# Package 'brglm2' 

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Title Bias Reduction in Generalized Linear Models
Version 0.8.2

## Description

Estimation and inference from generalized linear models based on various methods for bias reduction and maximum penalized likelihood with powers of the Jeffreys prior as penalty. The 'brglmFit' fitting method can achieve reduction of estimation bias by solving either the mean biasreducing adjusted score equations in Firth (1993) [doi:10.1093/biomet/80.1.27](doi:10.1093/biomet/80.1.27) and Kosmidis and Firth (2009) [doi:10.1093/biomet/asp055](doi:10.1093/biomet/asp055), or the median bias-reduction adjusted score equations in Kenne et al. (2017) [doi:10.1093/biomet/asx046](doi:10.1093/biomet/asx046), or through the direct subtraction of an estimate of the bias of the maximum likelihood estimator from the maximum likelihood estimates as in Cordeiro and McCul-
lagh (1991) <https: //www. jstor. org/stable/2345592>. See Kos-
midis et al (2020) [doi:10.1007/s11222-019-09860-6](doi:10.1007/s11222-019-09860-6) for more details. Estima-
tion in all cases takes place via a quasi Fisher scoring algorithm, and S3 methods for the construction of of confidence intervals for the reduced-bias estimates are provided. In the special case of generalized linear models for binomial and multinomial responses (both ordinal and nominal), the adjusted score approaches to mean and media bias reduc-
tion have been found to return estimates with improved frequentist properties, that are also always finite, even in cases where the maximum likelihood estimates are infinite (e.g. complete and quasi-complete separation; see Kos-
midis and Firth, 2020 [doi:10.1093/biomet/asaa052](doi:10.1093/biomet/asaa052), for a proof for mean bias reduction in logistic regression).

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$R$ topics documented:
alligators ..... 3
bracl ..... 3
brglm 2 ..... 6
brglm2-defunct ..... 7
brglmControl ..... 8
brglmFit ..... 11
brmultinom ..... 16
brnb ..... 19
coalition ..... 23
coef.brglmFit ..... 24
coef.brnb ..... 25
confint.brglmFit ..... 25
confint.brmultinom ..... 26
confint.brnb ..... 26
endometrial ..... 27
lizards ..... 28
mis ..... 29
ordinal_superiority ..... 30
predict.bracl ..... 31
predict.brmultinom ..... 33
print.summary.brnb ..... 34
residuals.brmultinom ..... 34
simulate.brmultinom ..... 35
simulate.brnb ..... 36
stemcell ..... 37
summary.brglmFit ..... 38
summary.brnb ..... 39
vcov.brglmFit ..... 40
vcov.brnb ..... 40
Index ..... 42

## Description

Alligator food choice data

## Usage

alligators

## Format

A data frame with 80 rows and 5 variables:
foodchoice primary food type, in volume, found in an alligator's stomach, with levels fish, invertebrate, reptile, bird, other
lake lake of capture with levels Hancock, Oklawaha, Trafford, George
gender gender of the alligator with levels Male and Female
size size of the alligator with levels $<=2.3$ meters long and $>2.3$ meters long
freq number of alligators for each foodchoice, lake, gender and size combination

## Source

The alligators data set is analyzed in Agresti (2002, Subsection~7.1.2).

## References

Agresti A (2002). *Categorical Data Analysis*. Wiley Series in Probability and Statistics. Wiley

## See Also

brmultinom
bracl
Bias reduction for adjacent category logit models for ordinal responses using the Poisson trick.

## Description

bracl is a wrapper of brglmFit that fits adjacent category logit models with or without proportional odds using implicit and explicit bias reduction methods. See Kosmidis \& Firth (2011) for details.

```
Usage
bracl(
formula,
data,
weights,
subset,
na.action,
parallel = FALSE,
contrasts = NULL,
model = TRUE,
\(x=\) TRUE,
control = list(...),
)
```


## Arguments

formula a formula expression as for regression models, of the form response $\sim$ predictors. The response should be a factor (preferably an ordered factor), which will be interpreted as an ordinal response, with levels ordered as in the factor. The model must have an intercept: attempts to remove one will lead to a warning and be ignored. An offset may be used. See the documentation of formula for other details.
data an optional data frame, list or environment in which to interpret the variables occurring in formula.
weights optional case weights in fitting. Default to 1.
subset expression saying which subset of the rows of the data should be used in the fit. All observations are included by default.
na.action a function to filter missing data.
parallel if FALSE (default), then a non-proportional odds adjacent category model is fit, assuming different effects per category; if TRUE then a proportional odds adjacent category model is fit. See Details.
contrasts a list of contrasts to be used for some or all of the factors appearing as variables in the model formula.
model logical for whether the model matrix should be returned.
x should the model matrix be included with in the result (default is TRUE).
control a list of parameters for controlling the fitting process. See brglmControl for details.
... arguments to be used to form the default 'control' argument if it is not supplied directly.

## Details

The bracl function fits adjacent category models, which assume multinomial observations with probabilities with proportional odds of the form

$$
\log \frac{\pi_{i j}}{\pi_{i j+1}}=\alpha_{j}+\beta^{T} x_{i}
$$

or with non-proportional odds of the form

$$
\log \frac{\pi_{i j}}{\pi_{i j+1}}=\alpha_{j}+\beta_{j}^{T} x_{i}
$$

where $x_{i}$ is a vector of covariates and $\pi_{i j}$ is the probability that category $j$ is observed at the covariate setting $i$.

## Author(s)

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## References

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59 doi: 10.1007/s11222019098606
Agresti, A (2010). *Analysis of Ordinal Categorical Data* (2nd edition). Wiley Series in Probability and Statistics. Wiley.
Albert A, Anderson J A (1984). On the Existence of Maximum Likelihood Estimates in Logistic Regression Models. *Biometrika*, **71**, 1-10 doi: 10.2307/2336390
Kosmidis I, Firth D (2011). Multinomial logit bias reduction via the Poisson log-linear model. *Biometrika*, **98**, 755-759 doi: 10.1093/biomet/asr026
Palmgren J (1981). The Fisher Information Matrix for Log Linear Models Arguing Conditionally on Observed Explanatory Variables. *Biometrika*, ${ }^{* *} 68 * *$, 563-566 doi: 10.1093/biomet/68.2.563

## See Also

multinom, brmultinom

## Examples

```
data("stemcell", package = "brglm2")
# Adjacent category logit (non-proportional odds)
fit_bracl <- bracl(research ~ as.numeric(religion) + gender, weights = frequency,
    data = stemcell, type = "ML")
# Adjacent category logit (proportional odds)
fit_bracl_p <- bracl(research ~ as.numeric(religion) + gender, weights = frequency,
    data = stemcell, type = "ML", parallel = TRUE)
```


## Description

Estimation and inference from generalized linear models using implicit and explicit bias reduction methods (Kosmidis, 2014), and other penalized maximum likelihood methods. Currently supported methods include the mean bias-reducing adjusted scores approach in Firth (1993) and Kosmidis \& Firth (2009), the median bias-reduction adjusted scores approach in Kenne Pagui et al. (2017), the correction of the asymptotic bias in Cordeiro \& McCullagh (1991), the mixed bias-reduction adjusted scores approach in Kosmidis et al (2020), maximum penalized likelihood with powers of the Jeffreys prior as penalty, and maximum likelihood.

## Details

In the special case of generalized linear models for binomial, Poisson and multinomial responses (both nominal and ordinal), mean and median bias reduction and maximum penalized likelihood return estimates with improved frequentist properties, that are also always finite, even in cases where the maximum likelihood estimates are infinite (e.g. complete and quasi-complete separation in multinomial regression; see also detect_separation and check_infinite_estimates for prefit and post-fit methods for the detection of infinite estimates in binomial response generalized linear models). Estimation in all cases takes place via a modified Fisher scoring algorithm, and S3 methods for the construction of confidence intervals for the reduced-bias estimates are provided.
The core model fitters are implemented by the functions brglm_fit (univariate generalized linear models), brmultinom (baseline category logit models for nominal multinomial responses), and bracl (adjacent category logit models for ordinal multinomial responses).
The similarly named **brglm** R package can only handle generalized linear models with binomial responses. Special care has been taken when developing **brglm2** in order not to have conflicts when the user loads ${ }^{* *}$ brglm $22^{* *}$ and ${ }^{* *}$ brglm ${ }^{* *}$ simultaneously. The development and maintenance of the two packages will continue in parallel, until $* * \operatorname{brglm} 2 * *$ incorporates all $* *$ brglm** functionality and gets an appropriate wrapper to the brglm: : brglm function.

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## References

Kosmidis I, Firth D (2021). Jeffreys-prior penalty, finiteness and shrinkage in binomial-response generalized linear models. *Biometrika*, ${ }^{* *}$ 108**, 71-82 doi: 10.1093/biomet/asaa0052
Cordeiro G M, McCullagh P (1991). Bias correction in generalized linear models. *Journal of the Royal Statistical Society. Series B (Methodological)*, **53**, 629-643 doi: 10.1111/j.25176161.1991.tb01852.x

Firth D (1993). Bias reduction of maximum likelihood estimates, Biometrika, **80**, 27-38 doi: 10.2307/2336755

Kenne Pagui E C, Salvan A, Sartori N (2017). Median bias reduction of maximum likelihood estimates. *Biometrika*, **104**, 923-938 doi: 10.1093/biomet/asx046

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59 doi: 10.1007/s11222019098606

Kosmidis I, Firth D (2009). Bias reduction in exponential family nonlinear models. *Biometrika*, **96**, 793-804 doi: 10.1093/biomet/asp055

Kosmidis I, Firth D (2010). A generic algorithm for reducing bias in parametric estimation. *Electronic Journal of Statistics*, **4**, 1097-1112 doi: 10.1214/10EJS579

Kosmidis I (2014). Bias in parametric estimation: reduction and useful side-effects. *WIRE Computational Statistics*, **6**, 185-196 doi: 10.1002/wics. 1296

## See Also

brglm_fit, brmultinom, bracl
brglm2-defunct Defunct Functions in package brglm2

## Description

The functions or variables listed here are no longer part of brglm2.

## Usage

check_infinite_estimates(...)
detect_separation(...)

## Arguments

... arguments to be passed to functions and methods.

## Details

- detect_separation: This function is defunct from brglm 2 since version 0.8.0. A new version of detect_separation is now maintained in the detectseparation $R$ package at https://cran.r-project.org/package=detectseparation.
- check_infinite_estimates is defunct from brglm2 since version 0.8.0. An new version of check_infinite_estimates is now maintained in the detectseparation R package at https://cran.r-project.org/package=detectseparation.


## Description

Typically only used internally by brglmFit, but may be used to construct a control argument.

## Usage

```
brglmControl(
        epsilon = 1e-06,
        maxit = 100,
        check_aliasing = TRUE,
        trace = FALSE,
        type = c("AS_mixed", "AS_mean", "AS_median", "correction", "MPL_Jeffreys", "ML"),
        transformation = "identity",
        slowit = 1,
        response_adjustment = NULL,
        max_step_factor = 12,
        a = 1/2,
    )
    brglm_control(
        epsilon = 1e-06,
        maxit = 100,
        check_aliasing = TRUE,
        trace = FALSE,
        type = c("AS_mixed", "AS_mean", "AS_median", "correction", "MPL_Jeffreys", "ML"),
        transformation = "identity",
        slowit = 1,
        response_adjustment = NULL,
        max_step_factor = 12,
        a = 1/2,
    )
```


## Arguments

epsilon positive convergence tolerance epsilon. Default is $1 \mathrm{e}-06$.
maxit integer giving the maximal number of iterations allowed. Default is 100.
check_aliasing logical indicating where a QR decomposition of the model matrix should be used to check for aliasing. Default is TRUE. See Details.
trace logical indicating if output should be produced for each iteration. Default is FALSE.

| type | the type of fitting method to be used. The options are AS_mean (mean-bias reducing adjusted scores), AS_median (median-bias reducing adjusted scores), AS_mixed (bias reduction using mixed score adjustments; default), correction (asymptotic bias correction), MPL_Jeffreys (maximum penalized likelihood with powers of the Jeffreys prior as penalty) andML (maximum likelihood). |
| :---: | :---: |
| transformation | the transformation of the dispersion to be estimated. Default is identity. See Details. |
| slowit | a positive real used as a multiplier for the stepsize. The smaller it is the smaller the steps are. Default is 1 . |
| response_adjustment |  |
|  | a (small) positive constant or a vector of such. Default is NULL. See Details. |
| max_step_factor |  |
|  | the maximum number of step halving steps to consider. Default is 12. |
| a | power of the Jeffreys prior penalty. See Details. |
|  | further arguments passed to brglmControl. Currently ignored in the output. |

## Details

brglmControl provides default values and sanity checking for the various constants that control the iteration and generally the behaviour of brglmFit.
When trace $=$ TRUE, calls to cat produce the output for each iteration. Hence, options(digits $=$ *) can be used to increase the precision.

When check_aliasing = TRUE (default), a QR decomposition of the model matrix is computed to check for aliasing. If the model matrix is known to be of full rank, then check_aliasing $=$ FALSE avoids the extra computational overhead of an additional QR decomposition, which can be substantial for large model matrices. However, setting check_aliasing = FALSE is tells brglmFit that the model matrix is full rank, and hard to trace back errors will result if it is rank deficient.
transformation sets the transformation of the dispersion parameter for which the bias reduced estimates are computed. Can be one of "identity", "sqrt", "inverse", "log" and "inverseSqrt". Custom transformations are accommodated by supplying a list of two expressions (transformation and inverse transformation). See the examples for more details.
The value of response_adjustment is only relevant if brglmFit is called with start = NULL, and family is binomial or poisson. For those models, an initial maximum likelihood fit is obtained on adjusted data to provide starting values for the iteration in brglmFit. The value of response_adjustment governs how the data is adjusted. Specifically, if family is binomial, then the responses and totals are adjusted by response_adjustment and 2 * response_adjustment, respectively; if family is poisson, then the responses are adjusted by and response_adjustment. response_adjustment $=$ NULL (default) is equivalent to setting it to "number of parameters"/"number of observations".

When type = "AS_mixed" (default), mean bias reduction is used for the regression parameters, and median bias reduction for the dispersion parameter, if that is not fixed. This adjustment has been developed based on equivariance arguments (see, Kosmidis et al, 2020, Section 4) in order to produce regression parameter estimates that are invariant to arbitrary contrasts, and estimates for the dispersion parameter that are invariant to arbitrary non-linear transformations. type = "AS_mixed" and type = "AS_mean" return the same results if brglmFit is called with family binomial or poisson (i.e. families with fixed dispersion).

When type $=$ "MPL_Jeffreys", brglmFit will maximize the penalized log-likelihood

$$
l(\beta, \phi)+a \log \operatorname{det} i(\beta, \phi)
$$

where $i(\beta, \phi)$ is the expected information matrix about the regression parameters $\beta$ and the dispersion parameter $\phi$. See, vignette("iteration", "brglm2") for more information. The argument $\$ a \$$ controls the amount of penalization and its default value is $a=1 / 2$, corresponding to maximum penalized likelihood using a Jeffreys-prior penalty. See, Kosmidis \& Firth (2021) for proofs and discussion about the finiteness and shrinkage properties of the maximum penalized likelihood estimators for binomial-response generalized linear models.
The estimates from type = "AS_mean" and type = "MPL_Jeffreys" with a = 1/2 (default) are identical for Poisson log-linear models and logistic regression models, i.e. for binomial and Poisson regression models with canonical links. See, Firth (1993) for details.
brglm_control is an alias to brglmControl.

## Value

a list with components named as the arguments, including symbolic expressions for the dispersion transformation (Trans) and its inverse (inverseTrans)

## Author(s)

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## References

Kosmidis I, Firth D (2021). Jeffreys-prior penalty, finiteness and shrinkage in binomial-response generalized linear models. *Biometrika*, **108**, 71-82 doi: 10.1093/biomet/asaa052
Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30**, 43-59 doi: 10.1007/s11222019098606

Firth D (1993). Bias reduction of maximum likelihood estimates. Biometrika, **80**, 27-38 doi: 10.2307/2336755

## See Also

brglm_fit and glm.fit

## Examples

```
data("coalition", package = "brglm2")
## The maximum likelihood fit with log link
coalitionML <- glm(duration ~ fract + numst2, family = Gamma, data = coalition)
## Bias reduced estimation of the dispersion parameter
coalitionBRi <- glm(duration ~ fract + numst2, family = Gamma, data = coalition,
    method = "brglmFit")
coef(coalitionBRi, model = "dispersion")
## Bias reduced estimation of log(dispersion)
```

```
coalitionBRl <- glm(duration ~ fract + numst2, family = Gamma, data = coalition,
    method = "brglmFit", transformation = "log")
coef(coalitionBRl, model = "dispersion")
## Just for illustration: Bias reduced estimation of dispersion^0.25
my_transformation <- list(expression(dispersion^0.25), expression(transformed_dispersion^4))
coalitionBRc <- update(coalitionBRi, transformation = my_transformation)
coef(coalitionBRc, model = "dispersion")
```

brglmFit Fitting function for glm for reduced-bias estimation and inference

## Description

brglmFit is a fitting method for glm that fits generalized linear models using implicit and explicit bias reduction methods (Kosmidis, 2014), and other penalized maximum likelihood methods. Currently supported methods include the mean bias-reducing adjusted scores approach in Firth (1993) and Kosmidis \& Firth (2009), the median bias-reduction adjusted scores approach in Kenne Pagui et al. (2017), the correction of the asymptotic bias in Cordeiro \& McCullagh (1991), the mixed biasreduction adjusted scores approach in Kosmidis et al (2020), maximum penalized likelihood with powers of the Jeffreys prior as penalty, and maximum likelihood. Estimation is performed using a quasi Fisher scoring iteration (see vignette("iteration", "brglm2")), which, in the case of mean-bias reduction, resembles an iterative correction of the asymptotic bias of the Fisher scoring iterates.

## Usage

> brglmFit(
x ,
$y$,
weights = rep(1, nobs),
start = NULL,
etastart = NULL,
mustart = NULL,
offset $=$ rep(0, nobs),
family = gaussian(),
control = list(),
intercept = TRUE,
fixed_totals = NULL,
singular.ok = TRUE
)
brglm_fit(
x ,
$y$,
weights $=$ rep(1, nobs),
start = NULL,

```
    etastart = NULL,
    mustart = NULL,
    offset = rep(0, nobs),
    family = gaussian(),
    control = list(),
    intercept = TRUE,
    fixed_totals = NULL,
    singular.ok = TRUE
)
```


## Arguments

\(\left.$$
\begin{array}{ll}\mathrm{x} & \mathrm{x} \text { is a design matrix of dimension } \mathrm{n} * \mathrm{p} . \\
\mathrm{y} & \mathrm{y} \text { is a vector of observations of length } \mathrm{n} . \\
\text { weights } & \begin{array}{l}\text { an optional vector of 'prior weights' to be used in the fitting process. Should be } \\
\text { NULL or a numeric vector. }\end{array} \\
\text { start } & \begin{array}{l}\text { starting values for the parameters in the linear predictor. If NULL (default) then } \\
\text { the maximum likelihood estimates are calculated and used as starting values. }\end{array} \\
\text { etastart } & \begin{array}{l}\text { applied only when start is not NULL. Starting values for the linear predictor to be } \\
\text { passed to glm. fit when computing starting values using maximum likelihood. }\end{array} \\
\text { mustart } & \begin{array}{l}\text { applied only when start is not NULL. Starting values for the vector of means to be } \\
\text { passed to glm.fit when computing starting values using maximum likelihood. } \\
\text { this can be used to specify an a priori known component to be included in the }\end{array}
$$ <br>

linear predictor during fitting. This should be NULL or a numeric vector of length\end{array}\right\}\)| equal to the number of cases. One or more offset terms can be included in the |
| :--- |
| formula instead or as well, and if more than one is specified their sum is used. |
| See model.offset. |

## Details

A detailed description of the supported adjustments and the quasi Fisher scoring iteration is given in the iteration vignette (see, vignette("iteration", "brglm2") or Kosmidis et al, 2020). A shorter description of the quasi Fisher scoring iteration is also given in one of the vignettes of the *enrichwith* R package (see, https://cran.r-project.org/package=enrichwith/vignettes/bias.
html). Kosmidis and Firth (2010) describe a parallel quasi Newton-Raphson iteration with the same stationary point.

In the special case of generalized linear models for binomial, Poisson and multinomial responses, the adjusted score equation approaches for type = "AS_mixed", type = "AS_mean", and type = "AS_median" (see below for what methods each type corresponds) return estimates with improved frequentist properties, that are also always finite, even in cases where the maximum likelihood estimates are infinite (e.g. complete and quasi-complete separation in multinomial regression). See, Kosmidis and Firth (2021) for a proof for binomial-response GLMs with Jeffreys-prior penalties to the log-likelihood, which is equivalent to mean bias reduction for logistic regression. See, also, detect_separation and check_infinite_estimates for pre-fit and post-fit methods for the detection of infinite estimates in binomial response generalized linear models.

The type of score adjustment to be used is specified through the type argument (see brglmControl for details). The available options are

- type = "AS_mixed": the mixed bias-reducing score adjustments in Kosmidis et al (2020) that result in mean bias reduction for the regression parameters and median bias reduction for the dispersion parameter, if any; default.
- type = "AS_mean": the mean bias-reducing score adjustments in Firth, 1993 and Kosmidis \& Firth, 2009. type = "AS_mixed" and type = "AS_mean" will return the same results when family is binomial or poisson, i.e. when the dispersion is fixed)
- type = "AS_median": the median bias-reducing score adjustments in Kenne Pagui et al. (2017)
- type = "MPL_Jeffreys": maximum penalized likelihood with powers of the Jeffreys prior as penalty.
- type = "ML": maximum likelihood.
- type = "correction": asymptotic bias correction, as in Cordeiro \& McCullagh (1991).

The null deviance is evaluated based on the fitted values using the method specified by the type argument (see brglmControl).
The family argument of the current version of brglmFit can accept any combination of family objects and link functions, including families with user-specified link functions, mis links, and power links, but excluding quasi, quasipoisson and quasibinomial families.

The description of method argument and the Fitting functions section in glm gives information on supplying fitting methods to glm.
fixed_totals to specify groups of observations for which the sum of the means of a Poisson model will be held fixed to the observed count for each group. This argument is used internally in brmultinom and bracl for baseline-category logit models and adjacent category logit models, respectively.
brglm_fit is an alias to brglmFit.

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## References

Kosmidis I, Firth D (2021). Jeffreys-prior penalty, finiteness and shrinkage in binomial-response generalized linear models. *Biometrika*, ${ }^{* *}$ 108**, 71-82 doi: 10.1093/biomet/asaa0052
Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. ${ }^{* S t a t i s t i c s ~ a n d ~ C o m p u t i n g *, ~}{ }^{* *} 30^{* *}, 43-59$ doi: 10.1007/s11222019098606
Cordeiro G M, McCullagh P (1991). Bias correction in generalized linear models. *Journal of the Royal Statistical Society. Series B (Methodological)*, **53**, 629-643 doi: 10.1111/j.25176161.1991.tb01852.x

Firth D (1993). Bias reduction of maximum likelihood estimates. *Biometrika*. **80**, 27-38 doi: 10.2307/2336755

Kenne Pagui E C, Salvan A, Sartori N (2017). Median bias reduction of maximum likelihood estimates. *Biometrika*, **104**, 923-938 doi: 10.1093/biomet/asx046
Kosmidis I, Firth D (2009). Bias reduction in exponential family nonlinear models. *Biometrika*, **96**, 793-804 doi: 10.1093/biomet/asp055

Kosmidis I, Firth D (2010). A generic algorithm for reducing bias in parametric estimation. *Electronic Journal of Statistics*, **4**, 1097-1112 doi: 10.1214/10EJS579
Kosmidis I (2014). Bias in parametric estimation: reduction and useful side-effects. *WIRE Computational Statistics*, **6**, 185-196 doi: 10.1002/wics. 1296

## See Also

brglmControl, glm.fit,glm

## Examples

```
## The lizards example from ?brglm::brglm
data("lizards")
# Fit the model using maximum likelihood
lizardsML <- glm(cbind(grahami, opalinus) ~ height + diameter +
    light + time, family = binomial(logit), data = lizards,
    method = "glm.fit")
# Mean bias-reduced fit:
lizardsBR_mean <- glm(cbind(grahami, opalinus) ~ height + diameter +
                            light + time, family = binomial(logit), data = lizards,
                            method = "brglmFit")
# Median bias-reduced fit:
lizardsBR_median <- glm(cbind(grahami, opalinus) ~ height + diameter +
    light + time, family = binomial(logit), data = lizards,
    method = "brglmFit", type = "AS_median")
summary(lizardsML)
summary(lizardsBR_median)
summary(lizardsBR_mean)
# Maximum penalized likelihood with Jeffreys prior penatly
lizards_Jeffreys <- glm(cbind(grahami, opalinus) ~ height + diameter +
    light + time, family = binomial(logit), data = lizards,
    method = "brglmFit", type = "MPL_Jeffreys")
# lizards_Jeffreys is the same fit as lizardsBR_mean (see Firth, 1993)
```

```
all.equal(coef(lizardsBR_mean), coef(lizards_Jeffreys))
# Maximum penalized likelihood with powers of the Jeffreys prior as
# penalty. See Kosmidis & Firth (2021) for the finiteness and
# shrinkage properties of the maximum penalized likelihood
# estimators in binomial response models
a <- seq(0, 20, 0.5)
coefs <- sapply(a, function(a) {
    out <- glm(cbind(grahami, opalinus) ~ height + diameter +
        light + time, family = binomial(logit), data = lizards,
        method = "brglmFit", type = "MPL_Jeffreys", a = a)
    coef(out)
})
# Illustration of shrinkage as a grows
matplot(a, t(coefs), type = "l", col = 1, lty = 1)
abline(0, 0, col = "grey")
## Another example from
## King, Gary, James E. Alt, Nancy Elizabeth Burns and Michael Laver
## (1990). "A Unified Model of Cabinet Dissolution in Parliamentary
## Democracies", _American Journal of Political Science_, **34**, 846-870
data("coalition", package = "brglm2")
# The maximum likelihood fit with log link
coalitionML <- glm(duration ~ fract + numst2, family = Gamma, data = coalition)
# The mean bias-reduced fit
coalitionBR_mean <- update(coalitionML, method = "brglmFit")
# The bias-corrected fit
coalitionBC <- update(coalitionML, method = "brglmFit", type = "correction")
# The median bias-corrected fit
coalitionBR_median <- update(coalitionML, method = "brglmFit", type = "AS_median")
## An example with offsets from Venables & Ripley (2002, p.189)
data("anorexia", package = "MASS")
anorexML <- glm(Postwt ~ Prewt + Treat + offset(Prewt),
    family = gaussian, data = anorexia)
anorexBC <- update(anorexML, method = "brglmFit", type = "correction")
anorexBR_mean <- update(anorexML, method = "brglmFit")
anorexBR_median <- update(anorexML, method = "brglmFit", type = "AS_median")
## All methods return the same estimates for the regression
## parameters because the maximum likelihood estimator is normally
## distributed around the `true` value under the model (hence, both
## mean and component-wise median unbiased). The Wald tests for
## anorexBC and anorexBR_mean differ from anorexML
## because the bias-reduced estimator of the dispersion is the
## unbiased, by degree of freedom adjustment (divide by n - p),
```

```
## estimator of the residual variance. The Wald tests from
## anorexBR_median are based on the median bias-reduced estimator
## of the dispersion that results from a different adjustment of the
## degrees of freedom (divide by n - p - 2/3)
summary (anorexML)
summary(anorexBC)
summary(anorexBR_mean)
summary (anorexBR_median)
## endometrial data from Heinze & Schemper (2002) (see ?endometrial)
data("endometrial", package = "brglm2")
endometrialML <- glm(HG ~ NV + PI + EH, data = endometrial,
    family = binomial("probit"))
endometrialBR_mean <- update(endometrialML, method = "brglmFit",
                    type = "AS_mean")
endometrialBC <- update(endometrialML, method = "brglmFit",
                            type = "correction")
endometrialBR_median <- update(endometrialML, method = "brglmFit",
                                    type = "AS_median")
summary(endometrialML)
summary(endometrialBC)
summary(endometrialBR_mean)
summary(endometrialBR_median)
```


## brmultinom

Bias reduction for multinomial response models using the Poisson trick.

## Description

brmultinom is a wrapper of brglmFit that fits multinomial regression models using implicit and explicit bias reduction methods. See Kosmidis \& Firth (2011) for details.

## Usage

```
brmultinom(
    formula,
    data,
    weights,
    subset,
    na.action,
    contrasts = NULL,
    ref = 1,
    model = TRUE,
    x = TRUE,
    control = list(...),
)
```


## Arguments

| formula | a formula expression as for regression models, of the form response $\sim$ predictors. The response should be a factor or a matrix with K columns, which will be interpreted as counts for each of K classes. A log-linear model is fitted, with coefficients zero for the first class. An offset can be included: it should be a numeric matrix with K columns if the response is either a matrix with K columns or a factor with $\mathrm{K}>=2$ classes, or a numeric vector for a response factor with 2 levels. See the documentation of formula() for other details. |
| :---: | :---: |
| data weights | an optional data frame in which to interpret the variables occurring in formula. optional case weights in fitting. |
| subset | expression saying which subset of the rows of the data should be used in the fit. All observations are included by default. |
| na.action | a function to filter missing data. |
| contrasts | a list of contrasts to be used for some or all of the factors appearing as variables in the model formula. |
| ref | the reference category to use for multinomial regression. Either an integer, in which case levels(response)[ref] is used as a baseline, or a character string. Default is 1 . |
| model | logical. If true, the model frame is saved as component model of the returned object. |
| x | should the model matrix be included with in the result (default is TRUE). |
| control | a list of parameters for controlling the fitting process. See brglmControl for details. |
|  | arguments to be used to form the default 'control' argument if it is not supplied directly. |

## Details

The models brmultinom handles are also known as baseline-category logit models (see, Agresti, 2002, Section 7.1), because they model the log-odds of every category against a baseline category. The user can control which baseline (or reference) category is used via the ref. By default brmultinom uses the first category as reference.
The maximum likelihood estimates for the parameters of baseline-category logit models have infinite components with positive probability, which can result in problems in their estimation and the use of inferential procedures (e.g. Wad tests). Albert and Andreson (1984) have categorized the possible data patterns for such models into the exclusive and exhaustive categories of complete separation, quasi-complete separation and overlap, and showed that infinite maximum likelihood estimates result when complete or quasi-complete separation occurs.
The adjusted score approaches to bias reduction that brmultinom implements for type = "AS_mean" and type = "AS_median" are alternatives to maximum likelihood that result in estimates with smaller asymptotic mean and median bias, respectively, that are also *always* finite, even in cases of complete or quasi-complete separation.
brmultinom is a wrapper of brglmFit that fits multinomial logit regression models through the 'Poisson trick' (see, for example, Palmgren, 1981; Kosmidis \& Firth, 2011).

The implementation relies on the construction of an 'extended' model matrix for the log-linear model and constraints on the sums of the Poisson means. Specifically, a log-linear model is fitted on a Kronecker product (https://en.wikipedia.org/wiki/Kronecker_product) of the original model matrix X implied by the formula, augmented by nrow $(X)$ dummy variables.
The extended model matrix is sparse, and the Matrix package is used for its effective storage.
While brmultinom can be used for analyses using multinomial regression models, the current implementation is more of a 'proof of concept' and is not expected to scale well with either of nrow $(X)$, $n \operatorname{col}(X)$ or the number of levels in the categorical response.

## Author(s)

Ioannis Kosmidis [ioannis.kosmidis@warwick.ac.uk](mailto:ioannis.kosmidis@warwick.ac.uk)

## References

Kosmidis I, Kenne Pagui E C, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing ${ }^{*}$, **30**, 43-59 doi: 10.1007/s11222019098606

Agresti A (2002). *Categorical data analysis* (2nd edition). Wiley Series in Probability and Statistics. Wiley.
Albert A, Anderson J A (1984). On the Existence of Maximum Likelihood Estimates in Logistic Regression Models. *Biometrika*, **71** 1-10, doi: 10.2307/2336390
Kosmidis I, Firth D (2011). Multinomial logit bias reduction via the Poisson log-linear model. *Biometrika*, **98**, 755-759 doi: 10.1093/biomet/asr026
Palmgren, J (1981). The Fisher Information Matrix for Log Linear Models Arguing Conditionally on Observed Explanatory Variables. *Biometrika*, **68**, 563-566 doi: 10.1093/biomet/68.2.563

## See Also

multinom, bracl for adjacent category logit models for ordinal responses

## Examples

```
data("housing", package = "MASS")
# Maximum likelihood using nnet::multinom
houseML1nnet <- nnet::multinom(Sat ~ Infl + Type + Cont, weights = Freq,
                    data = housing)
# Maximum likelihood using brmultinom with baseline category 'Low'
houseML1 <- brmultinom(Sat ~ Infl + Type + Cont, weights = Freq,
    data = housing, type = "ML", ref = 1)
# The estimates are numerically the same as houseML0
all.equal(coef(houseML1nnet), coef(houseML1), tolerance = 1e-04)
# Maximum likelihood using brmultinom with 'High' as baseline
houseML3 <- brmultinom(Sat ~ Infl + Type + Cont, weights = Freq,
    data = housing, type = "ML", ref = 3)
# The fitted values are the same as houseML1
all.equal(fitted(houseML3), fitted(houseML1), tolerance = 1e-10)
```

```
# Bias reduction
houseBR3 <- update(houseML3, type = "AS_mean")
# Bias correction
houseBC3 <- update(houseML3, type = "correction")
```

brnb Bias reduction for negative binomial regression models

## Description

brnb is a function that fits negative binomial regression models using implicit and explicit bias reduction methods.

## Usage

brnb(
formula,
data,
subset,
weights = NULL,
offset = NULL,
link = "log",
start = NULL,
etastart = NULL,
mustart = NULL,
control = list(...),
na.action,
model = TRUE,
$x$ = FALSE,
$y=$ TRUE,
contrasts $=$ NULL,
intercept = TRUE,
singular.ok = TRUE,
...
)

## Arguments

| formula | an object of class "formula" (or one that can be coerced to that class): a sym- <br> bolic description of the model to be fitted. The details of model specification are <br> given under 'Details'. |
| :--- | :--- |
| data | an optional data frame, list or environment (or object coercible by as.data.frame <br> to a data frame) containing the variables in the model. If not found in data, <br> the variables are taken from environment (formula), typically the environment <br> from which glm is called. |


| subset | an optional vector specifying a subset of observations to be used in the fitting <br> process. <br> an optional vector of 'prior weights' to be used in the fitting process. Should be <br> NULL or a numeric vector. <br> offset <br> this can be used to specify an a priori known component to be included in the <br> linear predictor during fitting. This should be NULL or a numeric vector of length <br> equal to the number of cases. One or more offset terms can be included in the <br> formula instead or as well, and if more than one is specified their sum is used. <br> See model. offset. |
| :--- | :--- |
| link | The link function. Currently must be one of log, sqrt or identity. <br> start |
| starting values for the parameters in the linear predictor. |  |
| etastart | starting values for the linear predictor. |
| starting values for the vector of means. |  |

## Details

A detailed description of the fitting procedure is given in the iteration vignette (see, vignette("iteration", "brglm2") and Kosmidis et al, 2020). The number of iterations when estimating parameters are controlled by the maxit argument of brglmControl.

The type of score adjustment to be used is specified through the type argument (see brglmControl for details).

The available options are:

- type = "AS_mixed": the mixed bias-reducing score adjustments in Kosmidis et al (2020) that result in mean bias reduction for the regression parameters and median bias reduction for the dispersion parameter, if any; default.
- type = "AS_mean": the mean bias-reducing score adjustments in Firth (1993) and Kosmidis \& Firth (2009).
- type = "AS_median": the median bias-reducing score adjustments in Kenne Pagui et al. (2017)
- type = "MPL_Jeffreys": maximum penalized likelihood with powers of the Jeffreys prior as penalty.
- type = "ML": maximum likelihood.
- type = "correction": asymptotic bias correction, as in Cordeiro \& McCullagh (1991).

The choice of the parameterization for the dispersion is controlled by the transformation argument (see brglmControl for details). The default is "identity". Using transformation = "inverse" uses the dispersion parameterization that glm.nb uses.

## Value

A fitted model object of class brnb inheriting from negbin and brglmFit. The object is similar to the output of brglmFit but contains four additional components: theta for the maximum likelihood estimate of the dispersion parameter as in glm.nb, vcov.mean for the estimated variancecovariance matrix of the regression coefficients, vcov.dispersion for the estimated variance of the dispersion parameter in the chosen parameterization (using the expected information), and twologlik for twice the log-likelihood function.

## Author(s)

Euloge Clovis Kenne Pagui [ctb] [kenne@stat.unipd.it](mailto:kenne@stat.unipd.it), Ioannis Kosmidis [aut, cre]<ioannis.kosmidis@warwick.ac.

## References

Cordeiro GM \& McCullagh, P (1991). Bias correction in generalized linear models. *Journal of the Royal Statistical Society. Series B(Methodological)*, **53**, 629-643.
Firth D (1993). Bias reduction of maximum likelihood estimates, Biometrika. ${ }^{* * 80 * *, ~ 27-38 . ~}$
Kenne Pagui EC, Salvan A, and Sartori N (2017). Median bias reduction of maximum likelihood estimates. *Biometrika*, **104**, 923-938

Kosmidis I, Kenne Pagui EC, Sartori N (2020). Mean and median bias reduction in generalized linear models. *Statistics and Computing*, **30** 43-59.
Kosmidis I and Firth D (2009). Bias reduction in exponential family nonlinear models. *Biometrika*, **96**, 793-804.

## Examples

```
# Example in Saha, K., & Paul, S. (2005). Bias-corrected maximum
# likelihood estimator of the negative binomial dispersion
# parameter. Biometrics, 61, 179--185.
#
# Number of revertant colonies of salmonella data
salmonella <- data.frame(freq = c(15, 16, 16, 27, 33, 20,
                                    21, 18, 26, 41, 38, 27,
                                    29, 21, 33, 60, 41, 42),
                                    dose = rep(c(0, 10, 33, 100, 333, 1000), 3),
                                    observation = rep(1:3, each = 6))
# Maximum likelihood fit with glm.nb of MASS
salmonella_fm <- freq ~ dose + log(dose + 10)
fitML_glmnb <- MASS::glm.nb(salmonella_fm, data = salmonella)
# Maximum likelihood fit with brnb
fitML <- brnb(salmonella_fm, data = salmonella,
            link = "log", transformation = "inverse", type = "ML")
# Mean bias-reduced fit
fitBR_mean <- update(fitML, type = "AS_mean")
# Median bias-reduced fit
fitBR_median <- update(fitML, type = "AS_median")
# Mixed bias-reduced fit
fitBR_mixed <- update(fitML, type = "AS_mixed")
# Mean bias-corrected fit
fitBC_mean <- update(fitML, type = "correction")
# Penalized likelihood with Jeffreys-prior penalty
fit_Jeffreys <- update(fitML, type = "MPL_Jeffreys")
# The parameter estimates from glm.nb and brnb with type = "ML" are
# numerically the same
all.equal(c(coef(fitML_glmnb), fitML_glmnb$theta),
    coef(fitML, model = "full"), check.attributes = FALSE)
# Because of the invariance properties of the maximum likelihood,
# median reduced-bias, and mixed reduced-bias estimators the
# estimate of a monotone function of the dispersion should be
# (numerically) the same as the function of the estimate of the
# dispersion:
# ML
coef(fitML, model = "dispersion")
1 / coef(update(fitML, transformation = "identity"), model = "dispersion")
# Median BR
coef(fitBR_median, model = "dispersion")
```

```
1 / coef(update(fitBR_median, transformation = "identity"), model = "dispersion")
# Mixed BR
coef(fitBR_mixed, model = "dispersion")
1 / coef(update(fitBR_mixed, transformation = "identity"), model = "dispersion")
## The same is not true for mean BR
coef(fitBR_mean, model = "dispersion")
1 / coef(update(fitBR_mean, transformation = "identity"), model = "dispersion")
## An example from Venables & Ripley (2002, p.169).
data("quine", package = "MASS")
quineML <- brnb(Days ~ Sex/(Age + Eth*Lrn), link = "sqrt", transformation="inverse",
    data = quine, type="ML")
quineBR_mean <- update(quineML, type = "AS_mean")
quineBR_median <- update(quineML, type = "AS_median")
quineBR_mixed <- update(quineML, type = "AS_mixed")
quine_Jeffreys <- update(quineML, type = "MPL_Jeffreys")
fits <- list(ML = quineML,
    AS_mean = quineBR_mean,
    AS_median = quineBR_median,
    AS_mixed = quineBR_mixed,
    MPL_Jeffreys = quine_Jeffreys)
sapply(fits, coef, model = "full")
```

coalition

## Coalition data

## Description

Coalition data

## Usage

coalition

## Format

An object of class data. frame with 314 rows and 7 columns.

## Note

Data is as provided in the Zeilig R package (https://cran.r-project.org/package=Zelig)

## References

King G, Alt J E, Burns N E, Laver M. (1990). A Unified Model of Cabinet Dissolution in Parliamentary Democracies. *American Journal of Political Science*, **34**, 846-870. doi: 10.2307/ 2111401

King G, Alt J E, Burns N E, Laver M. ICPSR Publication Related Archive, 1115.

## See Also

brglm_fit
coef.brglmFit Extract model coefficients from brglmFit objects

## Description

Extract model coefficients from brglmFit objects

## Usage

\#\# S3 method for class 'brglmFit'
coef(object, model = c("mean", "full", "dispersion"), ...)

## Arguments

object an object for which the extraction of model coefficients is meaningful.
model one of "mean" (default), "dispersion", "full", to return the estimates of the parameters in the linear prediction only, the estimate of the dispersion parameter only, or both, respectively.
... other arguments.

## Details

See coef for more details.

## See Also

coef
coef.brnb Extract model coefficients from brnb objects

## Description

Extract model coefficients from brnb objects

## Usage

```
## S3 method for class 'brnb'
coef(object, model = c("mean", "full", "dispersion"), ...)
```


## Arguments

object an object for which the extraction of model coefficients is meaningful.
model one of "mean" (default), "full", "dispersion", to return the estimates of the parameters in the linear prediction only, or both, the estimate of the dispersion parameter only, respectively.
... other arguments.

## Details

See coef for more details.
confint.brglmFit Method for computing confidence intervals for one or more regression parameters in a brglmFit object

## Description

Method for computing confidence intervals for one or more regression parameters in a brglmFit object

## Usage

\#\# S3 method for class 'brglmFit'
confint(object, parm, level $=0.95, \ldots$ )

## Arguments

object a fitted model object.
parm a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
level the confidence level required.
... additional argument(s) for methods.

```
confint.brmultinom Method for computing confidence intervals for one or more regression
parameters in a brmultinom object
```


## Description

Method for computing confidence intervals for one or more regression parameters in a brmultinom object

## Usage

\#\# S3 method for class 'brmultinom'
confint(object, parm, level = 0.95, ...)

## Arguments

object
parm
level
...
confint.brnb
a fitted model object.
a specification of which parameters are to be given confidence intervals, either a vector of numbers or a vector of names. If missing, all parameters are considered.
the confidence level required. additional argument(s) for methods.

## Description

Method for computing Wald confidence intervals for one or more regression parameters in a brnb object

## Usage

\#\# S3 method for class 'brnb'
confint(object, parm, level = 0.95, ...)

## Arguments

| object | a fitted model object. <br> a specification of which parameters are to be given confidence intervals, either <br> a vector of numbers or a vector of names. If missing, all parameters are consid- <br> ered. |
| :--- | :--- |
| level | the confidence level required. |
| $\ldots$ | additional argument(s) for methods. |

## Description

Histology grade and risk factors for 79 cases of endometrial cancer

## Usage

endometrial

## Format

A data frame with 79 rows and 4 variables:
NV neovasculization with coding 0 for absent and 1 for present
PI pulsality index of arteria uterina
EH endometrium height
HG histology grade with coding 0 for low grade and 1 for high grade

## Source

The packaged data set was downloaded in . dat format from http://www.stat.ufl.edu/~aa/ glm/data. The latter link provides the data sets used in Agresti (2015).
The endometrial data set was first analyzed in Heinze and Schemper (2002), and was originally provided by Dr E. Asseryanis from the Medical University of Vienna.

## References

Agresti A (2015). *Foundations of Linear and Generalized Linear Models*. Wiley Series in Probability and Statistics. Wiley
Heinze G, Schemper M (2002). A Solution to the Problem of Separation in Logistic Regression. *Statistics in Medicine*, **21**, 2409-2419 doi: 10.1002/sim. 1047

Kosmidis I, Firth D (2021). Jeffreys-prior penalty, finiteness and shrinkage in binomial-response generalized linear models. *Biometrika*, ${ }^{* *} 108^{* *}, 71-82$ doi: 10.1093/biomet/asaa0052

## See Also

brglm_fit

## Description

The lizards data frame has 23 rows and 6 columns. Variables grahami and opalinus are counts of two lizard species at two different perch heights, two different perch diameters, in sun and in shade, at three times of day.

## Usage

lizards

## Format

An object of class data. frame with 23 rows and 6 columns.

## Details

- grahami. count of grahami lizards
- opalinus. count of opalinus lizards
- height. a factor with levels $<5 \mathrm{ft},>=5 \mathrm{ft}$
- diameter. a factor with levels $<=2 \mathrm{in},>2 \mathrm{in}$
- light. a factor with levels sunny, shady
- time. a factor with levels early, midday, late


## Source

McCullagh P, Nelder J A (1989) _Generalized Linear Models_ (2nd Edition). London: Chapman and Hall.

Originally from
Schoener T W (1970) Nonsynchronous spatial overlap of lizards in patchy habitats. _Ecology_ *51*, 408-418.

## See Also

brglm_fit

## A link-glm object for misclassified responses in binomial regression

 models
## Description

mis is a link-glm object that specifies the link function in Neuhaus (1999, expression~(8)) for handling misclassified responses in binomial regression models using maximum likelihood. A prior specification of the sensitivity and specificity is required.

## Usage

```
mis(link = "logit", sensitivity = 1, specificity = 1)
```


## Arguments

link the baseline link to be used.
sensitivity the probability of observing a success given that a success actually took place given any covariate values.
specificity the probability of observing a failure given that a failure actually took place given any covariate values.

## Details

sensitivity + specificity should be greater or equal to 1 , otherwise it is implied that the procedure producing the responses performs worse than chance in terms of misclassification.

## References

Neuhaus J M (1999). Bias and efficiency loss due to misclassified responses in binary regression. Biometrika, **86**, 843-855 https://www.jstor.org/stable/2673589

## See Also

glm, brglm_fit

## Examples

```
## Define a few links with some misclassification
logit_mis <- mis(link = "logit", sensitivity = 0.9, specificity = 0.9)
lizards_f <- cbind(grahami, opalinus) ~ height + diameter + light + time
lizardsML <- glm(lizards_f, family = binomial(logit), data = lizards)
lizardsML_mis <- update(lizardsML, family = binomial(logit_mis),
    start = coef(lizardsML))
```

```
## A notable change is coefficients is noted here compared to when
## specificity and sensitity are 1
coef(lizardsML)
coef(lizardsML_mis)
## Bias reduction is also possible
update(lizardsML_mis, method = "brglmFit", type = "AS_mean",
    start = coef(lizardsML))
update(lizardsML_mis, method = "brglmFit", type = "AS_median",
    start = coef(lizardsML))
```

ordinal_superiority Ordinal superiority scores of Agresti and Kateri (2017)

## Description

ordinal_superiority is a method for the estimation and inference about model-based ordinal superiority scores introduced in Agresti and Kateri (2017, Section 5) from fitted objects. The mean bias of the estimates of the ordinal superiority scores can be corrected.

## Usage

```
    ordinal_superiority(
        object,
        formula,
        data,
        measure = c("gamma", "Delta"),
        level = 0.95,
        bc = FALSE
    )
    ## S3 method for class 'bracl'
    ordinal_superiority(
    object,
    formula,
    data,
    measure = c("gamma", "Delta"),
    level = 0.95,
    bc = FALSE
    )
```


## Arguments

object a fitted object from an ordinal regression model. Currently only models from class "bracl" are supported.

| formula | a RHS formula indicating the group variable to use. <br> an optional data frame in which to look for variables with which to compute <br> ordinal superiority measures. If omitted, an attempt is made to use the data that <br> produced object. |
| :--- | :--- |
| measure | either "gamma" (default) or "Delta", specifying the ordinal superiority measure <br> to be returned. <br> the confidence level required when computing confidence intervals for the ordi- <br> nal superiority measures. |
| bc | logical. If FALSE (default) then the ordinal superiority measures are computed <br> using the estimates in object. If TRUE then the ordinal superiority measure <br> estimates are corrected for mean bias. |

## References

Agresti, A., Kateri, M. (2017). Ordinal probability effect measures for group comparisons in multinomial cumulative link models. *Biometrics*, **73** 214-219 \#' doi: 10.1111/biom. 12565

## Examples

```
data("stemcell", package = "brglm2")
# Adjacent category logit (proportional odds)
stem <- within(stemcell, {nreligion = as.numeric(religion)})
fit_bracl_p <- bracl(research ~ nreligion + gender, weights = frequency,
            data = stem, type = "ML", parallel = TRUE)
# Estimates and 95% confidence intervals for the probabilities that the response
# category for gender "female" is higher than the response category for gender "male",
# while adjusting for religion.
ordinal_superiority(fit_bracl_p, ~ gender)
## Not run:
# And their (very-similar in value here) bias corrected versions
# with 99% CIs
ordinal_superiority(fit_bracl_p, ~ gender, bc = TRUE, level = 0.99)
# Note that the object is refitted with type = "AS_mean"
## End(Not run)
```

predict.bracl Predict method for bracl fits

## Description

Obtain class and probability predictions from a fitted adjacent category logits model.

## Usage

```
## S3 method for class 'bracl'
predict(object, newdata, type = c("class", "probs"), ...)
```


## Arguments

| object | a fitted object of class inheriting from "bracl". |
| :--- | :--- |
| newdata | optionally, a data frame in which to look for variables with which to predict. If <br> omitted, the fitted linear predictors are used. |
| type | the type of prediction required. The default is "class", which produces pre- <br> dictions of the response category at the covariate values supplied in "newdata", <br> selecting the category with the largest probability; the alternative "probs" re- <br> turns all category probabilities at the covariate values supplied in "newdata". |
| $\ldots$ | further arguments passed to or from other methods. |

## Details

If newdata is omitted the predictions are based on the data used for the fit.

## Value

If type = "class" a vector with the predicted response categories; if type = "probs" a matrix of probabilities for all response categories at newdata.

## Examples

```
data("stemcell", package = "brglm2")
# Adjacent category logit (non-proportional odds)
fit_bracl <- bracl(research ~ as.numeric(religion) + gender, weights = frequency,
    data = stemcell, type = "ML")
# Adjacent category logit (proportional odds)
fit_bracl_p <- bracl(research ~ as.numeric(religion) + gender, weights = frequency,
    data = stemcell, type = "ML", parallel = TRUE)
# New data
newdata <- expand.grid(gender = c("male", "female"),
                        religion = c("liberal", "moderate", "fundamentalist"))
# Predictions
sapply(c("class", "probs"), function(what) predict(fit_bracl, newdata, what))
sapply(c("class", "probs"), function(what) predict(fit_bracl_p, newdata, what))
```

```
predict.brmultinom Predict method for brmultinom fits
```


## Description

Obtain class and probability predictions from a fitted baseline category logits model.

## Usage

\#\# S3 method for class 'brmultinom'
predict(object, newdata, type $=c(" c l a s s ", " p r o b s "), \ldots)$

## Arguments

object a fitted object of class inheriting from "brmultinom".
newdata optionally, a data frame in which to look for variables with which to predict. If omitted, the fitted linear predictors are used.
type the type of prediction required. The default is "class", which produces predictions of the response category at the covariate values supplied in "newdata", selecting the category with the largest probability; the alternative "probs" returns all category probabilities at the covariate values supplied in "newdata".
... further arguments passed to or from other methods.

## Details

If newdata is omitted the predictions are based on the data used for the fit.

## Value

If type = "class" a vector with the predicted response categories; if type = "probs" a matrix of probabilities for all response categories at newdata.

## Examples

```
data("housing", package = "MASS")
# Maximum likelihood using brmultinom with baseline category 'Low'
houseML1 <- brmultinom(Sat ~ Infl + Type + Cont, weights = Freq,
        data = housing, type = "ML", ref = 1)
# New data
newdata <- expand.grid(Infl = c("Low", "Medium"),
    Type = c("Tower", "Atrium", "Terrace"),
    Cont = c("Low", NA, "High"))
## Predictions
sapply(c("class", "probs"), function(what) predict(houseML1, newdata, what))
```

```
print.summary.brnb Summarizing Linear Model Fits
```


## Description

print summary output for class "brnb".

## Usage

\#\# S3 method for class 'summary.brnb'
print(x, digits $=\max (3$, getOption("digits") -3$), \ldots$ )

## Arguments

X
digits
... an object of class "summary.brnb", usually, a result of a call to summary.brnb. the number of significant digits to use when printing. extra arguments to passed to methods. Not used currently.

## Details

print. summary.brnb tries to be smart about formatting the coefficients, standard errors, and additionally gives "significant stars". The coefficients components of the result gives the estimated coefficients and their estimated standard errors, together with their ratio ( $z$ ). A fourth column gives the two-tailed p-value corresponding to the $z$ statistica based on Normal reference distribution.

```
residuals.brmultinom Residuals for multinomial logistic regression and adjacent category
logit models
```


## Description

Residuals for multinomial logistic regression and adjacent category logit models

## Usage

```
## S3 method for class 'brmultinom'
residuals(object, type = c("pearson", "response", "deviance", "working"), ...)
## S3 method for class 'bracl'
residuals(object, type = c("pearson", "response", "deviance", "working"), ...)
```


## Arguments

object the object coming out of bracl and brmultinom.
type the type of residuals which should be returned. The options are: "pearson" (default), "response", "deviance", "working". See Details.
... Currently not used.

## Details

The residuals computed are the residuals from the equivalent Poisson log-linear model fit, organized in a form that matches the output of fitted(object, type = "probs"). As a result, the output is residuals defined in terms of the object and expected multinomial counts.

## See Also

brmultinom bracl
simulate.brmultinom Method for simulating a data set from brmultinom and bracl objects

## Description

Method for simulating a data set from brmultinom and bracl objects

## Usage

\#\# S3 method for class 'brmultinom'
simulate(object, ...)

## Arguments

$$
\begin{array}{ll}
\text { object } & \text { an object of class brmultinom or bracl. } \\
\ldots & \text { currently not used. }
\end{array}
$$

## Value

A data.frame with object\$ncat times the rows that model.frame (object) have and the same variables. If weights has been specified in the call that generated object, then the simulate frequencies will populate the weights variable. Otherwise, the resulting data.frame will have a ". weights" variable with the simulated multinomial counts.

## Examples

```
## Multinomial logistic regression
data("housing", package = "MASS")
houseML1 <- brmultinom(Sat ~ Infl + Type + Cont, weights = Freq,
                data = housing, type = "ML", ref = 1)
simulate(houseML1)
## Adjacent-category logits
data("stemcell", package = "brglm2")
stemML1 <- bracl(research ~ religion + gender, weights = frequency,
    data = stemcell, type = "ML")
simulate(stemML1)
```


## Description

Simulate one or more responses from the distribution corresponding to a fitted model brnb object.

## Usage

```
## S3 method for class 'brnb'
simulate(object, nsim = 1, seed = NULL, ...)
```


## Arguments

object an object representing a fitted model.
nsim number of response vectors to simulate. Defaults to 1.
seed an object specifying if and how the random number generator should be initialized ("seeded").
... extra arguments to be passed to methods. Not currently used.

## Examples

```
# Example in Saha, K., & Paul, S. (2005). Bias-corrected maximum
# likelihood estimator of the negative binomial dispersion
# parameter. Biometrics, 61, 179--185.
#
# Frequency distribution of red mites on apple leaves.
nomites <- 0:8
noleaves <- c(70, 38, 17, 10, 9, 3, 2, 1, 0)
fit_glmnb <- MASS::glm.nb(nomites~1,link="identity",weights = noleaves)
fit_brnb <- brnb(nomites ~ 1, link = "identity", transformation = "inverse",
    type = "ML",weights = noleaves)
## Let us simulate 10 response vectors
sim_glmnb <- simulate(fit_glmnb, nsim = 10, seed = 123)
sim_brnb <- simulate(fit_brnb, nsim = 10, seed = 123)
# The results from glm.nb and brnb with type = "ML" are
# exactly the same
all.equal(sim_glmnb, sim_brnb, check.attributes = FALSE)
```


## Description

A data set from the 2006 General Social Survey that shows the relationship in the United States between opinion about funding stem cell research and the fundamentalism/liberalism of one's religious beliefs, stratified by gender.

## Usage

stemcell

## Format

A data frame with 24 rows and 4 variables:
research opinion about funding stem cell research with levels definitely, probably, probably not, definitely not
gender the gender of the respondent with levels female and male
religion the fundamentalism/liberalism of one's religious beliefs with levels fundamentalist, moderate, liberal
frequency the number of times a respondent fell in each of the combinations of levels for research, religion and gender

## Source

The stemcell data set is analyzed in Agresti (2010, Subsection~4.1.5).

## References

Agresti A (2010). *Analysis of Ordinal Categorical Data* (2nd edition). Wiley Series in Probability and Statistics. Wiley

## See Also

bracl

## Description

summary method for brglmFit objects

## Usage

```
    ## S3 method for class 'brglmFit'
    summary(
        object,
        dispersion = NULL,
        correlation = FALSE,
        symbolic.cor = FALSE,
    )
    ## S3 method for class 'summary.brglmFit'
    print(
        x,
        digits = max(3L, getOption("digits") - 3L),
        symbolic.cor = x$symbolic.cor,
        signif.stars = getOption("show.signif.stars"),
    )
```


## Arguments

| object <br> dispersion | an object of class "glm", usually, a result of a call to glm. <br> the dispersion parameter for the family used. Either a single numerical value or <br> NULL (the default), when it is inferred from object (see 'Details'). |
| :--- | :--- |
| correlation | logical; if TRUE, the correlation matrix of the estimated parameters is returned <br> and printed. <br> logical. If TRUE, print the correlations in a symbolic form (see symnum) rather <br> than as numbers. |
| symbolic.cor |  |$\quad$| further arguments passed to or from other methods. |
| :--- |
| an object of class "summary.glm", usually, a result of a call to summary.glm. |
| x |$\quad$| the number of significant digits to use when printing. |
| :--- |
| digits |
| signif.stars |$\quad$| logical. If TRUE, 'significance stars' are printed for each coefficient. |
| :--- |

## Details

The interface of the summary method for brglmFit objects is identical to that of glm objects. The summary method for brglmFit objects computes the p-values of the individual Wald statistics based on the standard normal distribution, unless the family is Gaussian, in which case a t distribution with appropriate degrees of freedom is used.

## See Also

summary.glm and glm

## Examples

```
## For examples see examples(brglmFit)
```

```
summary.brnb summary method for brnb objects
```


## Description

summary method for brnb objects

## Usage

\#\# S3 method for class 'brnb'
summary (object, ...)

## Arguments

object an object of class "brnb", usually, a result of a call to brnb.
... further arguments passed to or from other methods.

## Details

The interface of the summary method for brnb objects is similar to that of brglmFit objects with minor additional informations. The summary method for brnb objects computes the p-values of the individual Wald statistics based on the standard normal distribution.

## See Also

summary.brglmFit and glm

## Examples

```
## For examples see examples(brnb)
```

```
vcov.brglmFit
```

Return the variance-covariance matrix for the regression parameters in a brglmFit object

## Description

Return the variance-covariance matrix for the regression parameters in a brglmFit object

## Usage

```
## S3 method for class 'brglmFit'
    vcov(object, model = c("mean", "full", "dispersion"), complete = TRUE, ...)
```


## Arguments

$$
\begin{array}{ll}
\text { object } & \begin{array}{l}
\text { a fitted model object, typically. Sometimes also a summary () object of such a } \\
\text { fitted model. } \\
\text { character specifying for which component of the model coefficients should be } \\
\text { extracted. }
\end{array} \\
\text { model } & \begin{array}{l}
\text { for the aov, lm, glm, mlm, and where applicable summary. lm etc methods: log- } \\
\text { ical indicating if the full variance-covariance matrix should be returned also }
\end{array} \\
\text { in case of an over-determined system where some coefficients are undefined } \\
\text { and coef(.) contains NAs correspondingly. When complete = TRUE, vcov() is } \\
\text { compatible with coef() also in this singular case. }
\end{array}
$$

## Details

The options for model are "mean" for mean regression parameters only (default), "dispersion" for the dispersion parameter (or the transformed dispersion; see brglm_control), and "mean" for both the mean regression and the (transformed) dispersion parameters.
vcov.brnb Extract model variance-covariance matrix from brnb objects

## Description

Extract model variance-covariance matrix from brnb objects

## Usage

\#\# S3 method for class 'brnb'
vcov(object, model = c("mean", "full", "dispersion"), complete = TRUE, ...)

## Arguments

object an object of class "brnb", usually, a result of a call to brnb.
model character specifying for which component of the model variance-covariance matrix should be extracted.
complete for the aov, lm, glm, mlm, and where applicable summary. lm etc methods: logical indicating if the full variance-covariance matrix should be returned also in case of an over-determined system where some coefficients are undefined and coef(.) contains NAs correspondingly. When complete = TRUE, vcov() is compatible with coef() also in this singular case.
... additional arguments for method functions. For the glm method this can be used to pass a dispersion parameter.

## Details

The options for model are "mean" for mean regression only (default), "dispersion" for the dispersion parameter (in a chosen transformation; see brglmControl), and "full" for both the mean regression and the (transformed) dispersion parameters. See vcov for more details.

## See Also

vcov

## Index

```
* datasets
    alligators, 3
    coalition, 23
    endometrial,27
    lizards,28
    stemcell, 37
```

alligators, 3
as.data.frame, 19
binomial, 9
bracl, 3, 6, 7, 13, 18, 34, 35, 37
brglm2, 6
brglm2-defunct, 7
brglm_control, 40
brglm_control (brglmControl), 8
brglm_fit, 6, 7, 10, 24, 27-29
brglm_fit (brglmFit), 11
brglmControl, 4, 8, 9, 12-14, 17, 20, 21, 41
brglmFit, 3, 8, 9, 11, 11, 16, 17, 21, 24, 25, 38-40
brmultinom, 3, 5-7, 12, 13, 16, 17, 18, 26, 34, 35
brnb, 19, 25, 26, 39-41
check_infinite_estimates, 6, 13
check_infinite_estimates
(brglm2-defunct), 7
coalition, 23
coef, $24,25,40,41$
coef.brglmFit, 24
coef.brnb, 25
confint.brglmFit, 25
confint.brmultinom, 26
confint.brnb, 26
data.frame, 35
detect_separation, 6, 7, 13
detect_separation (brglm2-defunct), 7
endometrial, 27
family, 12,13
formula, 4, 17, 19
glm, 8, 11, 13, 14, 29, 38-41
glm.fit, 10, 12, 14
glm.nb, 21
lizards, 28
mis, 13, 29, 29
model. offset, 12,20
multinom, 5, 18
na.exclude, 20
na.fail, 20
na.omit, 20
offset, 12,20
options, 20
ordinal_superiority, 30, 30
poisson, 9
power, 13
predict.bracl, 31
predict.brmultinom, 33
print.summary.brglmFit
(summary.brglmFit), 38
print. summary.brnb, 34
quasi, 13
quasibinomial, 13
quasipoisson, 13
residuals.bracl (residuals.brmultinom), 34
residuals.brmultinom, 34
simulate.brmultinom, 35
simulate.brnb, 36
stemcell, 37
summary, 40
summary.brglmFit, 38, 39
summary.brnb, 39
summary.glm, 39
symnum, 38
vcov, 41
vcov.brglmFit, 40
vcov.brnb, 40

