

# Package ‘arfima’

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**Title** Fractional ARIMA (and Other Long Memory) Time Series Modeling

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**Author** JQ (Justin) Veenstra [aut, cre], A.I. McLeod [aut]

**Maintainer** JQ Veenstra <jqveenstra@gmail.com>

**Depends** R (>= 3.0.0), ltsa

**Imports** parallel

**Description** Simulates, fits, and predicts long-memory and anti-persistent time series, possibly mixed with ARMA, regression, transfer-function components.

Exact methods (MLE, forecasting, simulation) are used.

Bug reports should be done via GitHub (at

<<https://github.com/JQVeenstra/arfima>>), where the development version of this package lives; it can be installed using devtools.

**License** MIT + file LICENSE

**RoxygenNote** 7.1.2

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arfima-package	<i>Simulates, fits, and predicts persistent and anti-persistent time series. arfima</i>
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## Description

Simulates with `arfima.sim`, fits with `arfima`, and predicts with a method for the generic function. Plots predictions and the original time series. Has the capability to fit regressions with ARFIMA/ARIMA-FGN/ARIMA-PLA errors, as well as transfer functions/dynamic regression.

## Details

Package:	arfima
Type:	Package
Version:	1.4-0
Date:	2017-06-20
License:	MIT

A list of functions:

`arfima.sim` - Simulates an ARFIMA, ARIMA-FGN, or ARIMA-PLA (three classes of mixed ARIMA hyperbolic decay processes) process, with possible seasonal components.

`arfima` - Fits an ARIMA-HD (default single-start) model to a series, with options for regression with ARIMA-HD errors and dynamic regression (transfer functions). Allows for fixed parameters as well as choices for the optimizer to be used.

`arfima0` - Simplified version of `arfima`

`weed` - Weeds out modes too close to each other in the same fit. The modes with the highest log-likelihoods are kept

`print.arfima` - Prints the relevant output of an `arfima` fitted object, such as parameter estimates, standard errors, etc.

`summary.arfima` - A much more detailed version of `print.arfima`

`coef.arfima` - Extracts the coefficients from a `arfima` object

`vcov.arfima` - Theoretical and observed covariance matrices of the coefficients

`residuals.arfima` - Extracts the residuals or regression residuals from a `arfima` object

`fitted.arfima` - Extracts the fitted values from a `arfima` object

`tacvfARFIMA` - Computes the theoretical autocovariance function of a supplied model. The model is checked for stationarity and invertibility.

`iARFIMA` - Computes the Fisher information matrix of all non-FGN components of the given model. Can be computed (almost) exactly or through a psi-weights approximation. The approximation takes more time.

`IdentInvertQ` - Checks whether the model is identifiable, stationary, and invertible. Identifiability is checked through the information matrix of all non-FGN components, as well as whether both types of fractional noise are present, both seasonally and non-seasonally.

`lARFIMA` and `lARFIMAwTF` - Computes the log-likelihood of a given model with a given series. The second admits transfer function data.

`predict.arfima` - Predicts from an `arfima` object. Capable of exact minimum mean squared error predictions even with integer  $d > 0$  and/or integer  $d_{seas} > 0$ . Does not include transfer function/leading indicators as of yet. Returns a `predarfima` object, which is composed of: predictions, and standard errors (exact and, if possible, limiting).

`print.predarfima` - Prints the relevant output from a `predarfima` object: the predictions and their standard deviations.

`plot.predarfima` - Plots a `predarfima` object. This includes the original time series, the forecasts and as default the standard 95% prediction intervals (exact and, if available, limiting).

`logLik.arfima`, `AIC.arfima`, `BIC.arfima` - Extracts the requested values from an `arfima` object

`distance` - Calculates the distances between the modes

`removeMode` - Removes a mode from a fit

`tacvf` - Calculates the theoretical autocovariance functions (`tacvfs`) from a fitted `arfima` object

`plot.tacvf` - Plots the `tacvfs`

`print.tacvf` - Prints the `tacvfs`

[tacfplot](#) - Plots the theoretical autocorrelation functions (tacfs) of different models on the same data

[SeriesJ](#), [tmpyr](#) - Two datasets included with the package

### Author(s)

JQ (Justin) Veenstra, A. I. McLeod

Maintainer: JQ (Justin) Veenstra <jqveenstra@gmail.com>

### References

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

### Examples

```
set.seed(8564)
sim <- arfima.sim(1000, model = list(phi = c(0.2, 0.1), dfrac = 0.4, theta = 0.9))
fit <- arfima(sim, order = c(2, 0, 1), back=TRUE)

fit

data(tmpyr)

fit1 <- arfima(tmpyr, order = c(1, 0, 1), numeach = c(3, 3), dmean = FALSE)
fit1

plot(tacvf(fit1), maxlag = 30, tacf = TRUE)

fit2 <- arfima(tmpyr, order = c(1, 0, 0), numeach = c(3, 3), autoweed = FALSE,
dmean = FALSE)

fit2

fit2 <- weed(fit2)

fit2

tacfplot(fits = list(fit1, fit2))

fit3 <- removeMode(fit2, 2)

fit3

coef(fit2)
vcov(fit2)

fit1fgn <- arfima(tmpyr, order = c(1, 0, 1), numeach = c(3, 3),
dmean = FALSE, lmodel = "g")
fit1fgn

fit1hd <- arfima(tmpyr, order = c(1, 0, 1), numeach = c(3, 3),
```

```

dmean = FALSE, lmodel = "h")
fit1hd

data(SeriesJ)
attach(SeriesJ)

fitTF <- arfima(YJ, order= c(2, 0, 0), xreg = XJ, reglist =
list(regpar = c(1, 2, 3)), lmodel = "n", dmean = FALSE)
fitTF

detach(SeriesJ)

set.seed(4567)

sim <- arfima.sim(1000, model = list(phi = 0.3, dfrac = 0.4, dint = 1),
sigma2 = 9)

X <- matrix(rnorm(2000), ncol = 2)

simreg <- sim + crossprod(t(X), c(2, 3))

fitreg <- arfima(simreg, order = c(1, 1, 0), xreg = X)

fitreg

plot(sim)

lines(residuals(fitreg, reg = TRUE)[[1]], col = "blue")
##pretty much a perfect match.

```

---

AIC.arfima

*Information criteria for arfima objects*


---

### Description

Computes information criteria for arfima objects. See [AIC](#) for more details.

### Usage

```

## S3 method for class 'arfima'
AIC(object, ..., k = 2)

```

### Arguments

object	An object of class "arfima". Note these functions can only be called on one object at a time because of possible multimodality.
...	Other models fit to data for which to extract the AIC/BIC. Not recommended, as an arfima object can be multimodal.
k	The penalty term to be used. See <a href="#">AIC</a> .

**Value**

The information criteria for each mode in a vector.

**Author(s)**

JQ (Justin) Veenstra

**Examples**

```
set.seed(34577)
sim <- arfima.sim(500, model = list(theta = 0.9, phi = 0.5, dfrac = 0.4))
fit1 <- arfima(sim, order = c(1, 0, 1), cpus = 2, back=TRUE)
fit2 <- arfima(sim, order = c(1, 0, 1), cpus = 2, lmodel = "g", back=TRUE)
fit3 <- arfima(sim, order = c(1, 0, 1), cpus = 2, lmodel = "h", back=TRUE)

AIC(fit1)
AIC(fit2)
AIC(fit3)
```

---

arfima

*Fit ARFIMA, ARIMA-FGN, and ARIMA-PLA (multi-start) models. Fits ARFIMA/ARIMA-FGN/ARIMA-PLA multi-start models to times series data. Options include fixing parameters, whether or not to fit fractional noise, what type of fractional noise (fractional Gaussian noise (FGN), fractionally differenced white noise (FDWN), or the newly introduced power-law autocovariance noise (PLA)), etc. This function can fit regressions with ARFIMA/ARIMA-FGN/ARIMA-PLA errors via the xreg argument, including dynamic regression (transfer functions).*

---

**Description**

Fits by direct optimization using optim. The optimizer choices are: 0 - BFGS; 1 - Nelder-Mead; 2 - SANN; otherwise CG.

**Usage**

```
arfima(
  z,
  order = c(0, 0, 0),
  numeach = c(1, 1),
  dmean = TRUE,
  whichopt = 0,
  itmean = FALSE,
  fixed = list(phi = NA, theta = NA, frac = NA, seasonal = list(phi = NA, theta = NA,
    frac = NA), reg = NA),
```

```

lmodel = c("d", "g", "h", "n"),
seasonal = list(order = c(0, 0, 0), period = NA, lmodel = c("d", "g", "h", "n"),
  numeach = c(1, 1)),
useC = 3,
cpus = 1,
rand = FALSE,
numrand = NULL,
seed = NA,
eps3 = 0.01,
xreg = NULL,
reglist = list(regpar = NA, minn = -10, maxx = 10, numeach = 1),
check = F,
autoweed = TRUE,
weedepts = 0.01,
adapt = TRUE,
weedtype = c("A", "P", "B"),
weedp = 2,
quiet = FALSE,
startfit = NULL,
back = FALSE
)

```

### Arguments

z	The data set (time series)
order	The order of the ARIMA model to be fit: $c(p, d, q)$ . We have that $p$ is the number of AR parameters ( $\phi$ ), $d$ is the amount of integer differencing, and $q$ is the number of MA parameters ( $\theta$ ). Note we use the Box-Jenkins convention for the MA parameters, in that they are the negative of <code>arima</code> : see "Details".
numeach	The number of starts to fit for each parameter. The first argument in the vector is the number of starts for each AR/MA parameter, while the second is the number of starts for the fractional parameter. When this is set to 0, no fractional noise is fit. Note that the number of starts in total is multiplicative: if we are fitting an ARFIMA(2, d, 2), and use the older number of starts ( $c(2, 2)$ ), we will have $2^2 * 2 * 2^2 = 32$ starting values for the fits. <b>Note that the default has changed from <math>c(2, 2)</math> to <math>c(1, 1)</math> since package version 1.4-0</b>
dmean	Whether the mean should be fit dynamically with the optimizer. Note that the likelihood surface will change if this is TRUE, but this is usually not worrisome. See the referenced thesis for details.
whichopt	Which optimizer to use in the optimization: see "Details".
itmean	This option is under investigation, and will be set to FALSE automatically until it has been decided what to do. Whether the mean should be fit iteratively using the function <code>TrenchMean</code> . Currently <code>itmean</code> , if set to TRUE, has higher priority than <code>dmean</code> : if both are TRUE, <code>dmean</code> will be set to FALSE, with a warning.
fixed	A list of parameters to be fixed. If we are to fix certain elements of the AR process, for example, <code>fixed\$phi</code> must have length equal to $p$ . Any numeric value

will fix the parameter at that value; for example, if we are modelling an AR(2) process, and we wish to fix only the first autoregressive parameter to 0, we would have `fixed = list(phi = c(0, NA))`. NA corresponds to that parameter being allowed to change in the optimization process. We can fix the fractional parameters, and unlike `arima`, can fix the seasonal parameters as well. Currently, fixing regression/transfer function parameters is disabled.

<code>lmodel</code>	The long memory model (noise type) to be used: "d" for FDWN, "g" for FGN, "h" for PLA, and "n" for none (i.e. ARMA short memory models). Default is "d".
<code>seasonal</code>	The seasonal components of the model we wish to fit, with the same components as above. The period must be supplied.
<code>useC</code>	How much interfaced C code to use: an integer between 0 and 3. The value 3 is strongly recommended. See "Details".
<code>cpus</code>	The number of CPUs used to perform the multi-start fits. A small number of fits and a high number of cpus (say both equal 4) with n not large can actually be slower than when cpus = 1. The number of CPUs should not exceed the number of threads available to R.
<code>rand</code>	Whether random starts are used in the multistart method. Defaults to FALSE.
<code>numrand</code>	The number of random starts to use.
<code>seed</code>	The seed for the random starts.
<code>eps3</code>	How far to start from the boundaries when using a grid for the multi-starts (i.e. when rand is FALSE.)
<code>xreg</code>	A matrix, data frame, or vector of regressors for regression or transfer functions.
<code>reglist</code>	A list with the following elements: <ul style="list-style-type: none"> <li><code>regpar</code> - either NA or a list, matrix, data frame, or vector with 3 columns. If regpar is a vector, the matrix xreg must have one row or column only. In order, the elements of regpar are: r, s, and b. The values of r are the orders of the delta parameters as in Box, Jenkins and Reinsel, the values of s are the orders of omega parameters, and the values of b are the backshifting to be done.</li> <li><code>minn</code> - the minimum value for the starting value of the search, if <code>reglist\$numeach &gt; 1</code>.</li> <li><code>maxx</code> - the maximum value for the starting value of the search, if <code>reglist\$numeach &gt; 1</code>.</li> <li><code>numeach</code> - the number of starts to try for each regression parameter.</li> </ul>
<code>check</code>	If TRUE, checks at each optim iteration whether the model is identifiable. This makes the optimization much slower.
<code>autoweed</code>	Whether to automatically (before the fit is returned) weed out modes found that are found that are close together (usually the same point.)
<code>weedeps</code>	The maximum distance between modes that are close together for the mode with the lower log-likelihood to be weeded out. If <code>adapt</code> is TRUE (default) this value changes.
<code>adapt</code>	If TRUE, if dim is the dimensionality of the search, <code>weedeps</code> is changed to $(1 + weedeps)^{dim} - 1$ .



weedtype	The type of weeding to be done. See <a href="#">weed</a> .
weedp	The p in the p-norm to be used in the weeding. p = 2 (default) is Euclidean distance.
quiet	If TRUE, no auxiliary output is generated. The default (FALSE) has information of fits being performed.
startfit	Meant primarily for debugging (for now), allows starting places for the fitting process. Overrides numeach.
back	Setting this to true will restore the defaults in numeach.

### Details

A word of warning: it is generally better to use the default, and only use Nelder-Mead to check for spurious modes. SANN takes a long time (and may only find one mode), and CG may not be stable.

If using Nelder-Mead, it must be stressed that Nelder-Mead can take out non-spurious modes or add spurious modes: we have checked visually where we could. Therefore it is wise to use BFGS as the default and if there are modes close to the boundaries, check using Nelder-Mead.

The moving average parameters are in the Box-Jenkins convention: they are the negative of the parameters given by [arima](#). That is, the model to be fit is, in the case of a non-seasonal ARIMA model,  $\phi(B) (1-B)^d z[t] = \theta(B) a[t]$ , where  $\phi(B) = 1 - \phi(1) B - \dots - \phi(p) B^p$  and  $\theta(B) = 1 - \theta(1) B - \dots - \theta(q) B^q$ .

For the useC parameter, a "0" means no C is used; a "1" means C is only used to compute the log-likelihood, but not the theoretical autocovariance function (tacvf); a "2" means that C is used to compute the tacvf and not the log-likelihood; and a "3" means C is used to compute everything.

### Value

An object of class "arfima". In it, full information on the fit is given, though not printed under the print.arfima method. The phis are the AR parameters, and the thetas are the MA parameters. Residuals, regression residuals, etc., are all available, along with the parameter values and standard errors. Note that the muHat returned in the arfima object is of the **differenced** series, if differencing is applied.

Note that if multiple modes are found, they are listed in order of log-likelihood value.

### Author(s)

JQ (Justin) Veenstra

### References

McLeod, A. I., Yu, H. and Krougly, Z. L. (2007) Algorithms for Linear Time Series Analysis: With R Package Journal of Statistical Software, Vol. 23, Issue 5

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

P. Borwein (1995) An efficient algorithm for Riemann Zeta function Canadian Math. Soc. Conf. Proc., 27, pp. 29-34.

**See Also**

[arfima.sim](#), [SeriesJ](#), [arfima-package](#)

**Examples**

```

set.seed(8564)
sim <- arfima.sim(1000, model = list(phi = c(0.2, 0.1),
dfrac = 0.4, theta = 0.9))
fit <- arfima(sim, order = c(2, 0, 1), back=TRUE)

fit

data(tmpyr)

fit <- arfima(tmpyr, order = c(1, 0, 1), numeach = c(3, 3))
fit

plot(tacvf(fit), maxlag = 30, tacf = TRUE)

data(SeriesJ)
attach(SeriesJ)

fitTF <- arfima(YJ, order= c(2, 0, 0), xreg = XJ, reglist =
list(regpar = c(2, 2, 3)), lmodel = "n")
fitTF

detach(SeriesJ)

```

---

arfima.sim

---

*Simulate an ARFIMA time series.*


---

**Description**

This function simulates an long memory ARIMA time series, with one of fractionally differenced white noise (FDWN), fractional Gaussian noise (FGN), power-law autocovariance (PLA) noise, or short memory noise and possibly seasonal effects.

**Usage**

```

arfima.sim(
  n,
  model = list(phi = numeric(0), theta = numeric(0), dint = 0, dfrac = numeric(0), H =
    numeric(0), alpha = numeric(0), seasonal = list(phi = numeric(0), theta = numeric(0),
    dint = 0, period = numeric(0), dfrac = numeric(0), H = numeric(0), alpha =
    numeric(0))),

```

```

    useC = 3,
    sigma2 = 1,
    rand.gen = rnorm,
    muHat = 0,
    zinit = NULL,
    innov = NULL,
    ...
)

```

## Arguments

n	The number of points to be generated.
model	The model to be simulated from. The phi and theta arguments should be vectors with the values of the AR and MA parameters. Note that Box-Jenkins notation is used for the MA parameters: see the "Details" section of <a href="#">arfima</a> . The dint argument indicates how much differencing should be required to make the process stationary. The dfrac, H, and alpha arguments are FDWN, FGN and PLA values respectively; note that only one (or none) of these can have a value, or an error is returned. The seasonal argument is a list, with the same parameters, and a period, as the model argument. Note that with a seasonal model, we can have mixing of FDWN/FGN/HD noise: one in the non-seasonal part, and the other in the seasonal part.
useC	How much interfaced C code to use: an integer between 0 and 3. The value 3 is strongly recommended. See the "Details" section of <a href="#">arfima</a> .
sigma2	The desired variance for the innovations of the series.
rand.gen	The distribution of the innovations. Any distribution recognized by R is possible
muHat	The theoretical mean of the series before integration (if integer integration is done)
zinit	Used for prediction; not meant to be used directly. This allows a start of a time series to be specified before inverse differencing (integration) is applied.
innov	Used for prediction; not meant to be used directly. This allows for the use of given innovations instead of ones provided by rand.gen.
...	Other parameters passed to the random variate generator; currently not used.

## Details

A suitably defined stationary series is generated, and if either of the dints (non-seasonal or seasonal) are greater than zero, the series is integrated (inverse-differenced) with zinit equalling a suitable amount of 0s if not supplied. Then a suitable amount of points are taken out of the beginning of the series (i.e. dint + period \* seasonal dint = the length of zinit) to obtain a series of length n. The stationary series is generated by calculating the theoretical autocovariance function and using it, along with the innovations to generate a series as in McLeod et. al. (2007). *Note:* if you would like to fit a function from a fitted arfima model, the function `sim_from_fitted` can be used.

**Value**

A sample from a multivariate normal distribution that has a covariance structure defined by the autocovariances generated for given parameters. The sample acts like a time series with the given parameters.

**Author(s)**

JQ (Justin) Veenstra

**References**

McLeod, A. I., Yu, H. and Krougly, Z. L. (2007) Algorithms for Linear Time Series Analysis: With R Package Journal of Statistical Software, Vol. 23, Issue 5

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

P. Borwein (1995) An efficient algorithm for Riemann Zeta function Canadian Math. Soc. Conf. Proc., 27, pp. 29-34.

**See Also**

[arfima](#), [sim\\_from\\_fitted](#)

**Examples**

```
set.seed(6533)
sim <- arfima.sim(1000, model = list(phi = .2, dfrac = .3, dint = 2))

fit <- arfima(sim, order = c(1, 2, 0))
fit
```

---

arfima0

*Exact MLE for ARFIMA The time series is corrected for the sample mean and then exact MLE is used for the other parameters. This is a simplified version of the arfima() function that may be useful in simulations and bootstrapping.*

---

**Description**

The sample mean is asymptotically efficient.

**Usage**

```
arfima0(z, order = c(0, 0, 0), lmodel = c("FD", "FGN", "PLA", "NONE"))
```

**Arguments**

z	time series
order	(p,d,q) where p=order AR, d=regular difference, q=order MA
lmodel	type of long-memory component: FD, FGN, PLA or NONE

**Value**

list with components:

bHat	transformed optimal parameters
alphaHat	estimate of alpha
HHat	estimate of H
dHat	estimate of d
phiHat	estimate of phi
thetaHat	estimate of theta
wLL	optimized value of Whittle approximate log-likelihood
LL	corresponding exact log-likelihood
convergence	convergence indicator

**Author(s)**

JQ (Justin) Veenstra and A. I. McLeod

**Examples**

```
z <- rnorm(100)
arfima0(z, lmodel="FGN")
```

---

arfimachanges      *Prints changes to the package since the last update. Started in 1.4-0*

---

**Description**

Prints changes to the package since the last update. Started in 1.4-0

**Usage**

```
arfimachanges()
```

---

ARToPacf	<i>Converts AR/MA coefficients from operator space to the PACF space</i>
----------	--

---

**Description**

Converts AR/MA coefficients from operator space to the PACF box-space; usually for internal use

**Usage**

ARToPacf(phi)

**Arguments**

phi	The AR/MA coefficients in operator space
-----	--

**Value**

The AR/MA coefficients in the PACF space

**Author(s)**

A. I. McLeod

**References**

Barndorff-Nielsen O. E., Schou G. (1973). "On the parametrization of autoregressive models by partial autocorrelations." *Journal of Multivariate Analysis*, 3, 408-419

McLeod A. I., Zhang Y (2006). "Partial autocorrelation parameterization for subset autoregression." *Journal of Time Series Analysis*, 27(4), 599-612

---

bestModes	<i>Finds the best modes of an arfima fit.</i>
-----------	---

---

**Description**

Finds the best modes of an arfima fit with respect to log-likelihood.

**Usage**

bestModes(object, bestn)

**Arguments**

object	An object of class "arfima".
bestn	The top number of modes to keep with respect to the log-likelihood.

**Details**

This is the easiest way to remove modes with lower log-likelihoods.

**Value**

The bestn "best" modes.

**Author(s)**

JQ (Justin) Veenstra

**See Also**

[arfima](#)

**Examples**

```
set.seed(8765)
sim <- arfima.sim(1000, model = list(phi = 0.4, theta = 0.9, dfrac = 0.4))
fit <- arfima(sim, order = c(1, 0, 1), back=TRUE)
fit
fit <- bestModes(fit, 2)
fit
```

---

coef.arfima

*Extract Model Coefficients*


---

**Description**

Extracts the coefficients from a arfima fit.

**Usage**

```
## S3 method for class 'arfima'
coef(object, tpacf = FALSE, digits = max(4, getOption("digits") - 3), ...)
```

**Arguments**

object	A fitted arfima object.
tpacf	If TRUE, the (ARMA) coefficients are in the transformed PACF space.
digits	The number of digits to print
...	Other optional arguments. Currently not used.

**Value**

A matrix of coefficients. The rows are for the modes, and the columns are for the model variables.

**Author(s)**

JQ (Justin) Veenstra

**Examples**

```
set.seed(8564)
sim <- arfima.sim(1000, model = list(phi = c(0.2, 0.1), dfrac = 0.4, theta = 0.9))
fit <- arfima(sim, order = c(2, 0, 1), back=TRUE)

fit
coef(fit)
```

---

distance

*The distance between modes of an arfima fit.*

---

**Description**

The distance between modes of an arfima fit.

**Usage**

```
distance(ans, p = 2, digits = 4)
```

**Arguments**

ans	An object of class "arfima".
p	The p in the p-norm to be used.
digits	The number of digits to print.

**Value**

A list of two data frames: one with distances in operator space, the second with distances in the transformed (PACF) space.

**Author(s)**

JQ (Justin) Veensta

**References**

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)



## Examples

```
set.seed(8564)
sim <- arfima.sim(1000, model = list(phi = c(0.2, 0.1), dfrac = 0.4, theta = 0.9))
fit <- arfima(sim, order = c(2, 0, 1), back=TRUE)

fit

distance(fit)
```

---

fitted.arfima	<i>Extract Model Fitted Values</i>
---------------	------------------------------------

---

## Description

Extract fitted values from an arfima object.

## Usage

```
## S3 method for class 'arfima'
fitted(object, ...)
```

## Arguments

object	A arfima object.
...	Optional parameters. Currently not used.

## Value

A list of vectors of fitted values, one for each mode.

## Author(s)

JQ (Justin) Veenstra

## References

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

## See Also

[arfima](#), [resid.arfima](#)

## Examples

```

set.seed(8564)
sim <- arfima.sim(1000, model = list(phi = c(0.2, 0.1), dfrac = 0.4, theta = 0.9))
fit <- arfima(sim, order = c(2, 0, 1), back=TRUE)

fit

resid <- resid(fit)
par(mfrow = c(1, 3))
fitted <- fitted(fit)
plot(fitted[[1]], resid[[1]])
plot(fitted[[2]], resid[[2]])
plot(fitted[[3]], resid[[3]])
par(mfrow = c(1, 1))

```

---

iARFIMA

*The Fisher information matrix of an ARFIMA process*


---

## Description

Computes the approximate or (almost) exact Fisher information matrix of an ARFIMA process

## Usage

```

iARFIMA(
  phi = numeric(0),
  theta = numeric(0),
  phiseas = numeric(0),
  thetaseas = numeric(0),
  period = 0,
  dfrac = TRUE,
  dfs = FALSE,
  exact = TRUE
)

```

## Arguments

phi	The autoregressive parameters in vector form.
theta	The moving average parameters in vector form. See Details for differences from <a href="#">arma</a> .
phiseas	The seasonal autoregressive parameters in vector form.
thetaseas	The seasonal moving average parameters in vector form. See Details for differences from <a href="#">arma</a> .
period	The periodicity of the seasonal components. Must be $\geq 2$ .

dfrac	TRUE if we include the fractional d parameter, FALSE otherwise
dfs	TRUE if we include the seasonal fractional d parameter, FALSE otherwise
exact	If FALSE, calculate the approximate information matrix via psi-weights. Otherwise the (almost) exact information matrix will be calculated. See "Details".

### Details

The matrices are calculated as outlined in Veenstra and McLeod (2012), which draws on many references. The psi-weights approximation has a fixed maximum lag for the weights as 2048 (to be changed to be adaptable.) The fractional difference(s) by AR/MA components have a fixed maximum lag of 256, also to be changed. Thus the exact matrix has some approximation to it. Also note that the approximate method takes much longer than the "exact" one.

The moving average parameters are in the Box-Jenkins convention: they are the negative of the parameters given by [arima](#).

### Value

The information matrix of the model.

### Author(s)

JQ (Justin) Veenstra

### References

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

### See Also

[IdentInvertQ](#)

### Examples

```
tick <- proc.time()
exactI <- iARFIMA(phi = c(.4, -.2), theta = c(.7), phiseas = c(.8, -.4),
d = TRUE, dfs = TRUE, period = 12)
proc.time() - tick
tick <- proc.time()
approxI <- iARFIMA(phi = c(.4, -.2), theta = c(.7), phiseas = c(.8, -.4),
d = TRUE, dfs = TRUE, period = 12, exact = FALSE)
proc.time() - tick
exactI
max(abs(exactI - approxI))
```

---

IdentInvertQ	<i>Checks invertibility, stationarity, and identifiability of a given set of parameters</i>
--------------	---

---

### Description

Computes whether a given long memory model is invertible, stationary, and identifiable.

### Usage

```
IdentInvertQ(
  phi = numeric(0),
  theta = numeric(0),
  phiseas = numeric(0),
  thetaseas = numeric(0),
  dfrac = numeric(0),
  dfs = numeric(0),
  H = numeric(0),
  Hs = numeric(0),
  alpha = numeric(0),
  alphas = numeric(0),
  delta = numeric(0),
  period = 0,
  debug = FALSE,
  ident = TRUE
)
```

### Arguments

phi	The autoregressive parameters in vector form.
theta	The moving average parameters in vector form. See Details for differences from <a href="#">arima</a> .
phiseas	The seasonal autoregressive parameters in vector form.
thetaseas	The seasonal moving average parameters in vector form. See Details for differences from <a href="#">arima</a> .
dfrac	The fractional differencing parameter.
dfs	The seasonal fractional differencing parameter.
H	The Hurst parameter for fractional Gaussian noise (FGN). Should not be mixed with dfrac or alpha: see "Details".
Hs	The Hurst parameter for seasonal fractional Gaussian noise (FGN). Should not be mixed with dfs or alphas: see "Details".
alpha	The decay parameter for power-law autocovariance (PLA) noise. Should not be mixed with dfrac or H: see "Details".
alphas	The decay parameter for seasonal power-law autocovariance (PLA) noise. Should not be mixed with dfs or Hs: see "Details".

delta	The delta parameters for transfer functions.
period	The periodicity of the seasonal components. Must be $\geq 2$ .
debug	When TRUE and model is not stationary/invertible or identifiable, prints some helpful output.
ident	Whether to test for identifiability.

### Details

This function tests for identifiability via the information matrix of the ARFIMA process. Whether the process is stationary or invertible amounts to checking whether all the variables fall in correct ranges.

The moving average parameters are in the Box-Jenkins convention: they are the negative of the parameters given by [arima](#).

If  $dfrac/H/\alpha$  are mixed and/or  $dfs/Hs/\alpha$ s are mixed, an error will not be thrown, even though only one of these can drive the process at either level. Note also that the FGN or PLA have no impact on the identifiability of the model, as information matrices containing these parameters currently do not have known closed form. These two parameters must be within their correct ranges ( $0 < H < 1$  for FGN and  $0 < \alpha < 3$  for PLA.)

### Value

TRUE if the model is stationary, invertible and identifiable. FALSE otherwise.

### Author(s)

Justin Veenstra

### References

McLeod, A.I. (1999) Necessary and sufficient condition for nonsingular Fisher information matrix in ARMA and fractional ARMA models *The American Statistician* 53, 71-72.

Veenstra, J. and McLeod, A. I. (2012, Submitted) Improved Algorithms for Fitting Long Memory Models: With R Package

### See Also

[iARFIMA](#)

### Examples

```
IdentInvertQ(phi = 0.3, theta = 0.3)
IdentInvertQ(phi = 1.2)
```

IARFIMA

*Exact log-likelihood of a long memory model***Description**

Computes the exact log-likelihood of a long memory model with respect to a given time series.

**Usage**

```
IARFIMA(
  z,
  phi = numeric(0),
  theta = numeric(0),
  dfrac = numeric(0),
  phiseas = numeric(0),
  thetaseas = numeric(0),
  dfs = numeric(0),
  H = numeric(0),
  Hs = numeric(0),
  alpha = numeric(0),
  alphas = numeric(0),
  period = 0,
  useC = 3
)
```

**Arguments**

<code>z</code>	A vector or (univariate) time series object, assumed to be (weakly) stationary.
<code>phi</code>	The autoregressive parameters in vector form.
<code>theta</code>	The moving average parameters in vector form. See Details for differences from <a href="#">arima</a> .
<code>dfrac</code>	The fractional differencing parameter.
<code>phiseas</code>	The seasonal autoregressive parameters in vector form.
<code>thetaseas</code>	The seasonal moving average parameters in vector form. See Details for differences from <a href="#">arima</a> .
<code>dfs</code>	The seasonal fractional differencing parameter.
<code>H</code>	The Hurst parameter for fractional Gaussian noise (FGN). Should not be mixed with <code>dfrac</code> or <code>alpha</code> : see "Details".
<code>Hs</code>	The Hurst parameter for seasonal fractional Gaussian noise (FGN). Should not be mixed with <code>dfs</code> or <code>alphas</code> : see "Details".
<code>alpha</code>	The decay parameter for power-law autocovariance (PLA) noise. Should not be mixed with <code>dfrac</code> or <code>H</code> : see "Details".
<code>alphas</code>	The decay parameter for seasonal power-law autocovariance (PLA) noise. Should not be mixed with <code>dfs</code> or <code>Hs</code> : see "Details".

period	The periodicity of the seasonal components. Must be $\geq 2$ .
useC	How much interfaced C code to use: an integer between 0 and 3. The value 3 is strongly recommended. See "Details".

### Details

The log-likelihood is computed for the given series  $z$  and the parameters. If two or more of `dfrac`, `H` or `alpha` are present and/or two or more of `dfs`, `Hs` or `alphas` are present, an error will be thrown, as otherwise there is redundancy in the model. Note that non-seasonal and seasonal components can be of different types: for example, there can be seasonal FGN with FDWN at the non-seasonal level.

The moving average parameters are in the Box-Jenkins convention: they are the negative of the parameters given by [arima](#).

For the `useC` parameter, a "0" means no C is used; a "1" means C is only used to compute the log-likelihood, but not the theoretical autocovariance function (`tacvf`); a "2" means that C is used to compute the `tacvf` and not the log-likelihood; and a "3" means C is used to compute everything.

Note that the time series is assumed to be stationary: this function does not do any differencing.

### Value

The exact log-likelihood of the model given with respect to  $z$ , up to an additive constant.

### Author(s)

Justin Veenstra

### References

Box, G. E. P., Jenkins, G. M., and Reinsel, G. C. (2008) Time Series Analysis: Forecasting and Control. 4th Edition. John Wiley and Sons, Inc., New Jersey.

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

### See Also

[arfima](#)

[IARFIMAwTF](#)

[tacvfARFIMA](#)

### Examples

```
set.seed(3452)
sim <- arfima.sim(1000, model = list(phi = c(0.3, -0.1)))
IARFIMA(sim, phi = c(0.3, -0.1))
```

---

IARFIMAwTF	<i>Exact log-likelihood of a long memory model with a transfer function model and series included Computes the exact log-likelihood of a long memory model with respect to a given time series as well as a transfer function model and series. This function is not meant to be used directly.</i>
------------	---

---

### Description

Once again, this function should not be used externally.

### Usage

```
IARFIMAwTF(
  z,
  phi = numeric(0),
  theta = numeric(0),
  dfrac = numeric(0),
  phiseas = numeric(0),
  thetaseas = numeric(0),
  dfs = numeric(0),
  H = numeric(0),
  Hs = numeric(0),
  alpha = numeric(0),
  alphas = numeric(0),
  xr = numeric(0),
  r = numeric(0),
  s = numeric(0),
  b = numeric(0),
  delta = numeric(0),
  omega = numeric(0),
  period = 0,
  useC = 3,
  meanval = 0
)
```

### Arguments

z	A vector or (univariate) time series object, assumed to be (weakly) stationary.
phi	The autoregressive parameters in vector form.
theta	The moving average parameters in vector form. See Details for differences from <a href="#">arima</a> .
dfrac	The fractional differencing parameter.
phiseas	The seasonal autoregressive parameters in vector form.
thetaseas	The seasonal moving average parameters in vector form. See Details for differences from <a href="#">arima</a> .



dfs	The seasonal fractional differencing parameter.
H	The Hurst parameter for fractional Gaussian noise (FGN). Should not be mixed with dfrac or alpha: see "Details".
Hs	The Hurst parameter for seasonal fractional Gaussian noise (FGN). Should not be mixed with dfs or alphas: see "Details".
alpha	The decay parameter for power-law autocovariance (PLA) noise. Should not be mixed with dfrac or H: see "Details".
alphas	The decay parameter for seasonal power-law autocovariance (PLA) noise. Should not be mixed with dfs or Hs: see "Details".
xr	The regressors in vector form
r	The order of the delta(s)
s	The order of the omegas(s)
b	The backshifting to be done
delta	Transfer function parameters as in Box, Jenkins, and Reinsel. Corresponds to the "autoregressive" part of the dynamic regression.
omega	Transfer function parameters as in Box, Jenkins, and Reinsel. Corresponds to the "moving average" part of the dynamic regression: note that omega_0 is not restricted to 1. See "Details" for issues.
period	The periodicity of the seasonal components. Must be $\geq 2$ .
useC	How much interfaced C code to use: an integer between 0 and 3. The value 3 is strongly recommended. See "Details".
meanval	If the mean is to be estimated dynamically, the mean.

**Value**

A log-likelihood value

**Author(s)**

Justin Veenstra

**References**

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

---

logLik.arfima	<i>Extract Log-Likelihood Values</i>
---------------	--------------------------------------

---

**Description**

Extracts log-likelihood values from a arfima fit.

**Usage**

```
## S3 method for class 'arfima'  
logLik(object, ...)
```

**Arguments**

object	A fitted arfima object
...	Optional arguments not currently used.

**Details**

Uses the function `DLoglikelihood` from the package [ltsa](#). The log-likelihoods returned are exact up to an additive constant.

**Value**

A vector of log-likelihoods, one for each mode, is returned, along with the degrees of freedom.

**Author(s)**

JQ (Justin) Veenstra

**References**

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

**See Also**

[AIC.arfima](#)

---

PacfToAR	<i>Converts AR/MA coefficients from the PACF space to operator space</i>
----------	--

---

**Description**

Converts AR/MA coefficients from PACF box-space to operator space; usually for internal use

**Usage**

PacfToAR(pi)

**Arguments**

pi                    The AR/MA coefficients in PACF box-space

**Value**

The AR/MA coefficients in operator space.

**Author(s)**

A. I. McLeod

**References**

Barndorff-Nielsen O. E., Schou G. (1973). "On the parametrization of autoregressive models by partial autocorrelations." *Journal of Multivariate Analysis*, 3, 408-419

McLeod A. I. , Zhang Y (2006). "Partial autocorrelation parameterization for subset autoregression." *Journal of Time Series Analysis*, 27(4), 599-612

---

plot.predarfima	<i>Plots the original time series, the predictions, and the prediction intervals for a predarfima object.</i>
-----------------	---

---

**Description**

This function takes a predarfima object generated by predict.arfima and plots all of the information contained in it. The colour code is as follows:

**Usage**

```
## S3 method for class 'predarfima'
plot(
  x,
  xlab = NULL,
  ylab = NULL,
  main = NULL,
  ylim = NULL,
  numback = 5,
  xlim = NULL,
  ...
)
```

**Arguments**

x	A predarfima object
xlab	Optional
ylab	Optional
main	Optional
ylim	Optional
numback	The number of last values of the original series to plot defined by the user. The default is five
xlim	Optional
...	Currently not used

**Details**

grey: exact prediction red: exact prediction intervals (PIs) orange: limiting PIs  
See [predict.arfima](#).

**Value**

None. Generates a plot

**Author(s)**

JQ (Justin) Veenstra

**References**

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

**See Also**

[predict.arfima](#), [print.predarfima](#)

**Examples**

```

set.seed(82365)
sim <- arfima.sim(1000, model = list(dfrac = 0.4, theta=0.9, dint = 1))
fit <- arfima(sim, order = c(0, 1, 1), back=TRUE)
fit
pred <- predict(fit, n.ahead = 5)
pred
plot(pred)
#Let's look at more context
plot(pred, numback = 50)

```

---

plot.tacvf

*Plots the output from a call to tacvf*


---

**Description**

Plots the theoretical autocovariance functions of the modes for a fitted arfima object

**Usage**

```

## S3 method for class 'tacvf'
plot(
  x,
  type = "o",
  pch = 20,
  xlab = NULL,
  ylab = NULL,
  main = NULL,
  xlim = NULL,
  ylim = NULL,
  tacf = FALSE,
  maxlag = NULL,
  lag0 = !tacf,
  ...
)

```

**Arguments**

x	A tacvf object from a call to said function
type	See <a href="#">plot</a> . The default is recommended for short maxlag
pch	See <a href="#">plot</a>
xlab	See <a href="#">plot</a>
ylab	See <a href="#">plot</a>
main	See <a href="#">plot</a>

xlim	See <a href="#">plot</a>
ylim	See <a href="#">plot</a>
tacf	If TRUE, plots the theoretical autocorrelations instead
maxlag	The maximum lag for the plot
lag0	Whether or not to plot lag 0 of the tacvfs/tacfs. Default !tacf. Used by <a href="#">tacfplot</a> .
...	Currently not used

### Details

Only plots up to nine tacvfs. It is highly recommended that the arfima object be weeded before calling tacvf

### Value

None. There is a plot as output.

### Author(s)

JQ (Justin) Veenstra

### References

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

### See Also

[tacvf](#)

### Examples

```
set.seed(1234)
sim <- arfima.sim(1000, model = list(theta = 0.99, dfrac = 0.49))
fit <- arfima(sim, order = c(0, 0, 1))
plot(tacvf(fit))
plot(tacvf(fit), tacf = TRUE)
```

---

predict.arfima	<i>Predicts from a fitted object.</i>
----------------	---------------------------------------

---

### Description

Performs prediction of a fitted arfima object. Includes prediction for each mode and exact and limiting prediction error standard deviations. **NOTE: the standard errors in beta are currently not taken into account in the prediction intervals shown. This will be updated as soon as possible.**

### Usage

```
## S3 method for class 'arfima'
predict(
  object,
  n.ahead = 1,
  prop.use = "default",
  newxreg = NULL,
  predint = 0.95,
  exact = c("default", T, F),
  setmuhat0 = FALSE,
  cpus = 1,
  trend = NULL,
  n.use = NULL,
  xreg = NULL,
  ...
)
```

### Arguments

object	A fitted arfima object
n.ahead	The number of steps ahead to predict
prop.use	The proportion (between 0 and 1) or percentage (between >1 and 100) of data points to use for prediction. Defaults to the string "default", which sets the number of data points n.use to the minimum of the series length and 1000. Overriden by n.use.
newxreg	If a regression fit, the new regressors
predint	The percentile to use for prediction intervals assuming normal deviations.
exact	Controls whether exact (based on the theoretical autocovariance matrix) prediction variances are calculated (which is recommended), as well as whether the exact prediction formula is used when the process is differenced (which can take a fair amount of time if the length of the series used to predict is large). Defaults to the string "default", which is TRUE for the first and FALSE for the second. A Boolean value (TRUE or FALSE) will set both to this value.
setmuhat0	Experimental. Sets muhat equal to zero

cpus	The number of CPUs to use for prediction. Currently not implemented
trend	An optional vector the length of n. ahead or longer to add to the predictions
n.use	Directly set the number mentioned in prop.use.
xreg	Alias for newxreg
...	Optional arguments. Currently not used

**Value**

A list of lists, ceiling(prop.use \* n)one for each mode with relevant details about the prediction

**Author(s)**

JQ (Justin) Veenstra

**References**

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

**See Also**

[arfima](#), [plot.predarfima](#), [print.predarfima](#)

**Examples**

```
set.seed(82365)
sim <- arfima.sim(1000, model = list(dfrac = 0.4, theta=0.9, dint = 1))
fit <- arfima(sim, order = c(0, 1, 1), back=TRUE)
fit
pred <- predict(fit, n.ahead = 5)
pred
plot(pred, numback=50)
#Predictions aren't really different due to the
#series. Let's see what happens when we regress!

set.seed(23524)
#Forecast 5 ahead as before
#Note that we need to integrate the regressors, since time series regression
#usually assumes that regressors are of the same order as the series.
n.fore <- 5
X <- matrix(rnorm(3000+3*n.fore), ncol = 3)
X <- apply(X, 2, cumsum)
Xnew <- X[1001:1005,]
X <- X[1:1000,]
beta <- matrix(c(2, -.4, 6), ncol = 1)
simX <- sim + as.vector(X%*%beta)
fitX <- arfima(simX, order = c(0, 1, 1), xreg = X, back=TRUE)
fitX
#Let's compare predictions.
predX <- predict(fitX, n.ahead = n.fore, xreg = Xnew)
```



```
predX
plot(predX, numback = 50)
#With the mode we know is really there, it looks better.
fitX <- removeMode(fitX, 2)
predXnew <- predict(fitX, n.ahead = n.fore, xreg = Xnew)
predXnew
plot(predXnew, numback=50)
```

---

print.arfima	<i>Prints a Fitted Object</i>
--------------	-------------------------------

---

### Description

Prints a fitted arfima object's relevant details

### Usage

```
## S3 method for class 'arfima'
print(x, digits = max(6, getOption("digits") - 3), ...)
```

### Arguments

x	A fitted arfima object
digits	The number of digits to print
...	Optional arguments. See <a href="#">print</a> .

### Value

The object is returned invisibly

### Author(s)

JQ (Justin) Veenstra

### References

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

---

print.predarfima      *Prints predictions and prediction intervals*

---

### Description

Prints the output of predict on an arfima object

### Usage

```
## S3 method for class 'predarfima'  
print(x, digits = max(6, getOption("digits") - 3), ...)
```

### Arguments

x	An object of class "predarfima"
digits	The number of digits to print
...	Currently not used

### Details

Prints all the relevant output of the prediction function of the arfima package

### Value

x is returned invisibly

### Author(s)

JQ (Justin) Veenstra

### See Also

[arfima](#), [predict.arfima](#), [predict](#), [plot.predarfima](#)

### Examples

```
set.seed(82365)  
sim <- arfima.sim(1000, model = list(dfrac = 0.4, theta=0.9, dint = 1))  
fit <- arfima(sim, order = c(0, 1, 1), back=TRUE)  
fit  
pred <- predict(fit, n.ahead = 5)  
pred  
plot(pred)
```

---

print.summary.arfima *Prints the output of a call to summary on an arfima object*

---

### Description

Prints the output of a call to summary on an arfima object

### Usage

```
## S3 method for class 'summary.arfima'
print(
  x,
  digits = max(6, getOption("digits") - 3),
  signif.stars = getOption("show.signif.stars"),
  ...
)
```

### Arguments

x	A summary.arfima object
digits	The number of digits to print
signif.stars	Whether to print stars on significant output
...	Currently not used

### Value

Returns the object x invisibly

### Author(s)

JQ (Justin) Veenstra

### References

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

### See Also

[arfima](#), [print.arfima](#), [summary.arfima](#), [print](#)

### Examples

```
set.seed(54678)
sim <- arfima.sim(1000, model = list(phi = 0.9, H = 0.3))
fit <- arfima(sim, order = c(1, 0, 0), lmodel = "g", back=TRUE)
summary(fit)
```

---

<code>print.tacvf</code>	<i>Prints a tacvf object.</i>
--------------------------	-------------------------------

---

**Description**

Prints the output of a call to `tacvf` on an `arfima` object

**Usage**

```
## S3 method for class 'tacvf'  
print(x, ...)
```

**Arguments**

<code>x</code>	The <code>tacvf</code> object.
<code>...</code>	Optional arguments. See <a href="#">print</a> .

**Value**

The object is returned invisibly

**Author(s)**

JQ (Justin) Veenstra

**See Also**

[tacvf](#), [plot.tacvf](#)

---

<code>removeMode</code>	<i>Removes a mode from an arfima fit.</i>
-------------------------	---

---

**Description**

This function is useful if one suspects a mode is spurious and does not want to call the `weed` function.

**Usage**

```
removeMode(object, num)
```

**Arguments**

<code>object</code>	An object of class "arfima".
<code>num</code>	The number of the mode as in the printed value of the object.

**Value**

The original object with the mode removed.

**Author(s)**

JQ (Justin) Veenstra

**See Also**

[arfima](#)

**Examples**

```
set.seed(8765)
sim <- arfima.sim(1000, model = list(phi = 0.4, theta = 0.9, dfrac = 0.4))
fit <- arfima(sim, order = c(1, 0, 1), back=TRUE)
fit
fit <- removeMode(fit, 3)
fit
```

---

residuals.arfima      *Extract the Residuals of a Fitted Object*

---

**Description**

Extracts the residuals or regression residuals from a fitted arfima object

**Usage**

```
## S3 method for class 'arfima'
residuals(object, reg = FALSE, ...)
```

**Arguments**

object	A fitted arfima object
reg	Whether to extract the regression residuals instead. If TRUE, throws an error if no regression was done.
...	Optional parameters. Currently not used.

**Value**

A list of vectors of residuals, one for each mode.

**Author(s)**

JQ (Justin) Veenstra

**References**

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

**See Also**

[arfima](#), [fitted.arfima](#)

**Examples**

```
set.seed(8564)
sim <- arfima.sim(1000, model = list(phi = c(0.2, 0.1), dfrac = 0.4, theta = 0.9))
fit <- arfima(sim, order = c(2, 0, 1), back=TRUE)

fit

resid <- resid(fit)
par(mfrow = c(1, 3))
plot(resid[[1]])
plot(resid[[2]])
plot(resid[[3]])
fitted <- fitted(fit)
plot(fitted[[1]], resid[[1]])
plot(fitted[[2]], resid[[2]])
plot(fitted[[3]], resid[[3]])
par(mfrow = c(1, 1))
```

---

SeriesJ

*Series J, Gas Furnace Data*

---

**Description**

Gas furnace data, sampling interval 9 seconds; observations for 296 pairs of data points.

**Format**

List with ts objects XJ and YJ.

**Details**

XJ is input gas rate in cubic feet per minute, YJ is percentage carbon dioxide (CO<sub>2</sub>) in outlet gas. X is the regressor.

Box, Jenkins, and Reinsel (2008) fit an AR(2) to YJ, with transfer function specifications  $r = 2$ ,  $s = 2$ , and  $b = 3$ , regressing on XJ. Our package agrees with their results.

**Source**

Box, Jenkins and Reinsel(2008). Time Series Analysis: Forecasting and Control.

**References**

Box, G. E. P., Jenkins, G. M., and Reinsel, G. C. (2008) Time Series Analysis: Forecasting and Control. 4th Edition. John Wiley and Sons, Inc., New Jersey.

Veenstra, J. and McLeod, A. I. (Working Paper). The arfima R package: Exact Methods for Hyperbolic Decay Time Series

**Examples**

```
data(SeriesJ)
attach(SeriesJ)

fitTF <- arfima(YJ, order= c(2, 0, 0), xreg = XJ, reglist =
list(regpar = c(2, 2, 3)), lmodel = "n")
fitTF ## agrees fairly closely with Box et. al.

detach(SeriesJ)
```

---

sim\_from\_fitted

*Simulate an ARFIMA time series from a fitted arfima object.*


---

**Description**

This function simulates an long memory ARIMA time series, with one of fractionally differenced white noise (FDWN), fractional Gaussian noise (FGN), power-law autocovariance (PLA) noise, or short memory noise and possibly seasonal effects.

**Usage**

```
sim_from_fitted(n, model, X = NULL, seed = NULL)
```

**Arguments**

n	The number of points to be generated.
model	The model to be simulated from. The phi and theta arguments should be vectors with the values of the AR and MA parameters. Note that Box-Jenkins notation is used for the MA parameters: see the "Details" section of <a href="#">arfima</a> .
X	The xreg matrix to add to the series, <i>required</i> if there is an xreg argument in model. An error will be thrown if there is a mismatch between this argument and whether model was called with a external regressor
seed	An optional seed that will be set before the simulation. If model is multimodal, a seed will be chosen randomly if not provided, and all modes will simulate a time series with said seed set.

**Details**

A suitably defined stationary series is generated, and if either of the dints (non-seasonal or seasonal) are greater than zero, the series is integrated (inverse-differenced) with zinit equalling a suitable amount of 0s if not supplied. Then a suitable amount of points are taken out of the beginning of the series (i.e. dint + period \* seasonal dint = the length of zinit) to obtain a series of length n. The stationary series is generated by calculating the theoretical autocovariance function and using it, along with the innovations to generate a series as in McLeod et. al. (2007). *Note*: if you would like to fit from parameters, use the function, `arfima.sim`.

**Value**

A sample (or list of samples) from a multivariate normal distribution that has a covariance structure defined by the autocovariances generated for given parameters. The sample acts like a time series with the given parameters. The returned value will be a list if the fit is multimodal.

**Author(s)**

JQ (Justin) Veenstra

**References**

McLeod, A. I., Yu, H. and Krougly, Z. L. (2007) Algorithms for Linear Time Series Analysis: With R Package Journal of Statistical Software, Vol. 23, Issue 5

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

P. Borwein (1995) An efficient algorithm for Riemann Zeta function Canadian Math. Soc. Conf. Proc., 27, pp. 29-34.

**See Also**

[arfima](#), [arfima.sim](#)

**Examples**

```
set.seed(6533)
sim <- arfima.sim(1000, model = list(phi = .2, dfrac = .3, dint = 2))

fit <- arfima(sim, order = c(1, 2, 0))
fit

sim2 <- sim_from_fitted(100, fit)

fit2 <- arfima(sim2, order = c(1, 2, 0))
fit2

set.seed(2266)
#Fairly pathological series to fit for this package
series = arfima.sim(500, model=list(phi = 0.98, dfrac = 0.46))
```



```

X = matrix(rnorm(1000), ncol = 2)
colnames(X) <- c('c1', 'c2')
series_added <- series + X*%c(2, 5)

fit <- arfima(series, order = c(1, 0, 0), numeach = c(2, 2))
fit_X <- arfima(series_added, order=c(1, 0, 0), xreg=X, numeach = c(2, 2))

from_series <- sim_from_fitted(1000, fit)

fit1a <- arfima(from_series[[1]], order = c(1, 0, 0), numeach = c(2, 2))
fit1a
fit1 <- arfima(from_series[[1]], order = c(1, 0, 0))
fit1
fit2 <- arfima(from_series[[1]], order = c(1, 0, 0))
fit2
fit3 <- arfima(from_series[[1]], order = c(1, 0, 0))
fit3
fit4 <- arfima(from_series[[1]], order = c(1, 0, 0))
fit4

Xnew = matrix(rnorm(2000), ncol = 2)
from_series_X <- sim_from_fitted(1000, fit_X, X=Xnew)

fit_X1a <- arfima(from_series_X[[1]], order=c(1, 0, 0), xreg=Xnew, numeach = c(2, 2))
fit_X1a
fit_X1 <- arfima(from_series_X[[1]], order=c(1, 0, 0), xreg=Xnew)
fit_X1
fit_X2 <- arfima(from_series_X[[2]], order=c(1, 0, 0), xreg=Xnew)
fit_X2
fit_X3 <- arfima(from_series_X[[3]], order=c(1, 0, 0), xreg=Xnew)
fit_X3
fit_X4 <- arfima(from_series_X[[4]], order=c(1, 0, 0), xreg=Xnew)
fit_X4

```

---

summary.arfima

*Extensive Summary of an Object*


---

## Description

Provides a very comprehensive summary of a fitted arfima object. Includes correlation and covariance matrices (observed and expected), the Fisher Information matrix of those parameters for which it is defined, and more, for each mode.

## Usage

```

## S3 method for class 'arfima'
summary(object, digits = max(4, getOption("digits") - 3), ...)

```

**Arguments**

object	A fitted arfima object
digits	The number of digits to print
...	Optional arguments, currently not used.

**Value**

A list of lists (one for each mode) of all relevant information about the fit that can be passed to `print.summary.arfima`.

**Author(s)**

JQ (Justin) Veenstra

**References**

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

**See Also**

[arfima](#), [iARFIMA](#), [vcov.arfima](#)

**Examples**

```
data(tmpyr)

fit <- arfima(tmpyr, order = c(1, 0, 1), back=TRUE)
fit

summary(fit)
```

---

tacfplot	<i>Plots the theoretical autocorrelation functions (tacfs) of one or more fits.</i>
----------	---

---

**Description**

Plots the theoretical autocorrelation functions (tacfs) of one or more fits.

**Usage**

```
tacfplot(  
  fits = list(),  
  modes = "all",  
  xlab = NULL,  
  ylab = NULL,  
  main = NULL,  
  xlim = NULL,  
  ylim = NULL,  
  maxlag = 20,  
  lag0 = FALSE,  
  ...  
)
```

**Arguments**

<code>fits</code>	A list of objects of class "arfima".
<code>modes</code>	Either "all" or a vector of the same length as fits for which the tacfs will be plotted.
<code>xlab</code>	Optional. Usually better to be generated by the function.
<code>ylab</code>	Optional. Usually better to be generated by the function.
<code>main</code>	Optional. Usually better to be generated by the function.
<code>xlim</code>	Optional. Usually better to be generated by the function.
<code>ylim</code>	Optional. Usually better to be generated by the function.
<code>maxlag</code>	Optional. Used to limit the length of tacfs. Highly recommended to be a value from 20 - 50.
<code>lag0</code>	Whether or not the lag 0 tacf should be printed. Since this is always 1 for all tacfs, recommended to be TRUE. It is easier to see the shape of the tacfs.
<code>...</code>	Optional. Currently not used.

**Value**

NULL. However, there is a plot output.

**Author(s)**

JQ (Justin) Veenstra

**References**

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

**See Also**

[tacvf](#), [plot.tacvf](#)

## Examples

```
set.seed(34577)
sim <- arfima.sim(500, model = list(theta = 0.9, phi = 0.5, dfrac = 0.4))
fit1 <- arfima(sim, order = c(1, 0, 1), cpus = 2, back=TRUE)
fit2 <- arfima(sim, order = c(1, 0, 1), cpus = 2, lmodel = "g", back=TRUE)
fit3 <- arfima(sim, order = c(1, 0, 1), cpus = 2, lmodel = "h", back=TRUE)
fit1
fit2
fit3
tacvplot(fits = list(fit1, fit2, fit3), maxlag = 30)
```

---

tacvf

*Extracts the tacvfs of a fitted object*

---

## Description

Extracts the theoretical autocovariance functions (tacvfs) from a fitted arfima or one of its modes (an ARFIMA) object.

## Usage

```
tacvf(obj, xmaxlag = 0, forPred = FALSE, n.ahead = 0, nuse = -1, ...)
```

## Arguments

obj	An object of class "arfima" or "ARFIMA". The latter class is a mode of the former.
xmaxlag	The number of extra points to be added on to the end. That is, if the original series has length 300, and xmaxlag = 5, the tacvfs will go from lag 0 to lag 304.
forPred	Should only be TRUE from a call to <code>predict.arfima</code> .
n.ahead	Only used internally.
nuse	Only used internally.
...	Optional arguments, currently not used.

## Value

A list of tacvfs, one for each mode, the length of the time series.

## Author(s)

JQ (Justin) Veenstra

## References

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

**See Also**

[plot.tacvf](#), [print.tacvf](#), [tacfplot](#), [arfima](#)

---

 tacvfARFIMA

*The theoretical autocovariance function of a long memory process.*


---

**Description**

Calculates the tacvf of a mixed long memory-ARMA (with possible seasonal components). Combines long memory and ARMA (and non-seasonal and seasonal) parts via convolution.

**Usage**

```
tacvfARFIMA(
  phi = numeric(0),
  theta = numeric(0),
  dfrac = numeric(0),
  phiseas = numeric(0),
  thetaseas = numeric(0),
  dfs = numeric(0),
  H = numeric(0),
  Hs = numeric(0),
  alpha = numeric(0),
  alphas = numeric(0),
  period = 0,
  maxlag,
  useCt = T,
  sigma2 = 1
)
```

**Arguments**

phi	The autoregressive parameters in vector form.
theta	The moving average parameters in vector form. See Details for differences from <a href="#">arima</a> .
dfrac	The fractional differencing parameter.
phiseas	The seasonal autoregressive parameters in vector form.
thetaseas	The seasonal moving average parameters in vector form. See Details for differences from <a href="#">arima</a> .
dfs	The seasonal fractional differencing parameter.
H	The Hurst parameter for fractional Gaussian noise (FGN). Should not be mixed with dfrac or alpha: see "Details".
Hs	The Hurst parameter for seasonal fractional Gaussian noise (FGN). Should not be mixed with dfs or alphas: see "Details".

alpha	The decay parameter for power-law autocovariance (PLA) noise. Should not be mixed with dfrac or H: see "Details".
alphas	The decay parameter for seasonal power-law autocovariance (PLA) noise. Should not be mixed with dfs or Hs: see "Details".
period	The periodicity of the seasonal components. Must be $\geq 2$ .
maxlag	The number of terms to compute: technically the output sequence is from lags 0 to maxlag, so there are maxlag + 1 terms.
useCt	Whether or not to use C to compute the (parts of the) tacvf.
sigma2	Used in <code>arfima.sim</code> : determines the value of the innovation variance. The tacvf sequence is multiplied by this value.

### Details

The log-likelihood is computed for the given series  $z$  and the parameters. If two or more of `dfrac`, `H` or `alpha` are present and/or two or more of `dfs`, `Hs` or `alphas` are present, an error will be thrown, as otherwise there is redundancy in the model. Note that non-seasonal and seasonal components can be of different types: for example, there can be seasonal FGN with FDWN at the non-seasonal level.

The moving average parameters are in the Box-Jenkins convention: they are the negative of the parameters given by `arima`.

### Value

A sequence of length maxlag + 1 (lags 0 to maxlag) of the tacvf of the given process.

### Author(s)

JQ (Justin) Veenstra and A. I. McLeod

### References

- Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)  
 P. Borwein (1995) An efficient algorithm for Riemann Zeta function Canadian Math. Soc. Conf. Proc., 27, pp. 29-34.

### Examples

```
t1 <- tacvfARFIMA(phi = c(0.2, 0.1), theta = 0.4, dfrac = 0.3, maxlag = 30)
t2 <- tacvfARFIMA(phi = c(0.2, 0.1), theta = 0.4, H = 0.8, maxlag = 30)
t3 <- tacvfARFIMA(phi = c(0.2, 0.1), theta = 0.4, alpha = 0.4, maxlag = 30)
plot(t1, type = "o", col = "blue", pch = 20)
lines(t2, type = "o", col = "red", pch = 20)
lines(t3, type = "o", col = "purple", pch = 20) #they decay at about the same rate
```

---

tmpyr

*Temperature Data*

---

### Description

Central England mean yearly temperatures from 1659 to 1976

### Format

A ts tmpyr

### Details

Hosking notes that while the ARFIMA(1, d, 1) has a lower AIC, it is not much lower than the AIC of the ARFIMA(1, d, 0).

Bhansali and Kobožka find:  $\mu_{\text{Hat}} = 9.14$ ,  $d = 0.28$ ,  $\phi = -0.77$ , and  $\theta = -0.66$  for the ARFIMA(1, d, 1), which is close to our result, although our result reveals trimodality if numeach is large enough. The third mode is close to Hosking's fit of an ARMA(1, 1) to these data, while the second is very antipersistent.

Our package gives a very close result to Hosking for the ARFIMA(1, d, 0) case, although there is also a second mode. Given how close it is to the boundary, it may or may not be spurious. A check with `dmean = FALSE` shows that it is not the optimized mean giving a spurious mode.

If, however, we use `whichopt = 1`, we only have one mode. Note that Nelder-Mead sometimes does take out non-spurious modes, or add spurious modes to the surface.

### Source

<https://hadleyserver.metoffice.gov.uk/hadobs/hadcet/>

### References

Parker, D.E., Legg, T.P., and Folland, C.K. (1992). A new daily Central England Temperature Series, 1772-1991. *Int. J. Clim.*, Vol 12, pp 317-342

Manley, G. (1974). Central England Temperatures: monthly means 1659 to 1973. *Q.J.R. Meteorol. Soc.*, Vol 100, pp 389-405.

Hosking, J. R. M. (1984). Modeling persistence in hydrological time series using fractional differencing, *Water Resour. Res.*, 20(12)

Bhansali, R. J. and Kobožka, P. S. (2003) Prediction of Long-Memory Time Series In Doukhan, P., Oppenheim, G. and Taqqu, M. S. (Eds) *Theory and Applications of Long-Range Dependence* (pp355-368) Birkhauser Boston Inc.

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

## Examples

```

data(tmpyr)

fit <- arfima(tmpyr, order = c(1, 0, 1), numeach = c(3, 3), dmean = TRUE, back=TRUE)
fit
##suspect that fourth mode may be spurious, even though not close to a boundary
##may be an induced mode from the optimization of the mean

fit <- arfima(tmpyr, order = c(1, 0, 1), numeach = c(3, 3), dmean = FALSE, back=TRUE)
fit

##perhaps so

plot(tacvf(fit), maxlag = 30, tacf = TRUE)

fit1 <- arfima(tmpyr, order = c(1, 0, 0), dmean = TRUE, back=TRUE)
fit1

fit2 <- arfima(tmpyr, order = c(1, 0, 0), dmean = FALSE, back=TRUE)
fit2 ##still bimodal. Second mode may or may not be spurious.

fit3 <- arfima(tmpyr, order = c(1, 0, 0), dmean = FALSE, whichopt = 1, numeach = c(3, 3))
fit3 ##Unimodal. So the second mode was likely spurious.

plot(tacvf(fit2), maxlag = 30, tacf = TRUE)
##maybe not spurious. Hard to tell without visualizing the surface.

##compare to plotted tacf of fit1: looks alike
plot(tacvf(fit1), maxlag = 30, tacf = TRUE)

tacfplot(list(fit1, fit2))

```

---

vcov.arfima

*Extracts the Variance-Covariance Matrix*


---

## Description

Extracts the variance-covariance matrices (one or two for each mode) from a fitted arfima object.

## Usage

```

## S3 method for class 'arfima'
vcov(
  object,
  type = c("b", "o", "e"),

```



```

    cor = FALSE,
    digits = max(4, getOption("digits") - 3),
    tapprox = FALSE,
    summ = FALSE,
    ...
  )

```

### Arguments

object	A fitted arfima object
type	Which type of covariance matrix to return: "o" is the observed matrix (from solving the Hessian), "e" is the expected matrix (from solving the information matrix), and "b" is both.
cor	Whether or not the correlation matrix should be returned instead.
digits	The number of digits to print.
tapprox	Whether or not to use an approximation to find the expected matrix. Highly recommended to be FALSE, as it takes much longer, and is an approximation.
summ	Whether the call is from the <code>summary.arfima</code> function. Should not be used except internally.
...	Optional arguments, currently not used.

### Value

A list of lists (one for each mode) with components observed and/or expected.

### Author(s)

JQ (Justin) Veenstra

### References

Veenstra, J.Q. Persistence and Antipersistence: Theory and Software (PhD Thesis)

### See Also

[summary.arfima](#), [arfima](#)

### Examples

```

set.seed(1234)
sim <- arfima.sim(1000, model = list(dfrac = 0.4, phi = .8, theta = -0.5))
fit1 <- arfima(sim, order = c(1, 0, 1), back=TRUE)
fit2 <- arfima(sim, order = c(1, 0, 1), lmodel = "g", back=TRUE)
fit3 <- arfima(sim, order = c(1, 0, 1), lmodel = "h", back=TRUE)
fit1
fit2
fit3
vcov(fit1)

```

```
vcov(fit2)
vcov(fit2)
```

---

```
weed
```

---

*Weeds out fits from a call to arfima that are too close to each other.*

---

### Description

Weeds out fits from a call to arfima that are too close to each other.

### Usage

```
weed(
  ans,
  type = c("A", "P", "B", "N"),
  walls = FALSE,
  eps2 = 0.025,
  eps3 = 0.01,
  adapt = TRUE,
  pn = 2
)
```

### Arguments

ans	The result from a call to arfima.
type	The space to perform the weeding in. "A" is for operating parameters. "P" is in the PACF space. "B" performs weeding in both. "N" performs no weeding and is only used internally.
walls	If more than one mode is on a wall in the PACF space, all modes but the one with the highest log-likelihood on the same wall are deleted.
eps2	The maximum distance between modes that are close together for the mode with the lower log-likelihood to be weeded out. If adapt is TRUE (default) this value changes.
eps3	The minimum distance from a wall for a secondary mode to be weeded out, if walls are TRUE.
adapt	If TRUE, if dim is the dimensionality of the search, eps2 is changed to $(1 + eps2)^{dim} - 1$ .
pn	The p in the p-norm to be used in the weeding. p = 2 (default) is Euclidean distance.

### Value

An object of class "arfima" with modes possibly weeded out.

**Author(s)**

JQ (Justin) Veenstra

**See Also**

[arfima](#), [distance](#)

**Examples**

```
set.seed(1234)
sim <- arfima.sim(1000, model = list(theta = 0.9, dfrac = 0.4))
fit <- arfima(sim, order = c(0, 0, 1), autoweed = FALSE, back=TRUE)
fit
distance(fit)
fit1 <- weed(fit)
fit1
distance(fit1)
```

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