Package 'gumbel'

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Type Package

Title The Gumbel-Hougaard Copula

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Description Provides probability functions (cumulative distribution and density functions), simulation function (Gumbel copula multivariate simulation) and estimation functions (Maximum Likelihood Estimation, Inference For Margins, Moment Based Estimation and Canonical Maximum Likelihood).

Depends R (>= 2.10.0)

License GPL (>= 2)

NeedsCompilation no

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`Gumbel` *The Gumbel Hougaard Copula*

Description

Density function, distribution function, random generation, generator and inverse generator function for the Gumbel Copula with parameters `alpha`. The 4 classic estimation methods for copulas.

Usage

```
dgumbel(u, v=NULL, alpha, dim=2, warning = FALSE)
pgumbel(u, v=NULL, alpha, dim=2)
rgumbel(n, alpha, dim=2, method=1)
phigumbel(t, alpha=1)
invphigumbel(t, alpha=1)
gumbel.MBE(x, y, marg = "exp")
gumbel.EML(x, y, marg = "exp")
gumbel.IFM(x, y, marg = "exp")
gumbel.CML(x, y)
```

Arguments

u	vector of quantiles if argument v is provided or matrix of quantiles if argument v is not provided
v	vector of quantiles, needed if u is not a matrix
n	number of observations. If length(n) > 1, the length is taken to be the number required.
alpha	parameter of the Copula. Must be greater than 1.
dim	an integer specifying the dimension of the copula.
t	dummy variable of the generator ϕ or the inverse generator ϕ^-1. could be a n-dimensional array.
method	an integer code for the method used in simulation. 1 is the common frailty approach, 2 uses the K function (only valid with dim=2).
x,y	vectors of observations, realizations of random variable X and Y.
marg	a character string specifying the marginals of vector (X, Y). It must be either "exp"(default value) or "gamma".
warning	a logical (default value FALSE) if you want warnings.

Details

The Gumbel Hougaard Copula with parameter alpha is defined by its generator

$$\phi(t) = (-ln(t))^a lpha.$$

The generator and inverse generator are implemented in phigumbel and invphigumbel respectively. As an Archimedean copula, its distribution function is

$$C(u_1, ..., u_n) = \phi^{-1}(\phi(u_1)+...+\phi(u_n)) = exp(-((-ln(u_1))^a lpha+...+(-ln(u_n))^a lpha)^1/\alpha).$$

pgumbel and dgumbel computes the distribution function (expression above) and the density (n times differentiation of expression above with respect to u_i). As there is no explicit formulas for the density of a Gumbel copula, dgumbel is not yet impemented for argument dim>3. This two functions works with a dim-dimensional array with the last dimension being equalled to dim or with a matrix with dim columns (see examples).

Random generation is carried out with 2 algorithms the common frailty algorithm (method=1) and the 'K' algorithm (method=2). The common frailty algorithm (cf. Marshall & Olkin(1988)) can be sum up in three lines

- generate y_1, y_2 from exponential distribution of mean 1,
- generate θ from a stable distribution with parameter$(1/alpha, 1, 1, 0)$,
- $u_1 < -phi(y_1/\theta)$ and $u_2 < -phi(y_2/\theta)$.

This algorithm works with any dimension. See Chambers et al(1976) for stable random generation. The 'K' algorithm use the fact the distribution function of random variable $C(U, V)$ is $K(t) = t - \phi(y)/\phi'(t)$. The algorithm is

- generate v_1, t from uniform distribution
- generate v_2 from the K distribution i.e. $v_2 < -K^{-1}(t)$.
- $u_1 < -\phi^{-1}(\phi(v_1)v_2)$ and $u_2 < -\phi^{-1}(\phi(v_1)(1 - v_2))$.

Warning, the 'K' algorithm does NOT work with `dim>2`.

We implements the 4 usual method of estimation for copulas, namely the Exact Maximum Likelihood (`gumbel.EML`), the Inference for Margins (`gumbel.IFM`), the Moment-base Estimation (`gumbel.MBE`) and the Canonical Maximum Likelihood (`gumbel.CML`). For the moment, only two types of marginals are available : exponential distribution (`marg="exp"`) and gamma distribution (`marg="gamma"`).

Value

`dgumbel` gives the density, `pgumbel` gives the distribution function, `rgumbel` generates random deviates, `phigumbel` gives the generator, `invphigumbel` gives the inverse generator.

`gumbel.EML`, `gumbel.IFM`, `gumbel.MBE` and `gumbel.CML` returns the vector of estimates.

Invalid arguments will result in return value `NaN`.

Author(s)

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References

Nelsen, R. (2006), *An Introduction to Copula, Second Edition*, Springer.

Marshall & Olkin(1988), *Families of multivariate distributions*, Journal of the American Statistical Association.

Chambers et al (1976), *A method for simulating stable random variables*, Journal of the American Statistical Association.

Nelsen, R. (2005), *Dependence Modeling with Archimedean Copulas*, booklet available at www.lclark.edu/~mathsci/brazil2.p

Examples

```
#dim=2
u<-seq(0,1, .1)
v<-u
#classic parametrization
#independance case (alpha=1)
dgumbel(u,v,1)
pgumbel(u,v,1)
#another parametrization
```

```
dgumbel(cbind(u,v), alpha=1)
pgumbel(cbind(u,v), alpha=1)

#dim=3 - equivalent parametrization
x <- cbind(u,u,u)
y <- array(u, c(1,11,3))
pgumbel(x, alpha=2, dim=3)
pgumbel(y, alpha=2, dim=3)
dgumbel(x, alpha=2, dim=3)
dgumbel(y, alpha=2, dim=3)

#dim=4
x <- cbind(x,u)
pgumbel(x, alpha=3, dim=4)
y <- array(u, c(2,1,11,4))
pgumbel(y, alpha=3, dim=4)


#independence case
rand <- t(rgumbel(200,1))
plot(rand[1,], rand[2,], col="green", main="Gumbel copula")

#positive dependence
rand <- t(rgumbel(200,2))
plot(rand[1,], rand[2,], col="red", main="Gumbel copula")

#comparison of random generation algorithms
nbsimu <- 10000
#Marshall Olkin algorithm
system.time(rgumbel(nbsimu, 2, dim=2, method=1))[3]
#K algortihm
system.time(rgumbel(nbsimu, 2, dim=2, method=2))[3]

#pseudo animation
## Not run:
anim <-function(n, max=50)
{
for(i in seq(1,max,length.out=n))
{
u <- t(rgumbel(10000, i, method=2))
plot(u[1,], u[2,], col="green", main="Gumbel copula",
            xlim=c(0,1), ylim=c(0,1), pch=".")
cat()
}
}
anim(20, 20)

## End(Not run)

#3D plots

#plot the density
  x <- seq(.05, .95, length = 30)
```

```
y <- x
z <- outer(x, y, dgumbel, alpha=2)

persp(x, y, z, theta = 30, phi = 30, expand = 0.5, col = "lightblue",
      ltheta = 100, shade = 0.25, ticktype = "detailed",
      xlab = "x", ylab = "y", zlab = "density")

#with wonderful colors
#code of P. Soutiras
zlim <- c(0, max(z))
ncol <- 100
nrz <- nrow(z)
ncz <- ncol(z)
jet.colors <- colorRampPalette(c("#00007F", "blue", "#007FFF",
"cyan", "#7FFF7F", "yellow", "#FF7F00", "red", "#7F0000"))
couleurs <- tail(jet.colors(1.2*ncol),ncol)
fcol <- couleurs[trunc(z/zlim[2]*(ncol-1))+1]
dim(fcol) <- c(nrz,ncz)
fcol <- fcol[-nrz,-ncz]
persp(x, y, z, col=fcol, zlim=zlim, theta=30, phi=30, ticktype = "detailed",
        box = TRUE,  xlab = "x", ylab = "y", zlab = "density")


#plot the distribution function
z <- outer(x, y, pgumbel, alpha=2)
persp(x, y, z, theta = 30, phi = 30, expand = 0.5, col = "lightblue",
        ltheta = 100, shade = 0.25, ticktype = "detailed",
        xlab = "u", ylab = "v", zlab = "cdf")



#parameter estimation
#true value : lambdaX=lambdaY=1, alpha=2
simu <- qexp(rgumbel(200, 2))
gumbel.MBE(simu[,1], simu[,2])
gumbel.IFM(simu[,1], simu[,2])
gumbel.EML(simu[,1], simu[,2])
gumbel.CML(simu[,1], simu[,2])

#true value : lambdaX=lambdaY=1, alphaX=alphaY=2, alpha=3
simu <- qgamma(rgumbel(200, 3), 2, 1)
gumbel.MBE(simu[,1], simu[,2], "gamma")
gumbel.IFM(simu[,1], simu[,2], "gamma")
gumbel.EML(simu[,1], simu[,2], "gamma")
gumbel.CML(simu[,1], simu[,2])
```

Description

Daily Climatological data recorded in two French cities: Echirolles and St Martin-En-Haut. Weather stations are located at Echirolles (ELEV: 237m, LAT: 45 06' 00" N LONG: 5 42' 00" E) and La Rafiliere (ELEV: 575m, LAT: 45 39' 00" N LONG: 4 33' 00" E), respectively.

Usage

```
data(windStMartin)
data(windEchirolles)
```

Format

`windStMartin` and `windEchirolles` are data frames of 15 columns:

`YEAR` Year.

`MONTH` Month number.

`DAY` Day number.

`TEMP.MEAN` Average temperature (Celsius degree).

`TEMP.HIGH` Maximum temperature.

`TIME.TH` Time of the maximum temperature (hh:mm).

`TEMP.LOW` Minimum temperature.

`TIME.TL` Time of the minimum temperature.

`HDD` Heating Degree Days.

`CDD` Cooling Degree Days.

`RAIN` Rain (mm).

`WIND.MEAN` Wind speed average (km/h).

`WIND.HIGH` Wind speed maximum.

`WIND.TH` Time of the wind speed maximum.

`DOM.DIR` Dominant direction of the wind, a character string where "N" for North, "NE" for North-East, etc...

Source

http://www.meteoisere.com/Vantage/ and http://hautsdulyonnais.free.fr/

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