Package 'geometry'

July 4, 2022

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Title Mesh Generation and Surface Tessellation

```
Description Makes the 'Qhull' library <a href="http://www.qhull.org">http://www.qhull.org</a>
      available in R, in a similar manner as in Octave and MATLAB. Qhull
      computes convex hulls, Delaunay triangulations, halfspace
      intersections about a point, Voronoi diagrams, furthest-site
      Delaunay triangulations, and furthest-site Voronoi diagrams. It
      runs in 2D, 3D, 4D, and higher dimensions. It implements the
      Quickhull algorithm for computing the convex hull. Qhull does not
      support constrained Delaunay triangulations, or mesh generation of
      non-convex objects, but the package does include some R functions
      that allow for this.
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```

37

Index

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R topics documented:

bary2cart 3

bary2cart

Conversion of Barycentric to Cartesian coordinates

Description

Given the barycentric coordinates of one or more points with respect to a simplex, compute the Cartesian coordinates of these points.

Usage

```
bary2cart(X, Beta)
```

Arguments

X Reference simplex in N dimensions represented by a N+1-by-N matrix

Beta M points in barycentric coordinates with respect to the simplex X represented by a M-by-N+1 matrix

Value

M-by-N matrix in which each row is the Cartesian coordinates of corresponding row of Beta

Author(s)

David Sterratt

See Also

```
cart2bary
```

4 cart2bary

cart2bary

Conversion of Cartesian to Barycentric coordinates.

Description

Given the Cartesian coordinates of one or more points, compute the barycentric coordinates of these points with respect to a simplex.

Usage

cart2bary(X, P)

Arguments

X Reference simplex in N dimensions represented by a N + 1-by-N matrix

P M-by-N matrix in which each row is the Cartesian coordinates of a point.

Details

Given a reference simplex in N dimensions represented by a N+1-by-N matrix an arbitrary point P in Cartesian coordinates, represented by a 1-by-N row vector, can be written as

$$P = \beta X$$

where β is an N+1 vector of the barycentric coordinates. A criterion on β is that

$$\sum_{i} \beta_i = 1$$

Now partition the simplex into its first N rows X_N and its N+1th row X_{N+1} . Partition the barycentric coordinates into the first N columns β_N and the N+1th column β_{N+1} . This allows us to write

$$P_{N+1} - X_{N+1} = \beta_N X_N + \beta_{N+1} X_{N+1} - X_{N+1}$$

which can be written

$$P_{N+1} - X_{N+1} = \beta_N (X_N - 1_N X_{N+1})$$

where 1_N is an N-by-1 matrix of ones. We can then solve for β_N :

$$\beta_N = (P_{N+1} - X_{N+1})(X_N - 1_N X_{N+1})^{-1}$$

and compute

$$\beta_{N+1} = 1 - \sum_{i=1}^{N} \beta_i$$

This can be generalised for multiple values of P, one per row.

Value

M-by-N+1 matrix in which each row is the barycentric coordinates of corresponding row of P. If the simplex is degenerate a warning is issued and the function returns NULL.

cart2pol 5

Note

Based on the Octave function by David Bateman.

Author(s)

David Sterratt

See Also

```
bary2cart
```

Examples

cart2pol

Transform Cartesian coordinates to polar or cylindrical coordinates.

Description

The inputs x, y (, and z) must be the same shape, or scalar. If called with a single matrix argument then each row of C represents the Cartesian coordinate (x, y (, z)).

Usage

```
cart2pol(x, y = NULL, z = NULL)
```

Arguments

```
    x x-coordinates or matrix with three columns
    y y-coordinates (optional, if x) is a matrix
    z z-coordinates (optional, if x) is a matrix
```

Value

A matrix P where each row represents one polar/(cylindrical) coordinate (theta, r, (, z)).

6 cart2sph

Author(s)

Kai Habel

David Sterratt

See Also

```
pol2cart, cart2sph, sph2cart
```

cart2sph

Transform Cartesian to spherical coordinates

Description

If called with a single matrix argument then each row of c represents the Cartesian coordinate (x, y, z).

Usage

```
cart2sph(x, y = NULL, z = NULL)
```

Arguments

x x-coordinates or matrix with three columns
 y y-coordinates (optional, if x) is a matrix
 z z-coordinates (optional, if x) is a matrix

Value

Matrix with columns:

theta the angle relative to the positive x-axis phi the angle relative to the xy-plane r the distance to the origin (0, 0, 0)

Author(s)

Kai Habel

David Sterratt

See Also

```
sph2cart, cart2pol, pol2cart
```

convhulln 7

convhulln

Compute smallest convex hull that encloses a set of points

Description

Returns information about the smallest convex complex of a set of input points in N-dimensional space (the convex hull of the points). By default, indices to points forming the facets of the hull are returned; optionally normals to the facets and the generalised surface area and volume can be returned. This function interfaces the Qhull library.

Usage

```
convhulln(
  p,
  options = "Tv",
  output.options = NULL,
  return.non.triangulated.facets = FALSE
)
```

Arguments

p An M-by-N matrix. The rows of p represent M points in N-dimensional space.

options String containing extra options for the underlying Qhull command; see de-

tails below and Qhull documentation at $\dots/doc/qhull/html/qconvex.html\#$

synopsis.

output.options String containing Qhull options to generate extra output. Currently n (normals)

and FA (generalised areas and volumes) are supported; see 'Value' for details. If

output.options is TRUE, select all supported options.

return.non.triangulated.facets

logical defining whether the output facets should be triangulated; FALSE by de-

fault.

Value

By default (return.non.triangulated.facets is FALSE), return an M-by-N matrix in which each row contains the indices of the points in p forming an N-1-dimensional facet. e.g In 3 dimensions, there are 3 indices in each row describing the vertices of 2-dimensional triangles.

If return.non.triangulated.facets is TRUE then the number of columns equals the maximum number of vertices in a facet, and each row defines a polygon corresponding to a facet of the convex hull with its vertices followed by NAs until the end of the row.

If the output.options or options argument contains FA or n, return a list with class convhulln comprising the named elements:

p The points passed to convnhulln

hull The convex hull, represented as a matrix indexing p, as described above

8 convhulln

```
area If FA is specified, the generalised area of the hull. This is the surface area of a 3D hull or the length of the perimeter of a 2D hull. See . . /doc/qhull/html/qh-optf.html#FA.
```

vol If FA is specified, the generalised volume of the hull. This is volume of a 3D hull or the area of a 2D hull. See .../doc/qhull/html/qh-optf.html#FA.

normals If n is specified, this is a matrix hyperplane normals with offsets. See ../doc/qhull/html/qh-opto.html#n.

Note

This function was originally a port of the Octave convhulln function written by Kai Habel. See further notes in delaunayn.

Author(s)

Raoul Grasman, Robert B. Gramacy, Pavlo Mozharovskyi and David Sterratt <david.c.sterratt@ed.ac.uk>

References

Barber, C.B., Dobkin, D.P., and Huhdanpaa, H.T., "The Quickhull algorithm for convex hulls," ACM Trans. on Mathematical Software, Dec 1996.

```
http://www.qhull.org
```

See Also

```
intersectn, delaunayn, surf.tri, convex.hull
```

```
## Points in a sphere
ps <- matrix(rnorm(3000), ncol=3)
ps <- sqrt(3)*ps/drop(sqrt((ps^2) %*% rep(1, 3)))
ts.surf <- t(convhulln(ps)) # see the qhull documentations for the options
## Not run:
rgl.triangles(ps[ts.surf,1],ps[ts.surf,2],ps[ts.surf,3],col="blue",alpha=.2)
for(i in 1:(8*360)) rgl.viewpoint(i/8)
## End(Not run)
## Square
pq < - rbox(0, C=0.5, D=2)
# Return indices only
convhulln(pq)
# Return convhulln object with normals, generalised area and volume
ch <- convhulln(pq, output.options=TRUE)</pre>
plot(ch)
## Cube
pc <- rbox(0, C=0.5, D=3)
# Return indices of triangles on surface
convhulln(pc)
```

9 delaunayn

Return indices of squares on surface convhulln(pc, return.non.triangulated.facets=TRUE)

delaunayn

Delaunay triangulation in N dimensions

Description

The Delaunay triangulation is a tessellation of the convex hull of the points such that no N-sphere defined by the N- triangles contains any other points from the set.

Usage

```
delaunayn(p, options = NULL, output.options = NULL, full = FALSE)
```

Arguments

р

An M-by-N matrix whose rows represent M points in N-dimensional space.

options

String containing extra control options for the underlying Qhull command; see the Qhull documentation (../doc/qhull/html/qdelaun.html) for the available options.

The Qbb option is always passed to Qhull. The remaining default options are Qcc Qc Qt Qz for N < 4 and Qcc Qc Qt Qx for N >= 4. If neither of the QJ or Qt options are supplied, the Qt option is passed to Qhull. The Qt option ensures all Delaunay regions are simplical (e.g., triangles in 2D). See ../doc/qhull/ html/qdelaun.html for more details. Contrary to the Qhull documentation, no degenerate (zero area) regions are returned with the Qt option since the R function removes them from the triangulation.

If options is specified, the default options are overridden. It is recommended to use output.options for options controlling the outputs.

output.options String containing Qhull options to control output. Currently Fn (neighbours) and Fa (areas) are supported. Causes an object of return value for details. If output.options is TRUE, select all supported options.

full

Deprecated and will be removed in a future release. Adds options Fa and Fn.

Value

If output options is NULL (the default), return the Delaunay triangulation as a matrix with Mrows and N+1 columns in which each row contains a set of indices to the input points p. Thus each row describes a simplex of dimension N, e.g. a triangle in 2D or a tetrahedron in 3D.

If the output.options argument is TRUE or is a string containing Fn or Fa, return a list with class delaunayn comprising the named elements:

tri The Delaunay triangulation described above

areas If TRUE or if Fa is specified, an M-dimensional vector containing the generalised area of each simplex (e.g. in 2D the areas of triangles; in 3D the volumes of tetrahedra). See . . /doc/ qhull/html/qh-optf.html#Fa.

10 delaunayn

neighbours If TRUE or if Fn is specified, a list of neighbours of each simplex. See ../doc/qhull/html/qh-optf.html#Fn

Note

This function interfaces the Qhull library and is a port from Octave (https://octave.org/) to R. Qhull computes convex hulls, Delaunay triangulations, halfspace intersections about a point, Voronoi diagrams, furthest-site Delaunay triangulations, and furthest-site Voronoi diagrams. It runs in 2D, 3D, 4D, and higher dimensions. It implements the Quickhull algorithm for computing the convex hull. Qhull handles round-off errors from floating point arithmetic. It computes volumes, surface areas, and approximations to the convex hull. See the Qhull documentation included in this distribution (the doc directory ../doc/qhull/index.html).

Qhull does not support constrained Delaunay triangulations, triangulation of non-convex surfaces, mesh generation of non-convex objects, or medium-sized inputs in 9D and higher. A rudimentary algorithm for mesh generation in non-convex regions using Delaunay triangulation is implemented in distmesh2d (currently only 2D).

Author(s)

Raoul Grasman and Robert B. Gramacy; based on the corresponding Octave sources of Kai Habel.

References

Barber, C.B., Dobkin, D.P., and Huhdanpaa, H.T., "The Quickhull algorithm for convex hulls," ACM Trans. on Mathematical Software, Dec 1996.

```
http://www.qhull.org
```

See Also

```
tri.mesh, convhulln, surf.tri, distmesh2d
```

```
# example delaunayn
d <- c(-1,1)
pc <- as.matrix(rbind(expand.grid(d,d,d),0))
tc <- delaunayn(pc)

# example tetramesh
## Not run:
rgl::rgl.viewpoint(60)
rgl::rgl.light(120,60)
tetramesh(tc,pc, alpha=0.9)

## End(Not run)

tc1 <- delaunayn(pc, output.options="Fa")
## sum of generalised areas is total volume of cube
sum(tc1$areas)</pre>
```

distmesh2d 11

distmesh2d

A simple mesh generator for non-convex regions

Description

An unstructured simplex requires a choice of mesh points (vertex nodes) and a triangulation. This is a simple and short algorithm that improves the quality of a mesh by relocating the mesh points according to a relaxation scheme of forces in a truss structure. The topology of the truss is reset using Delaunay triangulation. A (sufficiently smooth) user supplied signed distance function (fd) indicates if a given node is inside or outside the region. Points outside the region are projected back to the boundary.

Usage

```
distmesh2d(
  fd,
  fh,
 h0,
 bbox,
 p = NULL
 pfix = array(0, dim = c(0, 2)),
  dptol = 0.001,
  ttol = 0.1,
  Fscale = 1.2,
  deltat = 0.2,
  geps = 0.001 * h0,
  deps = sqrt(.Machine$double.eps) * h0,
 maxiter = 1000,
 plot = TRUE
)
```

Arguments

fd	Vectorized signed distance function, for example mesh.dcircle or mesh.diff, accepting an n-by-2 matrix, where n is arbitrary, as the first argument.
fh	Vectorized function, for example mesh.hunif, that returns desired edge length as a function of position. Accepts an n-by-2 matrix, where n is arbitrary, as its first argument.
h0	Initial distance between mesh nodes. (Ignored of p is supplied)
bbox	<pre>Bounding box cbind(c(xmin,xmax), c(ymin,ymax))</pre>
p	An n-by-2 matrix. The rows of p represent locations of starting mesh nodes.
pfix	nfix-by-2 matrix with fixed node positions.
	parameters to be passed to fd and/or fh
dptol	Algorithm stops when all node movements are smaller than dptol

12 distmesh2d

ttol Controls how far the points can move (relatively) before a retriangulation with

delaunayn.

Fscale "Internal pressure" in the edges.

deltat Size of the time step in Euler's method. geps Tolerance in the geometry evaluations.

deps Stepsize Δx in numerical derivative computation for distance function.

maxiter Maximum iterations.

plot logical. If TRUE (default), the mesh is plotted as it is generated.

Details

This is an implementation of original Matlab software of Per-Olof Persson.

Excerpt (modified) from the reference below:

'The algorithm is based on a mechanical analogy between a triangular mesh and a 2D truss structure. In the physical model, the edges of the Delaunay triangles of a set of points correspond to bars of a truss. Each bar has a force-displacement relationship $f(\ell,\ell_0)$ depending on its current length ℓ and its unextended length ℓ_0 .'

'External forces on the structure come at the boundaries, on which external forces have normal orientations. These external forces are just large enough to prevent nodes from moving outside the boundary. The position of the nodes are the unknowns, and are found by solving for a static force equilibrium. The hope is that (when fh = function(p) return(rep(1,nrow(p)))), the lengths of all the bars at equilibrium will be nearly equal, giving a well-shaped triangular mesh.'

See the references below for all details. Also, see the comments in the source file.

Value

n-by-2 matrix with node positions.

Wishlist

- Implement in C/Fortran
- Implement an nD version as provided in the Matlab package
- Translate other functions of the Matlab package

Author(s)

Raoul Grasman

References

http://persson.berkeley.edu/distmesh/

P.-O. Persson, G. Strang, A Simple Mesh Generator in MATLAB. SIAM Review, Volume 46 (2), pp. 329-345, June 2004

distmeshnd 13

See Also

tri.mesh, delaunayn, mesh.dcircle, mesh.drectangle, mesh.diff, mesh.union, mesh.intersect

Examples

distmeshnd

A simple mesh generator for non-convex regions in n-D space

Description

An unstructured simplex requires a choice of mesh points (vertex nodes) and a triangulation. This is a simple and short algorithm that improves the quality of a mesh by relocating the mesh points according to a relaxation scheme of forces in a truss structure. The topology of the truss is reset using Delaunay triangulation. A (sufficiently smooth) user supplied signed distance function (fd) indicates if a given node is inside or outside the region. Points outside the region are projected back to the boundary.

Usage

```
distmeshnd(
  fdist,
  fh,
  h,
  box,
  pfix = array(dim = c(0, ncol(box))),
    ...,
  ptol = 0.001,
  ttol = 0.1,
  deltat = 0.1,
  geps = 0.1 * h,
```

14 distmeshnd

```
deps = sqrt(.Machine$double.eps) * h
)
```

Arguments

fdist	Vectorized signed distance function, for example mesh.dsphere, accepting an m-by-n matrix, where m is arbitrary, as the first argument.
fh	Vectorized function, for example mesh.hunif, that returns desired edge length as a function of position. Accepts an m-by-n matrix, where n is arbitrary, as its first argument.
h	Initial distance between mesh nodes.
box	2-by-n matrix that specifies the bounding box. (See distmesh2d for an example.)
pfix	nfix-by-2 matrix with fixed node positions.
	parameters that are passed to fdist and fh
ptol	Algorithm stops when all node movements are smaller than dptol
ttol	Controls how far the points can move (relatively) before a retriangulation with delaunayn.
deltat	Size of the time step in Euler's method.
geps	Tolerance in the geometry evaluations.
deps	Stepsize Δx in numerical derivative computation for distance function.

Details

This is an implementation of original Matlab software of Per-Olof Persson.

Excerpt (modified) from the reference below:

'The algorithm is based on a mechanical analogy between a triangular mesh and a n-D truss structure. In the physical model, the edges of the Delaunay triangles of a set of points correspond to bars of a truss. Each bar has a force-displacement relationship $f(\ell,\ell_0)$ depending on its current length ℓ and its unextended length ℓ_0 .'

'External forces on the structure come at the boundaries, on which external forces have normal orientations. These external forces are just large enough to prevent nodes from moving outside the boundary. The position of the nodes are the unknowns, and are found by solving for a static force equilibrium. The hope is that (when fh = function(p) return(rep(1,nrow(p)))), the lengths of all the bars at equilibrium will be nearly equal, giving a well-shaped triangular mesh.'

See the references below for all details. Also, see the comments in the source file of distmesh2d.

Value

m-by-n matrix with node positions.

Wishlist

- Implement in C/Fortran
- Translate other functions of the Matlab package

dot 15

Author(s)

Raoul Grasman; translated from original Matlab sources of Per-Olof Persson.

References

```
http://persson.berkeley.edu/distmesh/
P.-O. Persson, G. Strang, A Simple Mesh Generator in MATLAB. SIAM Review, Volume 46 (2), pp. 329-345, June 2004
```

See Also

```
distmesh2d, tri.mesh, delaunayn, mesh.dsphere, mesh.hunif,
mesh.diff, mesh.union, mesh.intersect
```

Examples

dot

Compute the dot product of two vectors

Description

If x and y are matrices, calculate the dot-product along the first non-singleton dimension. If the optional argument d is given, calculate the dot-product along this dimension.

Usage

```
dot(x, y, d = NULL)
```

16 entry.value

Arguments

X	Matrix of vectors
у	Matrix of vectors

d Dimension along which to calculate the dot product

Value

Vector with length of dth dimension

Author(s)

David Sterratt

		1
entrv	. va	ııue

Retrieve or set a list of array element values

Description

entry.value retrieves or sets the values in an array a at the positions indicated by the rows of a matrix idx.

Usage

```
entry.value(a, idx)
```

Arguments

а	An	array	ν.

idx Numerical matrix with the same number of columns as the number of dimen-

sions of a. Each row indices a cell in a of which the value is to be retrieved or

set.

value An array of length nrow(idx).

Value

entry.value(a,idx) returns a vector of values at the indicated cells. entry.value(a,idx) <- val changes the indicated cells of a to val.

Author(s)

Raoul Grasman

extprod3d 17

Examples

```
a = array(1:(4^4),c(4,4,4,4))
entry.value(a,cbind(1:4,1:4,1:4,1:4))
entry.value(a,cbind(1:4,1:4,1:4,1:4)) <- 0
entry.value(a, as.matrix(expand.grid(1:4,1:4,1:4,1:4)))
    # same as `c(a[1:4,1:4,1:4,1:4])' which is same as `c(a)'</pre>
```

extprod3d

Compute external- or 'cross'- product of 3D vectors.

Description

Computes the external product

$$(x_2y_3-x_3y_2, x_3y_1-x_1y_3, x_1y_2-x_2y_1)$$

of the 3D vectors in \mathbf{x} and \mathbf{y} .

Usage

```
extprod3d(x, y, drop = TRUE)
```

Arguments

x n-by-3 matrix. Each row is one **x**-vector y n-by-3 matrix. Each row is one **y**-vector

drop logical. If TRUE and if the inputs are one row matrices or vectors, then delete the

dimensions of the array returned.

Value

If n is greater than 1 or drop is FALSE, n-by-3 matrix; if n is 1 and drop is TRUE, a vector of length 3.

Author(s)

Raoul Grasman

See Also

drop

halfspacen

feasible.point	Find point in intersection of convex hulls	

Description

Find point that lies somewhere in interesction of two convex hulls. If such a point does not exist, return NA. The feasible point is found using a linear program similar to the one suggested at ../doc/qhull/html/qhalf.html#notes

Usage

```
feasible.point(ch1, ch2, tol = 0)
```

Arguments

ch1	First convex hull with normals
ch2	Second convex hull with normals
tol	The point must be at least this far within the facets of both convex hulls

halfspacen Compute halfspace intersection about a point

Description

Compute halfspace intersection about a point

Usage

```
halfspacen(p, fp, options = "Tv")
```

Arguments

p	An M -by- $N+1$ matrix. Each row of p represents a halfspace by a N -dimensional normal to a hyperplane and the offset of the hyperplane.
fp	A "feasible" point that is within the space contained within all the halfspaces.
options	String containing extra options, separated by spaces, for the underlying Qhull command; see Qhull documentation at/doc/qhull/html/qhalf.html.

Value

A N-column matrix containing the intersection points of the hyperplanes ../doc/qhull/html/qhalf.html.

inhulln 19

Note

halfspacen was introduced in geometry 0.4.0, and is still under development. It is worth checking results for unexpected behaviour.

Author(s)

David Sterratt

References

Barber, C.B., Dobkin, D.P., and Huhdanpaa, H.T., "The Quickhull algorithm for convex hulls," ACM Trans. on Mathematical Software, Dec 1996.

```
http://www.qhull.org
```

See Also

convhul1n

Examples

```
p <- rbox(0, C=0.5) # Generate points on a unit cube centered around the origin ch <- convhulln(p, "n") # Generate convex hull, including normals to facets, with "n" option # Intersections of half planes # These points should be the same as the original points pn <- halfspacen(ch$normals, c(0, 0, 0))
```

inhulln

Test if points lie in convex hull

Description

Tests if a set of points lies within a convex hull, returning a boolean vector in which each element is TRUE if the corresponding point lies within the hull and FALSE if it lies outwith the hull or on one of its facets.

Usage

```
inhulln(ch, p)
```

Arguments

ch Convex hull produced using convhulln

p An M-by-N matrix of points to test. The rows of p represent M points in N-dimensional space.

20 intersectn

Value

A boolean vector with M elements

Note

inhulln was introduced in geometry 0.4.0, and is still under development. It is worth checking results for unexpected behaviour.

Author(s)

David Sterratt

See Also

```
convhulln, point.in.polygon in sp
```

Examples

intersectn

Compute convex hull of intersection of two sets of points

Description

Compute convex hull of intersection of two sets of points

Usage

```
intersectn(
  ps1,
  ps2,
  tol = 0,
  return.chs = TRUE,
  options = "Tv",
```

intersectn 21

```
fp = NULL,
autoscale = FALSE
)
```

Arguments

ps1	First set of points
ps2	Second set of points
tol	Tolerance used to determine if a feasible point lies within the convex hulls of both points and to round off the points generated by the halfspace intersection, which sometimes produces points very close together.
return.chs	If TRUE (default) return the convex hulls of the first and second sets of points, as well as the convex hull of the intersection.
options	Options passed to halfspacen. By default this is Tv.
fp	Coordinates of feasible point, i.e. a point known to lie in the hulls of ps1 and ps2. The feasible point is required for halfspacen to find the intersection. intersectn tries to find the feasible point automatically using the linear program in feasible.point, but currently the linear program fails on some examples where there is an obvious solution. This option overrides the automatic search for a feasible point
autoscale	Experimental in v0.4.2 Automatically scale the points to lie in a sensible numeric

Value

List containing named elements: ch1, the convex hull of the first set of points, with volumes, areas and normals (see convhulln; ch2, the convex hull of the first set of points, with volumes, areas and normals; ps, the intersection points of convex hulls ch1 and ch2; and ch, the convex hull of the intersection points, with volumes, areas and normals.

range. May help to correct some numerical issues.

Note

intersectn was introduced in geometry 0.4.0, and is still under development. It is worth checking results for unexpected behaviour.

Author(s)

David Sterratt

See Also

```
convhulln, halfspacen, inhulln, feasible.point
```

```
# Two overlapping boxes
ps1 <- rbox(0, C=0.5)
ps2 <- rbox(0, C=0.5) + 0.5
```

22 matmax

```
out <- intersectn(ps1, ps2)
message("Volume of 1st convex hull: ", out$ch1$vol)
message("Volume of 2nd convex hull: ", out$ch2$vol)
message("Volume of intersection convex hull: ", out$ch$vol)</pre>
```

matmax

Row-wise matrix functions

Description

Compute maximum or minimum of each row, or sort each row of a matrix, or a set of (equal length) vectors.

Usage

```
matmax(...)
```

Arguments

A numeric matrix or a set of numeric vectors (that are column-wise bind together into a matrix with cbind).

Value

matmin and matmax return a vector of length nrow(cbind(...)). matsort returns a matrix of dimension dim(cbind(...)) with in each row of cbind(...) sorted. matsort(x) is a lot faster than, e.g., t(apply(x,1,sort)), if x is tall (i.e., nrow(x) > ncol(x) and ncol(x) < 30. If ncol(x) > 30 then matsort simply calls 't(apply(x,1,sort))'. matorder returns a permutation which rearranges its first argument into ascending order, breaking ties by further arguments.

Author(s)

Raoul Grasman

```
example(Unique)
```

mesh.dcircle 23

mesh.	dcircle	

Circle distance function

Description

Signed distance from points p to boundary of circle to allow easy definition of regions in distmesh2d.

Usage

```
mesh.dcircle(p, radius = 1, ...)
```

Arguments

p A matrix with 2 columns (3 in mesh.dsphere), each row representing a point in

the plane.

radius radius of circle

... additional arguments (not used)

Value

A vector of length nrow(p) containing the signed distances to the circle

Author(s)

Raoul Grasman; translated from original Matlab sources of Per-Olof Persson.

References

```
http://persson.berkeley.edu/distmesh/
```

```
P.-O. Persson, G. Strang, A Simple Mesh Generator in MATLAB. SIAM Review, Volume 46 (2), pp. 329-345, June 2004
```

See Also

```
distmesh2d, mesh.drectangle, mesh.diff, mesh.intersect, mesh.union
```

```
example(distmesh2d)
```

24 mesh.drectangle

mesh.diff	Difference, union and intersection operation on two regions

Description

Compute the signed distances from points p to a region defined by the difference, union or intersection of regions specified by the functions regionA and regionB. regionA and regionB must accept a matrix p with 2 columns as their first argument, and must return a vector of length nrow(p) containing the signed distances of the supplied points in p to their respective regions.

Usage

```
mesh.diff(p, regionA, regionB, ...)
```

Arguments

р	A matrix with 2 columns (3 in mesh.dsphere), each row representing a point in the plane.
regionA	vectorized function describing region A in the union / intersection / difference
regionB	vectorized function describing region B in the union / intersection / difference
	additional arguments passed to regionA and regionB

Value

A vector of length nrow(p) containing the signed distances to the boundary of the region.

Author(s)

Raoul Grasman; translated from original Matlab sources of Per-Olof Persson.

See Also

distmesh2d, mesh.dcircle, mesh.drectangle mesh.dsphere

mesh.drectangle	Rectangle distance function	

Description

Signed distance from points p to boundary of rectangle to allow easy definition of regions in distmesh2d.

Usage

```
mesh.drectangle(p, x1 = -1/2, y1 = -1/2, x2 = 1/2, y2 = 1/2, ...)
```

mesh.dsphere 25

Arguments

р	A matrix with 2 columns, each row representing a point in the plane.
x1	lower left corner of rectangle
y1	lower left corner of rectangle
x2	upper right corner of rectangle
y2	upper right corner of rectangle
	additional arguments (not used)

Value

a vector of length nrow(p) containing the signed distances

Author(s)

Raoul Grasman; translated from original Matlab sources of Per-Olof Persson.

References

```
http://persson.berkeley.edu/distmesh/
P.-O. Persson, G. Strang, A Simple Mesh Generator in MATLAB. SIAM Review, Volume 46 (2), pp. 329-345, June 2004
```

See Also

```
distmesh2d, mesh.drectangle, mesh.diff, mesh.intersect, mesh.union
```

Examples

example(distmesh2d)

|--|

Description

Signed distance from points p to boundary of sphere to allow easy definition of regions in distmeshnd.

Usage

```
mesh.dsphere(p, radius = 1, ...)
```

Arguments

p	A matrix with 2 columns (3 in mesh.dsphere), each row representing a point in
	the plane.
radius	radius of sphere
	additional arguments (not used)

26 mesh.hunif

Value

A vector of length nrow(p) containing the signed distances to the sphere

Author(s)

Raoul Grasman; translated from original Matlab sources of Per-Olof Persson.

References

```
http://persson.berkeley.edu/distmesh/
```

P.-O. Persson, G. Strang, A Simple Mesh Generator in MATLAB. SIAM Review, Volume 46 (2), pp. 329-345, June 2004

See Also

distmeshnd

Examples

example(distmeshnd)

mesh.hunif

Uniform desired edge length

Description

Uniform desired edge length function of position to allow easy definition of regions when passed as the fh argument of distmesh2d or distmeshnd.

Usage

```
mesh.hunif(p, ...)
```

Arguments

p A n-by-m matrix, each row representing a point in an m-dimensional space.

... additional arguments (not used)

Value

Vector of ones of length n.

Author(s)

Raoul Grasman; translated from original Matlab sources of Per-Olof Persson.

pol2cart 27

See Also

distmesh2d and distmeshnd.

pol2cart

Transform polar or cylindrical coordinates to Cartesian coordinates.

Description

The inputs theta, r, (and z) must be the same shape, or scalar. If called with a single matrix argument then each row of P represents the polar/(cylindrical) coordinate (theta, r (, z)).

Usage

```
pol2cart(theta, r = NULL, z = NULL)
```

Arguments

theta describes the angle relative to the positive x-axis.

r is the distance to the z-axis (0, 0, z).

z (optional) is the z-coordinate

Value

a matrix C where each row represents one Cartesian coordinate (x, y (, z)).

Author(s)

Kai Habel

David Sterratt

See Also

```
cart2pol, sph2cart, cart2sph
```

28 polyarea

polyarea

Determines area of a polygon by triangle method.

Description

Determines area of a polygon by triangle method. The variables x and y define the vertex pairs, and must therefore have the same shape. They can be either vectors or arrays. If they are arrays then the columns of x and y are treated separately and an area returned for each.

Usage

```
polyarea(x, y, d = 1)
```

Arguments

- x X coordinates of vertices.
- y Y coordinates of vertices.
- d Dimension of array to work along.

Details

If the optional dim argument is given, then polyarea works along this dimension of the arrays x and y.

Value

```
Area(s) of polygon(s).
```

Author(s)

David Sterratt based on the octave sources by David M. Doolin

```
x \leftarrow c(1, 1, 3, 3, 1)

y \leftarrow c(1, 3, 3, 1, 1)

polyarea(x, y)

polyarea(cbind(x, x), cbind(y, y))  ##  c(4, 4)

polyarea(cbind(x, x), cbind(y, y), 1)  ##  c(4, 4)

polyarea(rbind(x, x), rbind(y, y), 2)  ##  c(4, 4)
```

rbox 29

rbox

Generate various point distributions

Description

Default is corners of a hypercube.

Usage

```
rbox(n = 3000, D = 3, B = 0.5, C = NA)
```

Arguments

n	number of random points in hypercube
D	number of dimensions of hypercube
В	bounding box coordinate - faces will be -B and B from origin
С	add a unit hypercube to the output - faces will be -C and C from origin

Value

Matrix of points

Author(s)

David Sterratt

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Transform spherical coordinates to Cartesian coordinates

Description

The inputs theta, phi, and r must be the same shape, or scalar. If called with a single matrix argument then each row of S represents the spherical coordinate (theta, phi, r).

Usage

```
sph2cart(theta, phi = NULL, r = NULL)
```

Arguments

theta describes the angle relative to the positive x-axis.

phi is the angle relative to the xy-plane.

r is the distance to the origin (0, 0, 0).

If only a single return argument is requested then return a matrix C where each

row represents one Cartesian coordinate (x, y, z).

30 surf.tri

Author(s)

Kai Habel

David Sterratt

See Also

```
cart2sph, pol2cart, cart2pol
```

surf.tri

Find surface triangles from tetrahedral mesh

Description

Find surface triangles from tetrahedral mesh typically obtained with delaunayn.

Usage

```
surf.tri(p, t)
```

Arguments

p An n-by-3 matrix. The rows of p represent n points in dim-dimensional space.

t Matrix with 4 columns, interpreted as output of delaunayn.

Details

surf.tri and convhulln serve a similar purpose in 3D, but surf.tri also works for non-convex meshes obtained e.g. with distmeshnd. It also does not produce currently unavoidable diagnostic output on the console as convhulln does at the Rterm console—i.e., surf.tri is silent.

Value

An m-by-3 index matrix of which each row defines a triangle. The indices refer to the rows in p.

Note

surf.tri was based on Matlab code for mesh of Per-Olof Persson (http://persson.berkeley.edu/distmesh/).

Author(s)

Raoul Grasman

See Also

```
tri.mesh, convhulln, surf.tri, distmesh2d
```

tetramesh 31

Examples

```
## Not run:
# more extensive example of surf.tri
# url's of publically available data:
data1.url = "http://neuroimage.usc.edu/USCPhantom/mesh_data.bin"
data2.url = "http://neuroimage.usc.edu/USCPhantom/CT_PCS_trans.bin"
meshdata = R.matlab::readMat(url(data1.url))
elec = R.matlab::readMat(url(data2.url))$eeg.ct2pcs/1000
brain = meshdata$mesh.brain[,c(1,3,2)]
scalp = meshdata mesh.scalp[,c(1,3,2)]
skull = meshdata mesh. skull[,c(1,3,2)]
tbr = t(surf.tri(brain, delaunayn(brain)))
tsk = t(surf.tri(skull, delaunayn(skull)))
tsc = t(surf.tri(scalp, delaunayn(scalp)))
rgl::rgl.triangles(brain[tbr,1], brain[tbr,2], brain[tbr,3],col="gray")
rgl::rgl.triangles(skull[tsk,1], skull[tsk,2], skull[tsk,3],col="white", alpha=0.3)
rgl::rgl.triangles(scalp[tsc,1], scalp[tsc,2], scalp[tsc,3],col="#a53900", alpha=0.6)
rgl::rgl.viewpoint(-40,30,.4,zoom=.03)
1x = c(-.025, .025); 1y = -c(.02, .02);
rgl::rgl.spheres(elec[,1],elec[,3],elec[,2],radius=.0025,col='gray')
rgl::rgl.spheres(lx,ly,.11,radius=.015,col="white")
rgl::rgl.spheres( lx, ly,.116,radius=.015*.7,col="brown")
rgl::rgl.spheres(lx,ly,.124,radius=.015*.25,col="black")
## End(Not run)
```

tetramesh

Render tetrahedron mesh (3D)

Description

tetramesh(T, X, col) uses the rgl package to display the tetrahedrons defined in the m-by-4 matrix T as mesh. Each row of T specifies a tetrahedron by giving the 4 indices of its points in X.

Usage

```
tetramesh(T, X, col = grDevices::heat.colors(nrow(T)), clear = TRUE, ...)
```

Arguments

T is a m-by-3 matrix in trimesh and m-by-4 in tetramesh. A row of T contains indices into X of the vertices of a triangle/tetrahedron. T is usually the output of delaunayn.

X is an n-by-2/n-by-3 matrix. The rows of X represent n points in 2D/3D space.

32 to.mesh3d

col The tetrahedron colour. See rgl documentation for details.

clear Should the current rendering device be cleared?

... Parameters to the rendering device. See the rgl package.

Author(s)

Raoul Grasman

See Also

```
trimesh, rgl, delaunayn, convhulln, surf.tri
```

Examples

```
## Not run:
# example delaunayn
d = c(-1,1)
pc = as.matrix(rbind(expand.grid(d,d,d),0))
tc = delaunayn(pc)

# example tetramesh
clr = rep(1,3) %o% (1:nrow(tc)+1)
rgl::rgl.viewpoint(60,fov=20)
rgl::rgl.light(270,60)
tetramesh(tc,pc,alpha=0.7,col=clr)

## End(Not run)
```

to.mesh3d

Convert convhulln object to RGL mesh

Description

Convert convhulln object to RGL mesh

Usage

```
to.mesh3d(x, ...)
```

Arguments

```
x convhulln object
```

... Arguments to qmesh3d or tmesh3d

Value

mesh3d object, which can be displayed in RGL with dot3d, wire3d or shade3d

trimesh 33

See Also

as.mesh3d

trimesh	Display triangles mesh (2D)	

Description

trimesh(T, p) displays the triangles defined in the m-by-3 matrix T and points p as a mesh. Each row of T specifies a triangle by giving the 3 indices of its points in X.

Usage

```
trimesh(T, p, p2, add = FALSE, axis = FALSE, boxed = FALSE, ...)
```

Arguments

Т	T is a m-by-3 matrix. A row of T contains indices into X of the vertices of a triangle. T is usually the output of delaunayn.
р	A vector or a matrix.
p2	if p is not a matrix p and p2 are bind to a matrix with cbind.
add	Add to existing plot in current active device?
axis	Draw axes?
boxed	Plot box?
	Parameters to the rendering device. See the rgl package.

Author(s)

Raoul Grasman

See Also

```
tetramesh, rgl, delaunayn, convhulln, surf.tri
```

```
#example trimesh
p = cbind(x=rnorm(30), y=rnorm(30))
tt = delaunayn(p)
trimesh(tt,p)
```

34 tsearch

tsearch

Search for the enclosing Delaunay convex hull

Description

For $t \leftarrow delaunay(cbind(x, y))$, where (x, y) is a 2D set of points, tsearch(x, y, t, xi, yi) finds the index in t containing the points (xi, yi). For points outside the convex hull the index is NA.

Usage

```
tsearch(x, y, t, xi, yi, bary = FALSE, method = "quadtree")
```

Arguments

X	X-coordinates of triangulation points
У	Y-coordinates of triangulation points
t	Triangulation, e.g. produced by t <-delaunayn(cbind(x, y))
xi	X-coordinates of points to test
yi	Y-coordinates of points to test
bary	If TRUE return barycentric coordinates as well as index of triangle.
method	One of "quadtree" or "orig". The Quadtree algorithm is much faster and new from version 0.4.0. The orig option uses the tsearch algorithm adapted from Octave code. Its use is deprecated and it may be removed from a future version of the package.

Value

If bary is FALSE, the index in t containing the points (xi, yi). For points outside the convex hull the index is NA. If bary is TRUE, a list containing:

```
list("idx") the index in t containing the points (xi, yi)
```

list("p") a 3-column matrix containing the barycentric coordinates with respect to the enclosing triangle of each point (xi, yi).

Note

The original Octave function is Copyright (C) 2007-2012 David Bateman

Author(s)

Jean-Romain Roussel (Quadtree algorithm), David Sterratt (Octave-based implementation)

See Also

tsearchn, delaunayn

tsearchn 35

tsearchn	Search for the enclosing Delaunay convex hull	

Description

For t = delaunayn(x), where x is a set of points in N dimensions, tsearchn(x, t, xi) finds the index in t containing the points xi. For points outside the convex hull, idx is NA. tsearchn also returns the barycentric coordinates p of the enclosing triangles.

Usage

```
tsearchn(x, t, xi, ...)
```

Arguments

х	An N -column matrix, in which each row represents a point in N -dimensional space.
t	A matrix with $N+1$ columns. A row of t contains indices into x of the vertices of an N -dimensional simplex. t is usually the output of delaunayn.
xi	An M -by- N matrix. The rows of xi represent M points in N -dimensional space whose positions in the mesh are being sought.
	Additional arguments

Details

If x is NA and the t is a delaunayn object produced by delaunayn with the full option, then use the Qhull library to perform the search. Please note that this is experimental in geometry version 0.4.0 and is only partly tested for 3D hulls, and does not yet work for hulls of 4 dimensions and above.

Value

A list containing:

idx An M-long vector containing the indices of the row of t in which each point in xi is found.

p An M-by-N+1 matrix containing the barycentric coordinates with respect to the enclosing simplex of each point in xi.

Note

Based on the Octave function Copyright (C) 2007-2012 David Bateman.

Author(s)

David Sterratt

36 Unique

See Also

tsearch, delaunayn

Unique

Extract Unique Rows

Description

'Unique' returns a vector, data frame or array like 'x' but with duplicate elements removed.

Usage

```
Unique(X, rows.are.sets = FALSE)
```

Arguments

Χ

Numerical matrix.

rows.are.sets If 'TRUE', rows are treated as sets - i.e., to define uniqueness, the order of the rows does not matter.

Value

Matrix of the same number of columns as x, with the unique rows in x sorted according to the columns of x. If rows.are.sets = TRUE the rows are also sorted.

Note

'Unique' is (under circumstances) much quicker than the more generic base function 'unique'.

Author(s)

Raoul Grasman

```
# `Unique' is faster than `unique'
x = matrix(sample(1:(4*8),4*8),ncol=4)
y = x[sample(1:nrow(x),3000,TRUE),]
gc(); system.time(unique(y))
gc(); system.time(Unique(y))
#
z = Unique(y)
x[matorder(x),]
z[matorder(z),]
```

Index

* arith	mesh.dsphere, 25
dot, 15	surf.tri,30
entry.value, 16	Unique, 36
extprod3d, 17	* optimize
matmax, 22	distmesh2d, 11
mesh.dcircle, 23	distmeshnd, 13
mesh.drectangle, 24	surf.tri,30
mesh.dsphere, 25	121.22
Unique, 36	as.mesh3d, <i>33</i>
* array	bary2cart, 3, 5
dot, 15	bar y2Car t, 3, 3
entry.value, 16	cart2bary, 3, 4
extprod3d, 17	cart2pol, 5, 6, 27, 30
matmax, 22	cart2sph, 6, 6, 27, 30
Unique, 36	convex.hull, 8
* dplot	convhulln, 7, 10, 19–21, 30, 32, 33
convhulln, 7	21, 30, 32, 33
delaunayn, 9	delaunayn, 8, 9, 12-15, 30, 32-36
distmesh2d, 11	distmesh2d, 10, 11, 14, 15, 23-27, 30
distmeshnd, 13	distmeshnd, 13, 25-27, 30
surf.tri,30	dot, 15
* graphs	dot3d, <i>32</i>
convhulln, 7	drop, <i>17</i>
delaunayn, 9	
distmesh2d, 11	entry.value, 16
distmeshnd, 13	entry.value<-(entry.value), 16
* hplot	extprod3d, 17
tetramesh, 31	
trimesh, 33	feasible.point, 18, 21
* math	halfspacen, 18, 21
convhulln, 7	11a11 Space11, 16, 21
delaunayn, 9	inhulln, 19, <i>21</i>
distmesh2d, 11	intersectn, 8, 20
distmeshnd, 13	111001 300011, 0, 20
dot, 15	matmax, 22
entry.value, 16	matmin (matmax), 22
extprod3d, 17	matorder (matmax), 22
mesh.dcircle, 23	matsort (matmax), 22
mesh.drectangle, 24	mesh.dcircle, 11, 13, 23, 24
- ·	• • • •

38 INDEX

```
mesh.diff, 11, 13, 15, 23, 24, 25
mesh.drectangle, 13, 23, 24, 24, 25
mesh.dsphere, 14, 15, 24, 25
mesh.hunif, 11, 14, 15, 26
mesh.intersect, 13, 15, 23, 25
mesh.intersect (mesh.diff), 24
mesh.union, 13, 15, 23, 25
mesh.union(mesh.diff), 24
mesh3d, 32
pol2cart, 6, 27, 30
polyarea, 28
qmesh3d, 32
rbox, 29
rgl, 31–33
shade3d, 32
sph2cart, 6, 27, 29
surf.tri, 8, 10, 30, 30, 32, 33
tetramesh, 31, 33
tmesh3d, 32
to.mesh3d, 32
tri.mesh, 10, 13, 15, 30
trimesh, 32, 33
tsearch, 34, 36
tsearchn, 34, 35
Unique, 36
wire3d, 32
```