Package ‘graphicalExtremes’

November 8, 2019

Title Statistical Methodology for Graphical Extreme Value Models

Version 0.1.0

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Description Statistical methodology for sparse multivariate extreme value models. Methods are provided for exact simulation and statistical inference for multivariate Pareto distributions on graphical structures as described in the paper 'Graphical Models for Extremes' by Engelke and Hitz (2018) <arXiv:1812.01734>.

Depends R (>= 3.6.0)

License GPL-3

Encoding UTF-8

LazyData true

Suggests testthat (>= 2.1.0)

RoxygenNote 6.1.1

Imports igraph (>= 1.2.4.1), mvtnorm (>= 1.0.10), matrixcalc (>= 1.0.3), Rdpack, stats (>= 3.6.0), grDevices (>= 3.6.0)

RdMacros Rdpack

NeedsCompilation no

Repository CRAN

Date/Publication 2019-11-08 09:40:02 UTC

R topics documented:

`censor` *Censor dataset*

Description

Censors each row of matrix `x` with vector `p`.

Usage

```
censor(x, p)
```

Arguments

`x`	Numeric matrix $n \times d$.
`p`	Numeric vector with d elements.

Value

Numeric matrix $n \times d$.

`chi2Gamma` *Transformation of extremal correlation χ to the Huesler–Reiss variogram Γ*

Description

Transforms the extremal correlation χ into the `Gamma` matrix from the definition of a Huesler–Reiss distribution.

Usage

```
chi2Gamma(chi)
```

Arguments

`chi`	Numeric or matrix, with entries between 0 and 1.

Details

The formula for transformation from `chi` to Γ that is applied element-wise is

$$\Gamma = (2\Phi^{-1}(1 - 0.5\chi))^2,$$

where Φ^{-1} is the inverse of the standard normal distribution function. This is the inverse of `Gamma2chi`.

Value

Numeric or matrix. The Γ parameters in the Huesler–Reiss distribution.

complete_Gamma *Completion of Gamma matrix on block graphs*

Description

Given a block graph and Gamma matrix with entries only specified on edges within the cliques of the graph, it returns the full Gamma matrix implied by the conditional independencies.

Usage

```
complete_Gamma(Gamma, graph)
```

Arguments

Gamma	Numeric $d \times d$ variogram matrix with entries only specified within the cliques of the graph. Alternatively, can be a vector containing the Gamma entries for each edge in the same order as in the graph object.
graph	Graph object from igraph package. The graph must be an undirected block graph, i.e., a decomposable, connected graph with singleton separator sets.

Details

For a block graph it suffices to specify the dependence parameters of the Huesler–Reiss distribution within the cliques of the graph, the remaining entries are implied by the conditional independence properties. For details see Engelke and Hitz (2018).

Value

Completed $d \times d$ Gamma matrix. s

References

Engelke S, Hitz AS (2018). “Graphical models for extremes.” Available from https://arxiv.org/abs/1812.01734.

Examples

```
## Complete a 4-dimensional HR distribution

my_graph <- igraph::graph_from_adjacency_matrix(rbind(
c(0, 1, 0, 0),
c(1, 0, 1, 1),
c(0, 1, 0, 1),
c(0, 1, 1, 0)),
mode = "undirected")

Gamma <- rbind(
c(0, .5, NA, NA),
```

```
c(.5, 0, 1, 1.5),
c(NA, 1, 0, .8),
c(NA, 1.5, .8, 0))

complete_Gamma(Gamma, my_graph)

## Alternative

Gamma_vec <- c(.5, 1, 1.5, .8)
complete_Gamma(Gamma_vec, my_graph)
```

---

data2mpareto *Data standardization to multivariate Pareto scale*

Description

Transforms the `data` matrix empirically to the multivariate Pareto scale.

Usage

```
data2mpareto(data, p)
```

Arguments

`data`	Numeric matrix of size $n \times d$, where n is the number of observations and d is the dimension.
`p`	Numeric between 0 and 1. Probability used for the quantile to threshold the data.

Details

The columns of the `data` matrix are first transformed empirically to standard Pareto distributions. Then, only the observations where at least one component exceeds the `p`-quantile of the standard Pareto distribution are kept. Those observations are finally divided by the `p`-quantile of the standard Pareto distribution to standardize them to the multivariate Pareto scale.

Value

Numeric matrix $m \times d$, where m is the number of rows in the original `data` matrix that are above the threshold.

Examples

```
n <- 20
d <- 4
p <- .8
G <-  cbind(c(0, 1.5, 1.5, 2),
            c(1.5, 0, 2, 1.5),
```

```
              c(1.5, 2, 0, 1.5),
              c(2, 1.5, 1.5, 0))

set.seed(123)
my_data = rmstable(n, "HR", d = d, par = G)
data2mpareto(my_data, p)
```

---

dim_Gamma *Is Gamma square matrix?*

Description

Check if Gamma matrix is square matrix. If so, return the dimension. Else, raise an error.

Usage

```
dim_Gamma(Gamma)
```

Arguments

`Gamma`	Numeric matrix. Matrix representing the variogram of an HR distribution.

Value

Numeric. The dimension of the matrix (number of rows and columns, if the matrix is symmetric). Else, raises an error.

---

emp_chi *Empirical estimation of extremal correlation χ*

Description

Estimates the d-dimensional extremal correlation coefficient χ empirically.

Usage

```
emp_chi(data, p)
```

Arguments

`data`	Numeric matrix of size $n \times d$, where n is the number of observations and d is the dimension.
`p`	Numeric between 0 and 1. Probability used for the quantile to compute the χ coefficient.

Value

Numeric. The empirical d-dimensional extremal correlation coefficient χ for the `data`.

Examples

```
n <- 100
d <- 2
p <- .8
G <-  cbind(c(0, 1.5),
            c(1.5, 0))

set.seed(123)
my_data = rmstable(n, "HR", d = d, par = G)
emp_chi(my_data, p)
```

---

emp_chi_mat — *Empirical estimation of extremal correlation matrix χ*

Description

Estimates empirically the matrix of bivariate extremal correlation coefficients χ.

Usage

```
emp_chi_mat(data, p)
```

Arguments

`data`	Numeric matrix of size $n \times d$, where n is the number of observations and d is the dimension.
`p`	Numeric between 0 and 1. Probability used for the quantile to compute the χ coefficient.

Value

Numeric matrix $d \times d$. The matrix contains the bivariate extremal coefficients χ_{ij}, for $i, j = 1, ..., d$.

Examples

```
n <- 100
d <- 4
p <- .8
Gamma <- cbind(c(0, 1.5, 1.5, 2),
               c(1.5, 0, 2, 1.5),
               c(1.5, 2, 0, 1.5),
               c(2, 1.5, 1.5, 0))
```

```
set.seed(123)
my_data = rmstable(n, "HR", d = d, par = Gamma)
emp_chi_mat(my_data, p)
```

---

emp_vario *Estimation of the variogram matrix Γ of the Huesler–Reiss distribution*

Description

Estimates the variogram of the Huesler–Reiss distribution empirically.

Usage

```
emp_vario(data, k = NULL, p = NULL)
```

Arguments

data	Numeric matrix of size $n \times d$, where n is the number of observations and d is the dimension.
k	Integer between 1 and d. Component of the multivariate observations that is conditioned to be larger than the threshold p. If NULL (default), then an average over all k is returned.
p	Numeric between 0 and 1 or NULL. If NULL (default), it is assumed that the data are already on multivariate Pareto scale. Else, p is used as the probability in the function data2mpareto to standardize the data.

Value

Numeric matrix $d \times d$. The estimated variogram of the Huesler–Reiss distribution.

---

fmpareto_graph_HR *Parameter fitting for multivariate Huesler–Reiss Pareto distributions on block graphs*

Description

Fits the parameters of a multivariate Huesler–Reiss Pareto distribution using (censored) likelihood estimation. Fitting is done separately on the cliques of the block graph. If edges_to_add are provided, then these edges are added in a greedy search to the original graph, such that in each step the likelihood is improved maximally and the new graph stays in the class of block graphs. See Engelke and Hitz (2018) for details.

Usage

```
fmpareto_graph_HR(data, graph, p = NULL, cens = FALSE,
  edges_to_add = NULL)
```

Arguments

- `data` — Numeric matrix of size $n \times d$, where n is the number of observations and d is the dimension.
- `graph` — Graph object from `igraph` package. The `graph` must be an undirected block graph, i.e., a decomposable, connected graph with singleton separator sets.
- `p` — Numeric between 0 and 1 or `NULL`. If `NULL` (default), it is assumed that the `data` are already on multivariate Pareto scale. Else, `p` is used as the probability in the function `data2mpareto` to standardize the `data`.
- `cens` — Logical. If true, then censored likelihood contributions are used for components below the threshold. By default, `cens = FALSE`.
- `edges_to_add` — Numeric matrix $m \times 2$, where m is the number of edges that are tried to be added in the greedy search. By default, `edges_to_add = NULL`.

Value

List consisting of:

- `graph`: Graph object from `igraph` package. If `edges_to_add` are provided, then this is a list of the resulting graphs in each step of the greedy search.
- `Gamma`: Numeric $d \times d$ estimated variogram matrix Γ. If `edges_to_add` are provided, then this is a list of the estimated variogram matrices in each step of the greedy search.
- `AIC`: (only if `edges_to_add` are provided) List of AIC values of the fitted models in each step of the greedy search.
- `edges_added`: (only if `edges_to_add` are provided) Numeric matrix $m' \times 2$, where the $m' \leq m$ rows contain the edges that were added in the greedy search.

References

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

Examples

```
## Fitting a 4-dimensional HR distribution

my_graph <- igraph::graph_from_adjacency_matrix(
  rbind(c(0, 1, 0, 0),
        c(1, 0, 1, 1),
        c(0, 1, 0, 0),
        c(0, 1, 0, 0)),
  mode = "undirected")
n <- 100
Gamma_vec <- c(.5,1.4,.8)
complete_Gamma(Gamma = Gamma_vec, graph = my_graph)  ## full Gamma matrix
edges_to_add <- rbind(c(1,3), c(1,4), c(3,4))

set.seed(123)
my_data <- rmpareto_tree(n, "HR", tree = my_graph, par = Gamma_vec)
```

```
my_fit <- fmpareto_graph_HR(my_data, graph = my_graph,
  p = NULL, cens = FALSE, edges_to_add = edges_to_add)
```

fmpareto_HR *Parameter fitting for multivariate Huesler–Reiss Pareto distribution*

Description

Fits the parameters of a multivariate Huesler–Reiss Pareto distribution using (censored) likelihood estimation.

Usage

```
fmpareto_HR(data, p = NULL, cens = FALSE, init, maxit = 100,
  graph = NULL, method = "BFGS")
```

Arguments

data	Numeric matrix of size $n \times d$, where n is the number of observations and d is the dimension.
p	Numeric between 0 and 1 or NULL. If NULL (default), it is assumed that the data are already on multivariate Pareto scale. Else, p is used as the probability in the function data2mpareto to standardize the data.
cens	Logical. If true, then censored likelihood contributions are used for components below the threshold. By default, cens = FALSE.
init	Numeric vector. Initial parameter values in the optimization. If graph is given, then the entries should correspond to the edges of the graph.
maxit	Positive integer. The maximum number of iterations in the optimization.
graph	Graph object from igraph package or NULL. If provided, the graph must be an undirected block graph, i.e., a decomposable, connected graph with singleton separator sets.
method	String. A valid optimization method used by the function optim. By default, method = "BFGS".

Details

If graph = NULL, then the parameters of a $d \times d$ parameter matrix Γ of a Huesler–Reiss Pareto distribution are fitted. If graph is provided, then the conditional independence structure of this graph is assumed and the parameters on the edges are fitted. In both cases the full likelihood is used and therefore this function should only be used for small dimensions, say, $d < 5$. For models in higher dimensions fitting can be done separately on the cliques; see fmpareto_graph_HR.

Value

List consisting of:

- `convergence`: Logical. Indicates whether the optimization converged or not.
- `par`: Numeric vector. Optimized parameters.
- `Gamma`: Numeric matrix $d \times d$. Fitted variogram matrix.
- `nllik`: Numeric. Optimized value of the negative log-likelihood function.
- `hessian`: Numeric matrix. Estimated Hessian matrix of the estimated parameters.

`Gamma2chi` *Transformation of the Huesler–Reiss variogram Γ to extremal correlation χ*

Description

Transforms the `Gamma` matrix from the definition of a Huesler–Reiss distribution into the corresponding extremal correlation χ.

Usage

```
Gamma2chi(Gamma)
```

Arguments

`Gamma`	Numeric or matrix, with positive entries.

Details

The formula for transformation from `Gamma` to χ that is applied element-wise is

$$\chi = 2 - 2\Phi(\sqrt{\Gamma}/2),$$

where Φ is the standard normal distribution function. This is the inverse of `chi2Gamma`.

Value

Numeric or matrix. The extremal correlation coefficient.

Gamma2chi_3D *Compute theoretical χ in 3D*

Description

Computes the theoretical χ coefficient in 3 dimensions.

Usage

```
Gamma2chi_3D(Gamma)
```

Arguments

Gamma	Numeric matrix 3×3.

Value

The 3-dimensional χ coefficient, i.e., the extremal correlation coefficient for the HR distribution. Note that $0 \le \chi \le 1$.

Gamma2graph *Transformation of Γ matrix to graph object*

Description

Transforms Gamma matrix to an igraph object for the corresponding Huesler–Reiss extremal graphical model, and plots it (optionally).

Usage

```
Gamma2graph(Gamma, to_plot = TRUE, ...)
```

Arguments

Gamma	Numeric $d \times d$ variogram matrix.
to_plot	Logical. If TRUE (default), it plots the resulting graph.
...	Graphical parameters for the plot.igraph function of the package igraph.

Details

The variogram uniquely determines the extremal graph structure of the corresponding Huesler–Reiss distribution. The conditional independencies can be identified from the inverses of the matrices $\Sigma^{(k)}$ defined in equation (10) in Engelke and Hitz (2018).

Value

Graph object from `igraph` package. An undirected graph.

References

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

Examples

```
Gamma <-  cbind(c(0, 1.5, 1.5, 2),
                c(1.5, 0, 2, 1.5),
                c(1.5, 2, 0, 1.5),
                c(2, 1.5, 1.5, 0))

Gamma2graph(Gamma, to_plot = TRUE)
```

`Gamma2par` *Extract upper triangular part of* Γ

Description

This function returns a vector containing the upper triangular part of the matrix `Gamma`. If `Gamma` is already a vector, it returns it as it is.

Usage

```
Gamma2par(Gamma)
```

Arguments

`Gamma`	Numeric $d \times d$ variogram matrix.

Value

Numeric vector with d elements. The upper triangular part of the given `Gamma` matrix.

Gamma2Sigma — *Transformation of Γ matrix to $\Sigma\hat{}(k)$ matrix*

Description

Transforms the Gamma matrix from the definition of a Huesler–Reiss distribution to the corresponding $\Sigma^{(k)}$ matrix.

Usage

```
Gamma2Sigma(Gamma, k = 1, full = FALSE)
```

Arguments

Gamma	Numeric $d \times d$ variogram matrix.
k	Integer between 1 (the default value) and d. Indicates which matrix $\Sigma^{(k)}$ should be produced.
full	Logical. If true, then the kth row and column in $\Sigma^{(k)}$ are included and the function returns a $d \times d$ matrix. By default, full = FALSE.

Details

Every $d \times d$ Gamma matrix in the definition of a Huesler–Reiss distribution can be transformed into a $(d-1) \times (d-1)$ $\Sigma^{(k)}$ matrix, for any k from 1 to d. The inverse of $\Sigma^{(k)}$ contains the graph structure corresponding to the Huesler–Reiss distribution with parameter matrix Gamma. If full = TRUE, then $\Sigma^{(k)}$ is returned as a $d \times d$ matrix with additional kth row and column that contain zeros. For details see Engelke and Hitz (2018). This is the inverse of function of Sigma2Gamma.

Value

Numeric $\Sigma^{(k)}$ matrix of size $(d-1) \times (d-1)$ if full = FALSE, and of size $d \times d$ if full = TRUE.

References

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

Examples

```
Gamma <-  cbind(c(0, 1.5, 1.5, 2),
                c(1.5, 0, 2, 1.5),
                c(1.5, 2, 0, 1.5),
                c(2, 1.5, 1.5, 0))
Gamma2Sigma(Gamma, k = 1, full = FALSE)
```

graphicalExtremes *graphicalExtremes: Statistical methodology for graphical extreme value models.*

Description

The graphicalExtremes package provides three categories of functions: simulation, estimation and transformation.

Simulation functions

- rmpareto
- rmpareto_tree
- rmstable
- rmstable_tree

Estimation functions

- fmpareto_graph_HR
- mst_HR
- emp_chi
- emp_chi_mat

Transformation functions

- Gamma2graph
- Gamma2Sigma
- Sigma2Gamma
- Gamma2chi
- chi2Gamma
- complete_Gamma
- data2mpareto

References

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

logdVK_HR *Compute censored exponent measure*

Description

Computes the censored exponent measure density of HR distribution.

Usage

```
logdVK_HR(x, K, par)
```

Arguments

x	Numeric vector with d positive elements where the censored exponent measure is to be evaluated.
K	Integer vector, subset of $\{1, \dots, d\}$. The index set that is not censored.
par	Numeric vector with $\frac{d(d-1)}{2}$ elements. It represents the upper triangular portion of a variogram matrix Γ.

Value

Numeric. The censored exponent measure of the HR distribution.

logdV_HR *Compute the exponent measure density of HR distribution*

Description

Computes the exponent measure density of HR distribution.

Usage

```
logdV_HR(x, par)
```

Arguments

x	Numeric matrix $n \times d$ or vector with d elements.
par	Numeric vector with $\frac{d(d-1)}{2}$ elements. It represents the upper triangular portion of a variogram matrix Γ.

Value

Numeric. The censored exponent measure of the HR distribution.

logLH_HR — *Full censored log-likelihood of HR model*

Description

Computes the full (censored) log-likelihood of HR model.

Usage

```
logLH_HR(data, Gamma, cens = FALSE)
```

Arguments

data	Numeric matrix $n \times d$. It contains observations following a multivariate HR Pareto distribution.
Gamma	Numeric matrix $n \times d$. It represents a variogram matrix Γ.
cens	Boolean. If true, then censored log-likelihood is computed. By default, `cens = FALSE`.

Value

Numeric. The full censored log-likelihood of HR model.

mparetomargins — *Marginalize multivariate Pareto dataset*

Description

Marginalize a multivariate Pareto dataset `data` with respect to the variables in `set_indices`.

Usage

```
mparetomargins(data, set_indices)
```

Arguments

data	Numeric matrix $n \times d$. A dataset containing observations following a multivariate Pareto distribution.
set_indices	Numeric vector with at most d different elements in 1, ..., d. The variables with respect to which to marginalize the multivariate distribution.

Value

Numeric matrix $n \times m$, where m is the length of `set_indices`. Marginalized multivariate Pareto data.

mst_HR — *Fitting of Huesler–Reiss minimum spanning tree*

Description

Fits the Huesler–Reiss minimum spanning tree, where the edge weights are are the negative maximized log-likelihoods of the bivariate Huesler–Reiss distributions. See Engelke and Hitz (2018) for details.

Usage

```
mst_HR(data, p = NULL, cens = FALSE)
```

Arguments

`data`	Numeric matrix of size $n \times d$, where n is the number of observations and d is the dimension.
`p`	Numeric between 0 and 1 or `NULL`. If `NULL` (default), it is assumed that the `data` are already on multivariate Pareto scale. Else, `p` is used as the probability in the function `data2mpareto` to standardize the `data`.
`cens`	Logical. If true, then censored likelihood contributions are used for components below the threshold. By default, `cens = FALSE`.

Value

List consisting of:

- `tree`: Graph object from `igraph` package. The fitted minimum spanning tree.
- `Gamma`: Numeric $d \times d$ estimated variogram matrix Γ corresponding to the fitted minimum spanning tree.

References

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

Examples

```
## Fitting a 4-dimensional HR MST tree

my_graph <- igraph::graph_from_adjacency_matrix(
  rbind(c(0, 1, 0, 0),
        c(1, 0, 1, 1),
        c(0, 1, 0, 0),
        c(0, 1, 0, 0)),
  mode = "undirected")
n <- 100
Gamma_vec <- c(.5,1.4,.8)
```

```
complete_Gamma(Gamma = Gamma_vec, graph = my_graph)  ## full Gamma matrix

set.seed(123)
my_data <- rmpareto_tree(n, "HR", tree = my_graph, par = Gamma_vec)
my_fit <- mst_HR(my_data, p = NULL, cens = FALSE)
```

`par2Gamma` *Create Γ from vector*

Description

This function takes the parameters in the vector `par` (upper triangular Gamma matrix) and returns full Gamma.

Usage

```
par2Gamma(par)
```

Arguments

`par`	Numeric vector with d elements. Upper triangular part of a Gamma matrix.

Value

Numeric matrix $d \times d$. Full Gamma matrix.

`rmpareto` *Sampling of a multivariate Pareto distribution*

Description

Simulates exact samples of a multivariate Pareto distribution.

Usage

```
rmpareto(n, model = c("HR", "logistic", "neglogistic", "dirichlet")[1],
  d, par)
```

Arguments

- `n` — Number of simulations.
- `model` — The parametric model type; one of:
  - `HR` (default),
  - `logistic`,
  - `neglogistic`,
  - `dirichlet`.
- `d` — Dimension of the multivariate Pareto distribution.
- `par` — Respective parameter for the given `model`, that is,
  - Γ, numeric $d \times d$ variogram matrix, if `model` = `HR`.
  - $\theta \in (0, 1)$, if `model` = `logistic`.
  - $\theta > 0$, if `model` = `neglogistic`.
  - α, numeric vector of size `d` with positive entries, if `model` = `dirichlet`.

Details

The simulation follows the algorithm in Engelke and Hitz (2018). For details on the parameters of the Huesler–Reiss, logistic and negative logistic distributions see Dombry et al. (2016), and for the Dirichlet distribution see Coles and Tawn (1991).

Value

Numeric matrix of size $n \times d$ of simulations of the multivariate Pareto distribution.

References

Coles SG, Tawn JA (1991). "Modelling extreme multivariate events." *J. R. Stat. Soc. Ser. B Stat. Methodol.*, **53**, 377–392.

Dombry C, Engelke S, Oesting M (2016). "Exact simulation of max-stable processes." *Biometrika*, **103**, 303–317.

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

Examples

```
## A 4-dimensional HR distribution
n <- 10
d <- 4
G <-  cbind(c(0, 1.5, 1.5, 2),
            c(1.5, 0, 2, 1.5),
            c(1.5, 2, 0, 1.5),
            c(2, 1.5, 1.5, 0))

rmpareto(n, "HR", d = d, par = G)
```

```
## A 3-dimensional logistic distribution
n <- 10
d <- 3
theta <- .6
rmpareto(n, "logistic", d, par = theta)

## A 5-dimensional negative logistic distribution
n <- 10
d <- 5
theta <- 1.5
rmpareto(n, "neglogistic", d, par = theta)

## A 4-dimensional Dirichlet distribution
n <- 10
d <- 4
alpha <- c(.8, 1, .5, 2)
rmpareto(n, "dirichlet", d, par = alpha)
```

`rmpareto_tree` *Sampling of a multivariate Pareto distribution on a tree*

Description

Simulates exact samples of a multivariate Pareto distribution that is an extremal graphical model on a tree as defined in Engelke and Hitz (2018).

Usage

```
rmpareto_tree(n, model = c("HR", "logistic", "dirichlet")[1], tree, par)
```

Arguments

`n` Number of simulations.

`model` The parametric model type; one of:
- `HR` (default),
- `logistic`,
- `dirichlet`.

`tree` Graph object from `igraph` package. This object must be a tree, i.e., an undirected graph that is connected and has no cycles.

`par` Respective parameter for the given `model`, that is,
- Γ, numeric $d \times d$ variogram matrix, where only the entries corresponding to the edges of the `tree` are used, if `model` = `HR`. Alternatively, can be a vector of length $d-1$ containing the entries of the variogram corresponding to the edges of the given `tree`.
- $\theta \in (0, 1)$, vector of length $d-1$ containing the logistic parameters corresponding to the edges of the given `tree`, if `model` = `logistic`.

- a matrix of size $(d-1) \times 2$, where the rows contain the parameters vectors α of size 2 with positve entries for each of the edges in `tree`, if `model = dirichlet`.

Details

The simulation follows the algorithm in Engelke and Hitz (2018). For details on the parameters of the Huesler–Reiss, logistic and negative logistic distributions see Dombry et al. (2016), and for the Dirichlet distribution see Coles and Tawn (1991).

Value

Numeric matrix of size $n \times d$ of simulations of the multivariate Pareto distribution.

References

Coles SG, Tawn JA (1991). "Modelling extreme multivariate events." *J. R. Stat. Soc. Ser. B Stat. Methodol.*, **53**, 377–392.

Dombry C, Engelke S, Oesting M (2016). "Exact simulation of max-stable processes." *Biometrika*, **103**, 303–317.

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

Examples

```
## A 4-dimensional HR tree model

my_tree <- igraph::graph_from_adjacency_matrix(rbind(
c(0, 1, 0, 0),
c(1, 0, 1, 1),
c(0, 1, 0, 0),
c(0, 1, 0, 0)),
mode = "undirected")
n <- 10
Gamma_vec <- c(.5,1.4,.8)
set.seed(123)
rmpareto_tree(n, "HR", tree = my_tree, par = Gamma_vec)

## A 4-dimensional Dirichlet model with asymmetric edge distributions

alpha = cbind(c(.2, 1, .5), c(1.5, .6, .8))
rmpareto_tree(n, model = "dirichlet", tree = my_tree, par = alpha)
```

rmstable *Sampling of a multivariate max-stable distribution*

Description

Simulates exact samples of a multivariate max-stable distribution.

Usage

```
rmstable(n, model = c("HR", "logistic", "neglogistic", "dirichlet")[1],
  d, par)
```

Arguments

`n`	Number of simulations.
`model`	The parametric model type; one of: • `HR` (default), • `logistic`, • `neglogistic`, • `dirichlet`.
`d`	Dimension of the multivariate Pareto distribution.
`par`	Respective parameter for the given `model`, that is, • Γ, numeric $d \times d$ variogram matrix, if `model` = `HR`. • $\theta \in (0, 1)$, if `model` = `logistic`. • $\theta > 0$, if `model` = `neglogistic`. • α, numeric vector of size `d` with positive entries, if `model` = `dirichlet`.

Details

The simulation follows the extremal function algorithm in Dombry et al. (2016). For details on the parameters of the Huesler–Reiss, logistic and negative logistic distributions see Dombry et al. (2016), and for the Dirichlet distribution see Coles and Tawn (1991).

Value

Numeric matrix of size $n \times d$ of simulations of the multivariate max-stable distribution.

References

Coles SG, Tawn JA (1991). "Modelling extreme multivariate events." *J. R. Stat. Soc. Ser. B Stat. Methodol.*, **53**, 377–392.

Dombry C, Engelke S, Oesting M (2016). "Exact simulation of max-stable processes." *Biometrika*, **103**, 303–317.

Examples

```
## A 4-dimensional HR distribution
n <- 10
d <- 4
G <-  cbind(c(0, 1.5, 1.5, 2),
            c(1.5, 0, 2, 1.5),
            c(1.5, 2, 0, 1.5),
            c(2, 1.5, 1.5, 0))

rmstable(n, "HR", d = d, par = G)

## A 3-dimensional logistic distribution
n <- 10
d <- 3
theta <- .6
rmstable(n, "logistic", d, par = theta)

## A 5-dimensional negative logistic distribution
n <- 10
d <- 5
theta <- 1.5
rmstable(n, "neglogistic", d, par = theta)

## A 4-dimensional Dirichlet distribution
n <- 10
d <- 4
alpha <- c(.8, 1, .5, 2)
rmstable(n, "dirichlet", d, par = alpha)
```

---

rmstable_tree — *Sampling of a multivariate max-stable distribution on a tree*

Description

Simulates exact samples of a multivariate max-stable distribution that is an extremal graphical model on a tree as defined in Engelke and Hitz (2018).

Usage

```
rmstable_tree(n, model = c("HR", "logistic", "dirichlet")[1], tree, par)
```

Arguments

`n` — Number of simulations.

`model` — The parametric model type; one of:

- `HR` (default),
- `logistic`,

- `dirichlet`.

`tree` Graph object from `igraph` package. This object must be a tree, i.e., an undirected graph that is connected and has no cycles.

`par` Respective parameter for the given `model`, that is,

- Γ, numeric $d \times d$ variogram matrix, where only the entries corresponding to the edges of the `tree` are used, if `model = HR`. Alternatively, can be a vector of length $d-1$ containing the entries of the variogram corresponding to the edges of the given `tree`.
- $\theta \in (0,1)$, vector of length $d-1$ containing the logistic parameters corresponding to the edges of the given `tree`, if `model = logistic`.
- a matrix of size $(d-1) \times 2$, where the rows contain the parameter vectors α of size 2 with positve entries for each of the edges in `tree`, if `model = dirichlet`.

Details

The simulation follows a combination of the extremal function algorithm in Dombry et al. (2016) and the theory in Engelke and Hitz (2018) to sample from a single extremal function. For details on the parameters of the Huesler–Reiss, logistic and negative logistic distributions see Dombry et al. (2016), and for the Dirichlet distribution see Coles and Tawn (1991).

Value

Numeric matrix of size $n \times d$ of simulations of the multivariate max-stable distribution.

References

Coles SG, Tawn JA (1991). "Modelling extreme multivariate events." *J. R. Stat. Soc. Ser. B Stat. Methodol.*, **53**, 377–392.

Dombry C, Engelke S, Oesting M (2016). "Exact simulation of max-stable processes." *Biometrika*, **103**, 303–317.

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

Examples

```
## A 4-dimensional HR tree model

my_tree <- igraph::graph_from_adjacency_matrix(rbind(
c(0, 1, 0, 0),
c(1, 0, 1, 1),
c(0, 1, 0, 0),
c(0, 1, 0, 0)),
mode = "undirected")
n <- 10
Gamma_vec <- c(.5,1.4,.8)
rmstable_tree(n, "HR", tree = my_tree, par = Gamma_vec)
```

```
## A 4-dimensional Dirichlet model with asymmetric edge distributions

alpha = cbind(c(.2, 1, .5), c(1.5, .6, .8))
rmstable_tree(n, model = "dirichlet", tree = my_tree, par = alpha)
```

---

select_edges *Select edges to add to a graph*

Description

This function selects all possible edges that can be added to the graph while still remaining in the class of block graphs.

Usage

```
select_edges(graph)
```

Arguments

graph	Graph object from igraph package. The graph must be an undirected block graph, i.e., a decomposable, connected graph with singleton separator sets.

Value

Numeric vector.

---

set_graph_parameters *Set graphical parameters*

Description

Set graphical parameters to graph which is an object from the igraph package.

Usage

```
set_graph_parameters(graph)
```

Arguments

graph	Graph object from igraph package.

Value

Graph object from igraph package.

`Sigma2Gamma` *Transformation of $\Sigma\hat{}(k)$ matrix to Γ matrix*

Description

Transforms the $\Sigma^{(k)}$ matrix from the definition of a Huesler–Reiss distribution to the corresponding Γ matrix.

Usage

```
Sigma2Gamma(S, k = 1, full = FALSE)
```

Arguments

`S`	Numeric $(d-1) \times (d-1)$ covariance matrix $\Sigma^{(k)}$ from the definition of a Huesler–Reiss distribution. Numeric $d \times d$ covariance matrix if `full = TRUE`, see `full` parameter.
`k`	Integer between `1` (the default value) and `d`. Indicates which matrix $\Sigma^{(k)}$ is represented by `S`.
`full`	Logical. If true, then the kth row and column in $\Sigma^{(k)}$ are included and the function returns a $d \times d$ matrix. By default, `full = FALSE`.

Details

For any `k` from `1` to `d`, the $\Sigma^{(k)}$ matrix of size $(d-1) \times (d-1)$ in the definition of a Huesler–Reiss distribution can be transformed into a the corresponding $d \times d$ Γ matrix. If `full = TRUE`, then $\Sigma^{(k)}$ must be a $d \times d$ matrix with kth row and column containing zeros. For details see Engelke and Hitz (2018). This is the inverse of function of `Gamma2Sigma`.

Value

Numeric $d \times d$ Γ matrix.

References

Engelke S, Hitz AS (2018). "Graphical models for extremes." Available from https://arxiv.org/abs/1812.01734.

Examples

```
Sigma1 <-  rbind(c(1.5, 0.5, 1),
                 c(0.5, 1.5, 1),
                 c(1, 1, 2))
Sigma2Gamma(Sigma1, k = 1, full = FALSE)
```

simu_px_dirichlet *Simulate Dirichlet extremal functions*

Description

Simulates Dirichlet extremal functions

Usage

```
simu_px_dirichlet(n, d, idx, alpha)
```

Arguments

n	Number of simulations.
d	Dimension of the multivariate Pareto distribution.
idx	Integer or numeric vector with n elements. Inde(x|ces) from 1 to d, that determine which extremal function to simulate.
alpha	Numeric vector of size d.

Value

Numeric matrix $n \times d$. Simulated data.

simu_px_HR *Simulate HR extremal functions*

Description

Simulates the Huessler–Reiss extremal functions

Usage

```
simu_px_HR(n, d, idx, trend, chol_mat)
```

Arguments

n	Number of simulations.
d	Dimension of the multivariate Pareto distribution.
idx	Integer. Index corresponding to the variable over which the extremal function is simulated.
trend	Numeric. Trend corresponding to the variable idx.
chol_mat	Numeric matrix $d \times d$. Cholesky decomposition of the desired covariance matrix.

Value

Numeric matrix $n \times d$. Simulated data.

simu_px_logistic *Simulate logistic extremal functions*

Description

Simulates logistic extremal functions

Usage

```
simu_px_logistic(n, d, idx, theta)
```

Arguments

n	Number of simulations.
d	Dimension of the multivariate Pareto distribution.
idx	Integer or numeric vector with n elements. Inde(x|ces) from 1 to d, that determine which extremal function to simulate.
theta	Numeric — assume $0 < \theta < 1$.

Value

Numeric matrix $n \times d$. Simulated data.

simu_px_neglogistic *Simulate negative logistic extremal functions*

Description

Simulates negative logistic extremal functions

Usage

```
simu_px_neglogistic(n, d, idx, theta)
```

Arguments

n	Number of simulations.
d	Dimension of the multivariate Pareto distribution.
idx	Integer or numeric vector with n elements. Inde(x|ces) from 1 to d, that determine which extremal function to simulate.
theta	Numeric — assume $\theta > 0$.

Value

Numeric matrix $n \times d$. Simulated data.

simu_px_tree_dirichlet *Simulate Dirichlet extremal functions on a tree*

Description

Simulates Dirichlet extremal functions on a tree

Usage

```
simu_px_tree_dirichlet(n, alpha.start, alpha.end, A)
```

Arguments

n	Number of simulations.
alpha.start	Numeric vector with $d-1$ elements, where d is the number of nodes in the tree (and $d-1$ is the number of edges).
alpha.end	Numeric vector with $d-1$ elements, where d is the number of nodes in the tree (and $d-1$ is the number of edges).
A	Numeric matrix $d \times (d-1)$; the rows represent the nodes in the tree, the columns represent the edges. For a fixed node $k = 1, \ldots, d$, each entry (i, j) is equal to 1 if the edge in position j is on the directed path from node k to node i in the polytree rooted at node k.

Value

Numeric matrix $n \times d$. Simulated data.

simu_px_tree_HR *Simulate HR extremal functions on a tree*

Description

Simulates the Huessler–Reiss extremal functions on a tree

Usage

```
simu_px_tree_HR(n, Gamma_vec, A)
```

Arguments

- `n` — Number of simulations.
- `Gamma_vec` — Numeric vector with $d-1$ elements, where d is the number of nodes in the tree (and $d-1$ is the number of edges).
- `A` — Numeric matrix $d \times (d-1)$; the rows represent the nodes in the tree, the columns represent the edges. For a fixed node $k = 1, \dots, d$, each entry (i, j) is equal to 1 if the edge in position j is on the directed path from node k to node i in the polytree rooted at node k.

Value

Numeric matrix $n \times d$. Simulated data.

`simu_px_tree_logistic` *Simulate logistic extremal functions on a tree*

Description

Simulates logistic extremal functions on a tree

Usage

```
simu_px_tree_logistic(n, theta, A)
```

Arguments

- `n` — Number of simulations.
- `theta` — Numeric vector with 1 or $d-1$ elements. Assume that the entry are such that $0 < \theta < 1$.
- `A` — Numeric matrix $d \times (d-1)$; the rows represent the nodes in the tree, the columns represent the edges. For a fixed node $k = 1, \dots, d$, each entry (i, j) is equal to 1 if the edge in position j is on the directed path from node k to node i in the polytree rooted at node k.

Value

Numeric matrix $n \times d$. Simulated data.

`unif` *Uniform margin*

Description

Rescale the vector `x` empirically to uniform margin.

Usage

```
unif(x)
```

Arguments

`x`	Numeric vector.

Value

Numeric vector with entries rescaled to uniform margins

`V_HR` *Compute exponent measure*

Description

Computes the exponent measure of HR distribution.

Usage

```
V_HR(x, par)
```

Arguments

`x`	Numeric vector with d positive elements where the exponent measure is to be evaluated.
`par`	Numeric vector with $\frac{d(d-1)}{2}$ elements. It represents the upper triangular portion of a variogram matrix Γ.

Value

Numeric. The exponent measure of the HR distribution.

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