# Package ‘delaunay’ 

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## Description

Delaunay tessellation of a set of 2D or 3D points.

## Usage

delaunay (points, elevation = FALSE, constraints = NULL, quick3d = FALSE)

## Arguments

points numeric matrix which stores the points, one point per row
elevation if points are three-dimensional and elevation=TRUE, then the function performs an elevated two-dimensional Delaunay triangulation, using the $z$ coordinates of the points for the elevations; see the example
constraints for $2 D$ only, some edges to perform a constrained Delaunay triangulation, given as an integer matrix with two columns (each row provides the indices of the two points forming the edge); NULL for no constraint
quick3d Boolean, for 3D only; if FALSE, there is more information in the output about the Delaunay tessellation; see the Value section for details

## Value

The Delaunay tessellation.

- If the dimension is 2 and constraints=NULL, the returned value is a list with three fields: faces, edges and area. The faces field contains an integer matrix with three columns; each row represents a triangle whose each vertex is given by the index (row number) of this point in the points matrix. The edges field also contains an integer matrix with three columns. The two first integers of a row are the indices of the two points which form the edge. The third column, named border, only contains some zeros and some ones; a border (exterior) edge is labelled by a 1 . The area field contains only a number: the area of the triangulated region (that is, the area of the convex hull of the points).
- If the dimension is 2 and constraints is not NULL, the returned value is a list with four fields: faces, edges, constraints, and area. The faces field contains an integer matrix with three columns; each row represents a triangle whose each vertex is given by the index (row number) of this point in the points matrix. The edges field is a dataframe with four columns. The first two columns provide the edges of the triangulation; they are given by row, the two integers of a row are the indices of the two points which form the edge. Each integer of the third column is the index of the face the corresponding edge belongs to. The fourth column, named border, only contains some zeros and some ones; a border edge is labelled by a 1. The constraints field is an integer matrix with two columns, it represents the constraint edges. Finally, the area field contains only a number: the area of the triangulated region.
- If the dimension is $\mathbf{3}$, the returned value is a list with four fields: cells, facets, edges, and volume. The cells field represents the tetrahedra which form the tessellation. The facets field represents the faces of these tetrahedra, some triangles. The edges field represents the edges of these triangles. The volume field provides only one number, the volume of the tessellation, in other words the volume of the convex hull of the given points. If quick3d=TRUE, then cells, facets and edges are integer matrices with four, three, and two columns respectively; each integer is a vertex index. If quick3d=FALSE, the cells field is a list of lists. Each sublist is composed of three fields: cell provides the indices of the four vertices of the corresponding tetrahedron, faces provides the indices of the four faces of the tetrahedron, that is to say the row number of the facets field which represents this face, and finally there is a volume field which provides the volume of the tetrahedron. The facets field is an integer matrix with four columns. The three first integers of a row are the indices of the points which form the corresponding facet. The fourth column, named onhull is composed of zeros and ones only, and a 1 means that the corresponding facet lies on the convex hull of the points. The edges field contains an integer matrix with three columns. Each row represents an edge, given by the two indices of the points which form this edge, and the third integer, in the column named onhull is a $0 / 1$ indicator of whether the edge lies on the convex hull. Finally the volume field provides only one number, the volume of the tessellation (i.e. the volume of the convex hull of the points).
- If elevation=TRUE, the returned value is a list with five fields: mesh, edges, faceVolumes, volume and area. The mesh field is an object of class mesh3d, ready for plotting with the rgl package. The edges field provides the indices of the edges, given as an integer matrix with two columns. The faceVolumes field is a numeric vector, it provides the volumes under the faces that can be found in the mesh field. The volume field provides the sum of these volumes, that is to say the total volume under the triangulated surface. Finally, the area field provides the sum of the areas of all triangles, thereby approximating the area of the triangulated surface.


## Examples

```
library(delaunay)
# elevated Delaunay triangulation ####
f <- function(x, y){
    2 * exp(-(x^2 + y^2)) # integrate to 2pi
}
x <- y <- seq(-4, 4, length.out = 50)
grd <- transform(expand.grid(x = x, y = y), z = f(x, y))
del <- delaunay(as.matrix(grd), elevation = TRUE)
# `del` is a list; its first component is a mesh representing the surface:
mesh <- del[["mesh"]]
library(rgl)
open3d(windowRect = c(50, 50, 562, 562))
shade3d(mesh, color = "limegreen")
wire3d(mesh)
# in `del` you can also found the volume under the surface, which should
# approximate the integral of the function:
del[["volume"]]
```


## Description

Makes a 'rgl' mesh (mesh3d object) from a 2D Delaunay triangulation, unconstrained or constrained.

## Usage

mesh2d(triangulation)

## Arguments

triangulation an output of delaunay executed with 2D points

## Value

A list with three fields; mesh, a mesh3d object, borderEdges, a numeric matrix that can be used with segments3d to plot the border edges, and constraintEdges, a numeric matrix that can be used with segments3d to plot the constraint edges which are not border edges.

## See Also

> plotDelaunay2D

## Examples

```
library(delaunay)
# outer and inner hexagons ####
nsides <- 6L
angles <- seq(0, 2*pi, length.out = nsides+1L)[-1L]
outer_points <- cbind(cos(angles), sin(angles))
inner_points <- outer_points / 2
points <- rbind(outer_points, inner_points)
# constraint edges
indices <- 1L:nsides
edges <- cbind(
    indices, c(indices[-1L], indices[1L])
)
edges <- rbind(edges, edges + nsides)
# constrained Delaunay triangulation
del <- delaunay(points, constraints = edges)
# mesh
m2d <- mesh2d(del)
mesh <- m2d[["mesh"]]
# plot all edges with `wire3d`
library(rgl)
open3d(windowRect = c(100, 100, 612, 612))
```

```
shade3d(mesh, color = "red", specular = "orangered")
wire3d(mesh, color = "black", lwd = 3, specular = "black")
# plot only the border edges
open3d(windowRect = c(100, 100, 612, 612))
shade3d(mesh, color = "darkred", specular = "firebrick")
segments3d(m2d[["borderEdges"]], lwd = 3)
```

plotDelaunay2D Plot 2D Delaunay triangulation

## Description

Plot a constrained or unconstrained 2D Delaunay triangulation.

## Usage

```
plotDelaunay2D(
    triangulation,
    col_edges = "black",
    col_borders = "red",
    col_constraints = "green",
    fillcolor = "distinct",
    hue = "random",
    luminosity = "light",
    lty_edges = par("lty"),
    lwd_edges = par("lwd"),
    lty_borders = par("lty"),
    lwd_borders = par("lwd"),
    lty_constraints = par("lty"),
    lwd_constraints = par("lwd"),
    ...
)
```


## Arguments

triangulation an output of delaunay without constraints (constraints=NULL) or with constraints
col_edges the color of the edges of the triangles which are not border edges nor constraint edges; NULL for no color
col_borders the color of the border edges; note that the border edges can contain the constraint edges for a constrained Delaunay tessellation; NULL for no color
col_constraints
for a constrained Delaunay tessellation, the color of the constraint edges which are not border edges; NULL for no color
fillcolor controls the filling colors of the triangles, either NULL for no color, a single color, "random" to get multiple colors with randomColor, or "distinct" get multiple colors with distinctColorPalette
hue, luminosity
if color = "random", these arguments are passed to randomColor
lty_edges, lwd_edges
graphical parameters for the edges which are not border edges nor constraint edges
lty_borders, lwd_borders
graphical parameters for the border edges
lty_constraints, lwd_constraints
in the case of a constrained Delaunay triangulation, graphical parameters for the
constraint edges which are not border edges
$\ldots \quad$ arguments passed to points for the vertices, such as type="n" or asp=1

## Value

No value, just renders a 2D plot.

## See Also

mesh2d for an interactive plot

## Examples

```
library(delaunay)
# random points in a square ####
square <- rbind(
    c(-1, 1), c(1, 1), c(1, -1), c(-1, -1)
)
library(uniformly)
set.seed(314)
ptsinsquare <- runif_in_cube(10L, d = 2L)
pts <- rbind(square, ptsinsquare)
d <- delaunay(pts)
opar <- par(mar = c(0, 0, 0, 0))
plotDelaunay2D(
    d, type = "n", xlab = NA, ylab = NA, axes = FALSE, asp = 1,
    fillcolor = "random", luminosity = "dark", lwd_borders = 3
)
par(opar)
# a constrained Delaunay triangulation: outer and inner hexagons ####
nsides <- 6L
angles <- seq(0, 2*pi, length.out = nsides+1L)[-1L]
outer_points <- cbind(cos(angles), sin(angles))
inner_points <- outer_points / 2
points <- rbind(outer_points, inner_points)
# constraint edges
indices <- 1L:nsides
edges <- cbind(
    indices, c(indices[-1L], indices[1L])
)
edges <- rbind(edges, edges + nsides)
```

```
# constrained Delaunay triangulation
d <- delaunay(points, constraints = edges)
opar <- par(mar = c(0, 0, 0, 0))
plotDelaunay2D(
    d, type = "p", pch = 19, xlab = NA, ylab = NA, axes = FALSE, asp = 1,
    fillcolor = "orange", lwd_borders = 3
)
par(opar)
# another constrained Delaunay tesselation: a face ####
V <- as.matrix(read.table(
    system.file("extdata", "face_vertices.txt", package = "delaunay")
))[, c(2L, 3L)]
E <- as.matrix(read.table(
    system.file("extdata", "face_edges.txt", package = "delaunay")
))[, c(2L, 3L)]
d <- delaunay(points = V, constraints = E)
opar <- par(mar = c(0, 0, 0, 0))
plotDelaunay2D(
    d, type = "n", xlab = NA, ylab = NA, axes = FALSE, asp = 1,
    fillcolor = "salmon", col_borders = "black",
    lwd_borders = 3, lwd_constraints = 2, lty_edges = "dashed"
)
par(opar)
```

plotDelaunay3D Plot 3D Delaunay tessellation

## Description

Plot a 3D Delaunay tessellation with rgl.

## Usage

```
plotDelaunay3D(
        tessellation,
        color = "distinct",
        hue = "random",
        luminosity = "light",
        alpha = 0.3,
)
```


## Arguments

tessellation the output of delaunay with 3D points
color controls the filling colors of the tetrahedra, either FALSE for no color, "random" to use randomColor, or "distinct" to use distinctColorPalette
hue, luminosity
if color="random", these arguments are passed to randomColor
alpha opacity, number between 0 and 1
... arguments passed to material3d

## Value

No value, just renders a 3D plot.

## Examples

```
library(delaunay)
pts <- rbind(
        \(c(-5,-5,16)\),
        \(c(-5,8,3)\),
        \(\mathrm{c}(4,-1,3)\),
        c(4, -5, 7),
        \(c(4,-1,-10)\),
        \(c(4,-5,-10)\),
        \(c(-5,8,-10)\),
        \(c(-5,-5,-10)\)
)
tess <- delaunay (pts)
library(rgl)
open3d(windowRect = c(50, 50, 562, 562))
plotDelaunay3D(tess)
```


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