)
```

Arguments

as3d	An object of class "ashape3d" that represents the α-shape of a given sample of points in the three-dimensional space, see ashape3d.
indexAlpha	A single value or vector with the indexes of as3d$alpha that should be used for the computation, see Details.
points	A 3-column matrix with the coordinates of the input points.

Details

The function inashape3d checks whether each point in points is inside the α-shape for each value of α in as3d$alpha[indexAlpha].

If indexAlpha="all" or indexAlpha="ALL" then the function checks whether each point in points is inside the α-shape for all values of α in as3d$alpha.

Value

If indexAlpha is a single value then the function returns a vector of boolean of length the number of input points. The element at position i is TRUE if the point in points[i,] is inside the α-shape.

Otherwise inashape3d returns a list of vectors of boolean values (each object in the list as described above).

See Also

ashape3d

Examples

```
T1 <- rtorus(2000, 0.5, 2)
T2 <- rtorus(2000, 0.5, 2, ct = c(2, 0, 0), rotx = pi/2)
x <- rbind(T1, T2)
ashape3d.obj <- ashape3d(x, alpha = 0.4)
# Random sample of points in a plane
points <- matrix(c(5*runif(10000) - 2.5, rep(0.01, 5000)), nc = 3)
in3d <- inashape3d(ashape3d.obj, points = points)
plot(ashape3d.obj, transparency = 0.2)
colors <- ifelse(in3d, "blue", "green")
rgl.points(points, col = colors)
```

---

plot.ashape3d   *Plot the α-shape in 3D*

Description

This function plots the α-shape in 3D using the package rgl.

Usage

```
## S3 method for class 'ashape3d'
plot(x, clear = TRUE, col = c(2, 2, 2),
  byComponents = FALSE, indexAlpha = 1, transparency = 1,
  walpha = FALSE, triangles = TRUE, edges = TRUE, vertices = TRUE, ...)
```

Arguments

x	An object of class "ashape3d" that represents the α-shape of a given sample of points in the three-dimensional space, see ashape3d.
clear	Logical, specifying whether the current rgl device should be cleared.
col	A vector of length three specifying the colors of the triangles, edges and vertices composing the α-shape, respectively.
byComponents	Logical, if TRUE the connected components of the α-shape are represented in different colors, see components_ashape3d.
indexAlpha	A single value or vector with the indexes of x$alpha that should be used for the computation, see Details.
transparency	The coefficient of transparency, from 0 (transparent) to 1 (opaque), used to plot the α-shape.
walpha	Logical, if TRUE the value of α is displayed in the rgl device.

triangles	Logical, if TRUE triangles are plotted.
edges	Logical, if TRUE edges are plotted.
vertices	Logical, if TRUE vertices are plotted.
...	Material properties. See `rgl.material` for details.

Details

The function `plot.ashape3d` opens a rgl device for each value of α in `x$alpha[indexAlpha]`. Device information is displayed in the console.

If `indexAlpha="all"` or `indexAlpha="ALL"` then the function represents the α-shape for all values of α in `as3d$alpha`.

See Also

`ashape3d`, `components_ashape3d`

Examples

```
T1 <- rtorus(1000, 0.5, 2)
T2 <- rtorus(1000, 0.5, 2, ct = c(2, 0, 0), rotx = pi/2)
x <- rbind(T1, T2)
alpha <- c(0.15, 0.25, 1)
ashape3d.obj <- ashape3d(x, alpha = alpha)

# Plot the alpha-shape for all values of alpha
plot(ashape3d.obj, indexAlpha = "all")

# Plot the connected components of the alpha-shape for alpha=0.25
plot(ashape3d.obj, byComponents = TRUE, indexAlpha = 2)
```

---

rtorus   *Generate points in the torus*

Description

This function generates n random points within the torus whose minor radius is r, major radius is R and center is ct.

Usage

```
rtorus(n, r, R, ct = c(0, 0, 0), rotx = NULL)
```

Arguments

n	Number of observations.
r	Minor radius (radius of the tube).
R	Major radius (distance from the center of the tube to the center of the torus).
ct	A vector with the coordinates of the center of the torus.
rotx	If not NULL, a rotation through an angle rotx (in radians) about the x-axis is performed.

Examples

```
T1 <- rtorus(2000, 0.5, 2.5)
rgl.bbox()
rgl.points(T1, col = 4)

T2 <- rtorus(2000, 0.5, 2.5, ct = c(2, 0, 0.5), rotx = pi/2)
rgl.points(T2, col = 2)
```

---

surfaceNormals	*Normal vectors computation*

Description

This function calculates the normal vectors of all the triangles which belong to the boundary of the α-shape.

Usage

```
surfaceNormals(x, indexAlpha = 1, display = FALSE, col = 3, scale = 1,
  ...)
```

Arguments

x	An object of class "ashape3d" that represents the α-shape of a given sample of points in the three-dimensional space, see [ashape3d]().
indexAlpha	A single value or vector with the indexes of as3d$alpha that should be used for the computation, see Details.
display	Logical, if TRUE, surfaceNormals open a new rgl device and display the related α-shape and its normals vectors.
col	Color of the normal vectors.
scale	Scale parameter to control the length of the surface normals, only affect display.
...	Material properties. See [rgl.material]() for details.

Details

The function `surfaceNormals` computes the normal vectors of all the triangles which belong to the boundary of the α-shape for each value of α in `x$alpha[indexAlpha]`. The magnitude of each vector is equal to the area of its associated triangle.

If `indexAlpha="all"` or `indexAlpha="ALL"` then the function computes the normal vectors for all values of α in `as3d$alpha`.

Value

If `indexAlpha` is a single value then the function returns an object of class `"normals"` with the following components:

normals   Three-column matrix with the euclidean coordinates of the normal vectors.

centers   Three-column matrix with the euclidean coordinates of the centers of the triangles that form the α-shape.

Otherwise `surfaceNormals` returns a list of class `"normals-List"` (each object in the list as described above).

See Also

ashape3d

Examples

```
x <- rtorus(1000, 0.5, 1)
alpha <- 0.25
ashape3d.obj <- ashape3d(x, alpha = alpha)
surfaceNormals(ashape3d.obj, display = TRUE)
```

---

volume_ashape3d   *Volume computation*

Description

This function calculates the volume of the α-shape of a given sample of points in the three-dimensional space.

Usage

```
volume_ashape3d(as3d, byComponents = FALSE, indexAlpha = 1)
```

Arguments

as3d	An object of class "ashape3d" that represents the α-shape of a given sample of points in the three-dimensional space, see ashape3d.
byComponents	Logical, if FALSE (default) volume_ashape3d computes the volume of the whole α-shape. If TRUE, volume_ashape3d computes the volume of each connected component of the α-shape separately.
indexAlpha	A single value or vector with the indexes of as3d$alpha that should be used for the computation, see Details.

Details

The function volume_ashape3d computes the volume of the α-shape for each value of α in as3d$alpha[indexAlpha] when indexAlpha is numeric.

If indexAlpha="all" or indexAlpha="ALL" then the function computes the volume of the α-shape for all values of α in as3d$alpha.

Value

If indexAlpha is a single value then the function returns the volume of the α-shape (if the argument byComponents is set to FALSE) or a vector with the volumes of each connected component of the α-shape (if the argument byComponents is set to TRUE).

Otherwise volume_ashape3d returns a list (each object in the list as described above).

See Also

ashape3d, components_ashape3d

Examples

```
C1 <- matrix(runif(6000), nc = 3)
C2 <- matrix(runif(6000), nc = 3) + 2
x <- rbind(C1, C2)
ashape3d.obj <- ashape3d(x, alpha = 0.75)
plot(ashape3d.obj, byComponents = TRUE)

# Compute the volume of the alpha-shape
volume_ashape3d(ashape3d.obj)
# Compute the volumes of the connected components of the alpha-shape
volume_ashape3d(ashape3d.obj, byComponents = TRUE)
```

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