

# Package ‘cmvnorm’

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**Type** Package

**Title** The Complex Multivariate Gaussian Distribution

**Version** 1.0-7

**Depends** emulator ( $\geq 1.2-21$ )

**Suggests** knitr

**Imports** elliptic

**Maintainer** Robin K. S. Hankin <hankin.robin@gmail.com>

**Description** Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes.

**VignetteBuilder** knitr

**License** GPL-2

**URL** <https://github.com/RobinHankin/cmvnorm>

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**NeedsCompilation** no

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cmvnorm-package

*The Complex Multivariate Gaussian Distribution*


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## Description

Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes.

## Details

The DESCRIPTION file:

```

Package:      cmvnorm
Type:         Package
Title:        The Complex Multivariate Gaussian Distribution
Version:      1.0-7
Authors@R:    person(given=c("Robin", "K. S."), family="Hankin", role = c("aut","cre"), email="hankin.robin@gmail.co
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Maintainer:   Robin K. S. Hankin <hankin.robin@gmail.com>
Description:  Various utilities for the complex multivariate Gaussian distribution and complex Gaussian processes.
VignetteBuilder: knitr
License:      GPL-2
URL:          https://github.com/RobinHankin/cmvnorm
BugReports:   https://github.com/RobinHankin/cmvnorm/issues
Author:       Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>)

```

Index of help topics:

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var           Variance and standard deviation of complex
               vectors
wishart       The complex Wishart distribution

```

Generalizing the real multivariate Gaussian distribution to the complex case is not straightforward but one common approach is to replace the real symmetric variance matrix with a Hermitian positive-definite matrix. The **cmvnorm** package provides some functionality for the resulting density function.

**Author(s)**

NA

Maintainer: Robin K. S. Hankin &lt;hankin.robin@gmail.com&gt;

**References**

- N. R. Goodman 1963. “Statistical analysis based on a certain multivariate complex Gaussian distribution”. *The Annals of Mathematical Statistics*. 34(1): 152–177
- R. K. S. Hankin 2015. “The complex multivariate Gaussian distribution”. *R News*, volume 7, number 1.

**Examples**

```
S1 <- 4+diag(5)
S2 <- S1
S2[1,5] <- 4+1i
S2[5,1] <- 4-1i # Hermitian

rcmvnorm(10, sigma=S1)
rcmvnorm(10, mean=rep(1i, 5), sigma=S2)

dcmvnorm(rep(1, 5), sigma=S2)
```

corr\_complex

*Complex Gaussian processes***Description**

Various utilities for investigating complex Gaussian processes

**Usage**

```
corr_complex(z1, z2 = NULL, distance.function = complex_CF, means =
NULL, scales = NULL, pos.def.matrix = NULL)
complex_CF(z1, z2, means, pos.def.matrix)
scales.likelihood.complex(pos.def.matrix, scales, means, zold, z,
give_log = TRUE, func = regressor.basis)
interpolant.quick.complex(x, d, zold, Ainv, scales = NULL, pos.def.matrix = NULL,
means=NULL, func = regressor.basis, give.Z = FALSE,
distance.function = corr_complex, ...)
```

## Arguments

`z, z1, z2` Points in  $C^n$   
`distance.function` Function giving the (complex) covariance between two points in  $C^n$   
`means, pos.def.matrix, scales` In function `complex_CF()`, the mean and covariance matrix of the distribution whose characteristic function is used to give the covariance matrix; `scales` is used to specify the diagonal of `pos.def.matrix` if the off-diagonal elements are zero  
`zold, d, give_log, func, x, Ainv, give.Z, ...` Direct analogues of the arguments in `interpolant()` and `scales.likelihood()` in the **emulator** package

## Details

- Function `complex_CF()` returns a (slightly reparameterized) characteristic function of a complex Gaussian distribution. The covariance is given by

$$c(\mathbf{t}) = \exp(i\text{Re}(\mathbf{t}^* \mu) - \mathbf{t}^* B \mathbf{t})$$

where  $\mathbf{t} = \mathbf{x} - \mathbf{x}'$  is interpreted as the distance between two observations,  $\mu$  is the mean of the distribution (which is in general a complex vector), and  $B$  a positive-definite matrix.

- Function `corr_complex()` is the complex analogue of `corr.matrix()`. It returns a matrix with entry  $(i, j)$  equal to the covariance of the process at observation  $i$  and observation  $j$ , or  $\text{cov}(\eta(\mathbf{x}_i), \eta(\mathbf{x}_j))$ . The elements are calculated by `complex_CF()`.

This function includes only a single method, that of nested calls to `apply()`. I could not figure out how to generalize method 1 of `corr.matrix()` to the complex case.

- Function `scales.likelihood.complex()` is a complex version of `scales.likelihood()` which takes a positive definite matrix and a mean. The formula used is

$$(\sigma^2)^{-(n-q)} |A|^{-1} |H^* A^{-1} H|^{-1}$$

. Here and elsewhere,  $A^*$  means the complex conjugate of the transpose.

- Function `interpolant.quick.complex()` is a complex version of `interpolant.quick()`.

$$\mathbf{h}(\mathbf{x})^* \hat{\beta} + \mathbf{t}(\mathbf{x})^* A^{-1} (\mathbf{y} - H \hat{\beta})$$

This is the complex version of Oakley's equation 2.30 or Hankin's equation 5.

More details are given in the package vignette.

## Author(s)

Robin K. S. Hankin

## References

- Hankin, R. K. S. 2005. "Introducing BACCO, an R bundle for Bayesian Analysis of Computer Code Output", *Journal of Statistical Software*, 14(15)
- J. Oakley 1999. *Bayesian uncertainty analysis for complex computer codes*, PhD thesis, University of Sheffield.

**Examples**

```

complex_CF(c(1,1i),c(1,-1i),means=c(1i,1i),pos.def.matrix=diag(2))

V <- latin.hypercube(7,2,complex=TRUE)

cm <- c(1,1+1i)           # "complex mean"
cs <- matrix(c(2,1i,-1i,1),2,2) # "complex scales"
tb <- c(1,1i,1-1i)       # "true beta"

A <- corr_complex(V,means=cm,pos.def.matrix=cs)
Ainv <- solve(A)
z <- drop(rcmvnorm(n=1,mean=regressor.multi(V) %*% tb, sigma=A))

betahat.fun(V,Ainv,z)    # should be close to 'tb'

#scales.likelihood.complex(cs,cm,V,z) # log-likelihood evaluated true parameters

interpolant.quick.complex(x=0.1i+V[1:3,],d=z,zold=V,Ainv=Ainv,pos.def.matrix=cs,means=cm)

```

---

isHermitian

*Is a Matrix Hermitian?*


---

**Description**

Returns TRUE if a matrix is Hermitian or Hermitian positive-definite

**Usage**

```

isHermitian(x, tol = 100 * .Machine$double.eps)
ishpd(x,tol= 100 * .Machine$double.eps)
zapim(x,tol= 100 * .Machine$double.eps)

```

**Arguments**

x	A square matrix
tol	Tolerance for numerical scruff

**Details**

Functions `isHermitian()` and `ishpd()` return a Boolean, indicating whether the argument is Hermitian or Hermitian positive definite respectively. Function `zapim()` zaps small imaginary parts of a vector, returning real if all elements are so zapped.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
v <- 2^(1:30)
zapim(v+1i*exp(-v))
```

```
ishpd(matrix(c(1,0.1i,-0.1i,1),2,2)) # should be TRUE
isHermitian(matrix(c(1,3i,-3i,1),2,2)) # should be TRUE
ishpd(rcwis(6,2)) # should be TRUE
```

Mvnorm

*Multivariate complex Gaussian density and random deviates***Description**

Density function and a random number generator for the multivariate complex Gaussian distribution.

**Usage**

```
rcnorm(n)
dcmvnorm(z, mean, sigma, log = FALSE)
rcmvnorm(n, mean = rep(0, nrow(sigma)), sigma = diag(length(mean)),
  method = c("svd", "eigen", "chol"),
  tol= 100 * .Machine$double.eps)
```

**Arguments**

z	Complex vector or matrix of quantiles. If a matrix, each row is taken to be a quantile
n	Number of observations
mean	Mean vector
sigma	Covariance matrix, Hermitian positive-definite
tol	numerical tolerance term for verifying positive definiteness
log	In dcmvnorm(), Boolean with default FALSE meaning to return the Gaussian density function, and TRUE meaning to return the logarithm
method	Specifies the decomposition used to determine the positive-definite matrix square root of sigma. Possible methods are eigenvalue decomposition ("eigen", default), and singular value decomposition ("svd")

**Details**

Function `dcmvnorm()` is the density function of the complex multivariate normal (Gaussian) distribution:

$$p(\mathbf{z}) = \frac{\exp(-\mathbf{z}^* \Gamma \mathbf{z})}{|\pi \Gamma|}$$

Function `rcnorm()` is a low-level function designed to generate observations drawn from a standard complex Gaussian. Function `rcmvnorm()` is a user-friendly wrapper for this.

**Author(s)**

Robin K. S. Hankin

**References**

N. R. Goodman 1963. "Statistical analysis based on a certain multivariate complex Gaussian distribution". *The Annals of Mathematical Statistics*. 34(1): 152–177

**Examples**

```
S <- emulator::cprod(rcmvnorm(3,mean=c(1,1i),sigma=diag(2)))

rcmvnorm(10,sigma=S)
rcmvnorm(10,mean=c(0,1+10i),sigma=S)

# Now try and estimate the mean (viz 1,1i) and variance (S) from a
# random sample:

n <- 101
z <- rcmvnorm(n,mean=c(0,1+10i),sigma=S)
xbar <- colMeans(z)
Sbar <- cprod(sweep(z,2,xbar))/n
```

**Description**

Manipulate real or imaginary components of an object

**Usage**

```
Im(x) <- value  
Re(x) <- value
```

**Arguments**

x	Complex-valued object
value	Real-valued object

**Author(s)**

Robin K. S. Hankin

**Examples**

```
A <- matrix(c(1,0.1i,-0.1i,1),2,2)  
Im(A) <- Im(A)*3  
Re(A) <- matrix(c(5,2,2,5),2,2)
```

---

var

*Variance and standard deviation of complex vectors*

---

**Description**

Complex generalizations of `stats::sd()` and `stats::var()`

**Usage**

```
var(x, y=NULL, na.rm=FALSE, use)  
sd(x, na.rm=FALSE)
```

**Arguments**

x,y	Complex vector or matrix
na.rm	Boolean with default FALSE meaning to leave NA values present and TRUE meaning to remove them
use	Ignored



**Details**

Intended to be broadly compatible with `stats::sd()` and `stats::var()`.

If given real values, `var()` and `sd()` return the variance and standard deviation as per ordinary real analysis. If given complex values, returns the complex generalization in which Hermitian transposes are used.

If `z` is a complex matrix, `var(z)` returns the variance of the rows.

These functions use  $n - 1$  on the denominator purely for consistency with `stats::var()` (for the record, I disagree with the rationale for  $n - 1$ ).

**Author(s)**

Robin K. S. Hankin

**Examples**

```
sd(rcnorm(10)) # imaginary component suppressed by zapim()
var(rcmvnorm(1e5,mean=c(0,0)))
```

---

wishart

*The complex Wishart distribution*

---

**Description**

Returns an observation drawn from the complex Wishart distribution. To sample from the inverse complex Wishart distribution (or indeed the complex inverse Wishart distribution), use `solve(rcwis(...))`.

**Usage**

```
rcwis(n, S)
```

**Arguments**

<code>n</code>	Integer; degrees of freedom
<code>S</code>	Variance matrix. If an integer, use <code>diag(nrow=S)</code>

**Value**

Returns a (semi-) positive definite Hermitian matrix the same size as argument `S`

**Note**

The first argument of `rcwis()` is `n`, by universal statistics convention. But in the R world, functions returning random observations (such as `runif()`) generally reserve argument `n` for the number of observations to return. Although `rchisq()` uses `df` for the number of degrees of freedom.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
rcwis(10,2)
eigen(rcwis(7,3),TRUE,TRUE) # all positive
eigen(rcwis(3,7),TRUE,TRUE) # 4 positive, 3 zero

rcwis(10,rcwis(10,3))
```

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