# Package 'hal9001' 

February 9, 2022
Title The Scalable Highly Adaptive Lasso

## Version 0.4.3

Description A scalable implementation of the highly adaptive lasso algorithm, including routines for constructing sparse matrices of basis functions of the observed data, as well as a custom implementation of Lasso regression tailored to enhance efficiency when the matrix of predictors is composed exclusively of indicator functions. For ease of use and increased flexibility, the Lasso fitting routines invoke code from the 'glmnet' package by default. The highly adaptive lasso was first formulated and described by MJ van der Laan (2017) [doi:10.1515/ijb-2015-0097](doi:10.1515/ijb-2015-0097), with practical demonstrations of its performance given by Benkeser and van der Laan (2016) [doi:10.1109/DSAA.2016.93](doi:10.1109/DSAA.2016.93). This implementation of the highly adaptive lasso algorithm was described by Hejazi, Coyle, and van der Laan (2020) [doi:10.21105/joss.02526](doi:10.21105/joss.02526).
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BugReports https://github.com/tlverse/hal9001/issues

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## $R$ topics documented:

+.formula_hal9001 ..... 3
apply_copy_map ..... 3
as_dgCMatrix ..... 4
basis_list_cols ..... 5
basis_of_degree ..... 6
cv_lasso ..... 7
cv_lasso_early_stopping ..... 7
enumerate_basis ..... 8
evaluate_basis ..... 9
fit_hal ..... 10
formula_hal ..... 13
formula_helpers ..... 14
h ..... 14
hal9000 ..... 16
hal9001 ..... 16
hal_quotes ..... 16
index_first_copy ..... 17
lassi_fit_module ..... 17
lassi_origami ..... 17
lassi_predict ..... 18
make_basis_list ..... 18
make_copy_map ..... 19
make_design_matrix ..... 20
make_reduced_basis_map ..... 21
meets_basis ..... 21
predict.hal9001 ..... 22
predict.SL.hal9001 ..... 23
print.formula_hal9001 ..... 24
print.summary.hal9001 ..... 24
SL.hal9001 ..... 25
squash_hal_fit ..... 26
summary.hal9001 ..... 27
Index ..... 29
+. formula_hal9001 HAL Formula addition: Adding formula term object together into a single formula object term.

## Description

HAL Formula addition: Adding formula term object together into a single formula object term.

```
Usage
\#\# S3 method for class 'formula_hal9001'
\(x+y\)
```


## Arguments

$x \quad$ A formula_hal9001 object as outputted by $h$.
y A formula_hal9001 object as outputted by $h$.
apply_copy_map Apply copy map

## Description

OR duplicate training set columns together

## Usage

apply_copy_map(X, copy_map)

## Arguments

$\begin{array}{ll}X & \text { Sparse matrix containing columns of indicator functions. } \\ \text { copy_map } & \text { the copy map }\end{array}$

## Value

A dgCMatrix sparse matrix corresponding to the design matrix for a zero-th order highly adaptive lasso, but with all duplicated columns (basis functions) removed.

## Examples

```
gendata <- function(n) {
    W1 <- runif(n, -3, 3)
    W2 <- rnorm(n)
    W3 <- runif(n)
    W4 <- rnorm(n)
    g0 <- plogis(0.5 * (-0.8 * W1 + 0.39 * W2 + 0.08 * W3 - 0.12 * W4))
    A <- rbinom(n, 1, g0)
    Q0 <- plogis(0.15 * (2 * A + 2 * A * W1 + 6 * A * W3 * W4 - 3))
    Y <- rbinom(n, 1, Q0)
    data.frame(A, W1, W2, W3, W4, Y)
}
set.seed(1234)
data <- gendata(100)
covars <- setdiff(names(data), "Y")
X <- as.matrix(data[, covars, drop = FALSE])
basis_list <- enumerate_basis(X)
x_basis <- make_design_matrix(X, basis_list)
copy_map <- make_copy_map(x_basis)
x_basis_uniq <- apply_copy_map(x_basis, copy_map)
```

as_dgCMatrix Fast Coercion to Sparse Matrix

## Description

Fast and efficient coercion of standard matrix objects to sparse matrices. Borrowed from http://gallery.rcpp.org/articles/sparse matrix-coercion/. INTERNAL USE ONLY.

## Usage

as_dgCMatrix(XX_)

## Arguments

XX_ An object of class Matrix that has a sparse structure suitable for coercion to a sparse matrix format of dgCMatrix.

## Value

An object of class dgCMatrix, coerced from input $X_{X}$.

```
basis_list_cols List Basis Functions
```


## Description

Build a list of basis functions from a set of columns

## Usage

```
basis_list_cols(
        cols,
        x,
        smoothness_orders,
        include_zero_order,
        include_lower_order = FALSE
    )
```


## Arguments

cols Index or indices (as numeric) of covariates (columns) of interest in the data matrix $x$ for which basis functions ought to be generated. Note that basis functions for interactions of these columns are computed automatically.
x
A matrix containing observations in the rows and covariates in the columns. Basis functions are computed for these covariates.
smoothness_orders
An integer vector of length $n \operatorname{col}(x)$ specifying the desired smoothness of the function in each covariate. $\mathrm{k}=0$ is no smoothness (indicator basis), $\mathrm{k}=1$ is first order smoothness, and so on. For an additive model, the component function for each covariate will have the degree of smoothness as specified by smoothness_orders. For non-additive components (tensor products of univariate basis functions), the univariate basis functions in each tensor product have smoothness degree as specified by smoothness_orders.
include_zero_order
A logical, indicating whether the zeroth order basis functions are included for each covariate (if TRUE), in addition to the smooth basis functions given by smoothness_orders. This allows the algorithm to data-adaptively choose the appropriate degree of smoothness.
include_lower_order
A logical, like include_zero_order, except including all basis functions of lower smoothness degrees than specified via smoothness_orders.

## Value

A list containing the basis functions generated from a set of input columns.

## Description

Find the full list of basis functions up to a particular degree

## Usage

basis_of_degree(
x ,
degree,
smoothness_orders,
include_zero_order,
include_lower_order
)

## Arguments

x
degree The highest order of interaction terms for which the basis functions ought to be generated. The default (NULL) corresponds to generating basis functions for the full dimensionality of the input matrix.
smoothness_orders
An integer vector of length $n \operatorname{col}(x)$ specifying the desired smoothness of the function in each covariate. $k=0$ is no smoothness (indicator basis), $k=1$ is first order smoothness, and so on. For an additive model, the component function for each covariate will have the degree of smoothness as specified by smoothness_orders. For non-additive components (tensor products of univariate basis functions), the univariate basis functions in each tensor product have smoothness degree as specified by smoothness_orders.
include_zero_order
A logical, indicating whether the zeroth order basis functions are included for each covariate (if TRUE), in addition to the smooth basis functions given by smoothness_orders. This allows the algorithm to data-adaptively choose the appropriate degree of smoothness.
include_lower_order
A logical, like include_zero_order, except including all basis functions of lower smoothness degrees than specified via smoothness_orders.

## Value

A list containing basis functions and cutoffs generated from a set of input columns up to a particular pre-specified degree.

## Description

Fits Lasso regression using a customized procedure, with cross-validation based on origami

## Usage

cv_lasso(x_basis, y, n_lambda = 100, n_folds = 10, center = FALSE)

## Arguments

| x_basis | A dgCMatrix object corresponding to a sparse matrix of the basis functions <br> generated for the HAL algorithm. |
| :--- | :--- |
| y | A numeric vector of the observed outcome variable values. |
| n_lambda | A numeric scalar indicating the number of values of the L1 regularization pa- <br> rameter (lambda) to be obtained from fitting the Lasso to the full data. Cross- <br> validation is used to select an optimal lambda (that minimizes the risk) from <br> among these. |
| n_folds | A numeric scalar for the number of folds to be used in the cross-validation <br> procedure to select an optimal value of lambda. <br> binary. If TRUE, covariates are centered. This is much slower, but matches the <br> glmnet implementation. Default FALSE. |

```
cv_lasso_early_stopping
```

Cross-validated LASSO on Indicator Bases

## Description

Fits the LASSO regression using a customized procedure with cross-validation based on origami

## Usage

cv_lasso_early_stopping(x_basis, y, n_lambda = 100, n_folds = 10)

## Arguments

x_basis A dgCMatrix object corresponding to a sparse matrix of the basis functions generated for the HAL algorithm.
$y \quad$ A numeric vector of the observed outcome variable values.

$$
\begin{array}{ll}
\text { n_lambda } & \begin{array}{l}
\text { A numeric scalar indicating the number of values of the L1 regularization pa- } \\
\text { rameter (lambda) to be obtained from fitting the LASSO to the full data. Cross- } \\
\text { validation is used to select an optimal lambda (that minimizes the risk) from } \\
\text { among these. }
\end{array} \\
\text { n_folds } & \begin{array}{l}
\text { A numeric scalar for the number of folds to be used in the cross-validation } \\
\text { procedure to select an optimal value of lambda. }
\end{array}
\end{array}
$$

```
enumerate_basis Enumerate Basis Functions
```


## Description

Generate basis functions for all covariates and interaction terms thereof up to a specified order/degree.

## Usage

enumerate_basis(
x ,
max_degree $=$ NULL,
smoothness_orders $=$ rep (0, ncol(x)),
include_zero_order = FALSE,
include_lower_order = FALSE,
num_knots = NULL
)

## Arguments

x
An input matrix containing observations and covariates following standard conventions in problems of statistical learning.
max_degree The highest order of interaction terms for which the basis functions ought to be generated. The default (NULL) corresponds to generating basis functions for the full dimensionality of the input matrix.
smoothness_orders
An integer vector of length $n \operatorname{col}(x)$ specifying the desired smoothness of the function in each covariate. $\mathrm{k}=0$ is no smoothness (indicator basis), $\mathrm{k}=1$ is first order smoothness, and so on. For an additive model, the component function for each covariate will have the degree of smoothness as specified by smoothness_orders. For non-additive components (tensor products of univariate basis functions), the univariate basis functions in each tensor product have smoothness degree as specified by smoothness_orders.
include_zero_order
A logical, indicating whether the zeroth order basis functions are included for each covariate (if TRUE), in addition to the smooth basis functions given by smoothness_orders. This allows the algorithm to data-adaptively choose the appropriate degree of smoothness.

A logical, like include_zero_order, except including all basis functions of lower smoothness degrees than specified via smoothness_orders.
num_knots A vector of length max_degree, which determines how granular the knot points to generate basis functions should be for each degree of basis function. The first entry of num_knots determines the number of knot points to be used for each univariate basis function. More generally, The kth entry of num_knots determines the number of knot points to be used for the kth degree basis functions. Specifically, for a kth degree basis function, which is the tensor product of k univariate basis functions, this determines the number of knot points to be used for each univariate basis function in the tensor product.

## Value

A list of basis functions generated for all covariates and interaction thereof up to a pre-specified degree.

## Examples

```
gendata <- function(n) {
    W1 <- runif(n, -3, 3)
    W2 <- rnorm(n)
    W3 <- runif(n)
    W4 <- rnorm(n)
    g0 <- plogis(0.5 * (-0.8 * W1 + 0.39 * W2 + 0.08 * W3 - 0.12 * W4))
    A <- rbinom(n, 1, g0)
    Q0 <- plogis(0.15 * (2 * A + 2 * A * W1 + 6 * A * W3 * W4 - 3))
    Y <- rbinom(n, 1, Q0)
    data.frame(A, W1, W2, W3, W4, Y)
}
set.seed(1234)
data <- gendata(100)
covars <- setdiff(names(data), "Y")
X <- as.matrix(data[, covars, drop = FALSE])
basis_list <- enumerate_basis(X)
```

evaluate_basis Generate Basis Functions

## Description

Populates a column (indexed by basis_col) of $x$ _basis with basis indicators.

## Usage

evaluate_basis(basis, X, x_basis, basis_col)

## Arguments

basis The basis function.
X
The design matrix, containing the original data.
x_basis The HAL design matrix, containing indicator functions.
basis_col Numeric indicating which column to populate.

```
fit_hal HAL: The Highly Adaptive Lasso
```


## Description

Estimation procedure for HAL, the Highly Adaptive Lasso

## Usage

```
fit_hal(
        X,
        Y,
        formula = NULL,
        X_unpenalized = NULL,
        max_degree = ifelse(ncol(X) >= 20, 2, 3),
        smoothness_orders = 1,
        num_knots = num_knots_generator(max_degree = max_degree, smoothness_orders =
            smoothness_orders, base_num_knots_0 = 200, base_num_knots_1 = 50),
    reduce_basis = 1/sqrt(length(Y)),
    family = c("gaussian", "binomial", "poisson", "cox"),
    lambda = NULL,
    id = NULL,
    offset = NULL,
    fit_control = list(cv_select = TRUE, n_folds = 10, foldid = NULL, use_min = TRUE,
        lambda.min.ratio = 1e-04, prediction_bounds = "default"),
    basis_list = NULL,
    return_lasso = TRUE,
    return_x_basis = FALSE,
    yolo = FALSE
    )
```


## Arguments

X

Y A numeric vector of observations of the outcome variable.
formula A character string formula to be used in formula_hal. See its documentation for details.

| X_unpenalized | An input matrix with the same number of rows as $X$, for which no $L 1$ penalization will be performed. Note that $X_{\text {_unpenal ized is directly appended to the }}$ design matrix; no basis expansion is performed on X_unpenalized. |
| :---: | :---: |
| max_degree | The highest order of interaction terms for which basis functions ought to be generated. |
| smoothness_orders |  |
|  | An integer, specifying the smoothness of the basis functions. See details for smoothness_orders for more information. |
| num_knots | An integer vector of length 1 or max_degree, specifying the maximum number of knot points (i.e., bins) for any covariate for generating basis functions. If num_knots is a unit-length vector, then the same num_knots are used for each degree (this is not recommended). The default settings for num_knots are recommended, and these defaults decrease num_knots with increasing max_degree and smoothness_orders, which prevents (expensive) combinatorial explosions in the number of higher-degree and higher-order basis functions generated. This allows the complexity of the optimization problem to grow scalably. See details of num_knots more information. |
| reduce_basis | A numeric value bounded in the open unit interval indicating the minimum proportion of 1's in a basis function column needed for the basis function to be included in the procedure to fit the lasso. Any basis functions with a lower proportion of 1's than the cutoff will be removed. When reduce_basis is set to NULL, all basis functions are used in the lasso-fitting stage of fit_hal. |
| family | A character or a family object (supported by glmnet) specifying the error/link family for a generalized linear model. character options are limited to "gaussian" for fitting a standard penalized linear model, "binomial" for penalized logistic regression, "poisson" for penalized Poisson regression, and "cox" for a penalized proportional hazards model. Note that passing in family objects leads to slower performance relative to passing in a character family (if supported). For example, one should set family = "binomial" instead of family = binomial () when calling fit_hal. |
| lambda | User-specified sequence of values of the regularization parameter for the lasso L1 regression. If NULL, the default sequence in cv.glmnet will be used. The cross-validated optimal value of this regularization parameter will be selected with cv.glmnet. If fit_control's cv_select argument is set to FALSE, then the lasso model will be fit via glmnet, and regularized coefficient values for each lambda in the input array will be returned. |
| id | A vector of ID values that is used to generate cross-validation folds for cv .glmnet. This argument is ignored when fit_control's cv_select argument is FALSE. |
| offset | a vector of offset values, used in fitting. |
| fit_control | List of arguments for fitting. Includes the following arguments, and any others to be passed to cv .glmnet or glmnet. <br> - cv_select: A logical specifying if the sequence of specified lambda values should be passed to cv.glmnet in order for a single, optimal value of lambda to be selected according to cross-validation. When cv_select = FALSE, a glmnet model will be used to fit the sequence of (or single) lambda. |

- n _folds: Integer for the number of folds to be used when splitting the data for V-fold cross-validation. Only used when cv_select = TRUE.
- foldid: An optional numeric containing values between 1 and $n \_f o l d s$, identifying the fold to which each observation is assigned. If supplied, $\mathrm{n}_{\mathrm{f}}$ folds can be missing. In such a case, this vector is passed directly to $\mathrm{cv} . g l m n e t$. Only used when cv_select = TRUE.
- use_min: Specify the choice of lambda to be selected by cv.glmnet. When TRUE, "lambda.min" is used; otherwise, "lambda. 1 se ". Only used when cv_select = TRUE.
- lambda.min. ratio: A glmnet argument specifying the smallest value for lambda, as a fraction of lambda.max, the (data derived) entry value (i.e. the smallest value for which all coefficients are zero). We've seen that not setting lambda.min. ratio can lead to no lambda values that fit the data sufficiently well.
- prediction_bounds: A vector of size two that provides the lower and upper bounds for predictions. When prediction_bounds = "default", the predictions are bounded between $\min (Y)-s d(Y)$ and $\max (Y)+s d(Y)$. Bounding ensures that there is no extrapolation, and it is necessary for cross-validation selection and/or Super Learning.
basis_list The full set of basis functions generated from $X$.
return_lasso A logical indicating whether or not to return the glmnet fit object of the lasso model.
return_x_basis A logical indicating whether or not to return the matrix of (possibly reduced) basis functions used in fit_hal.
yolo A logical indicating whether to print one of a curated selection of quotes from the HAL9000 computer, from the critically acclaimed epic science-fiction film "2001: A Space Odyssey" (1968).


## Details

The procedure uses a custom C++ implementation to generate a design matrix of spline basis functions of covariates and interactions of covariates. The lasso regression is fit to this design matrix via cv.glmnet or a custom implementation derived from origami. The maximum dimension of the design matrix is $n$-by- $\left.\left(n * 2^{( } d-1\right)\right)$, where where $n$ is the number of observations and $d$ is the number of covariates.

For smoothness_orders = 0, only zero-order splines (piece-wise constant) are generated, which assume the true regression function has no smoothness or continuity. When smoothness_orders $=1$, first-order splines (piece-wise linear) are generated, which assume continuity of the true regression function. When smoothness_orders $=2$, second-order splines (piece-wise quadratic and linear terms) are generated, which assume a the true regression function has a single order of differentiability.
num_knots argument specifies the number of knot points for each covariate and for each max_degree. Fewer knot points can significantly decrease runtime, but might be overly simplistic. When considering smoothness_orders $=0$, too few knot points (e.g., $<50$ ) can significantly reduce performance. When smoothness_orders = 1 or higher, then fewer knot points (e.g., 10-30) is actually better for performance. We recommend specifying num_knots with respect to smoothness_orders,
and as a vector of length max_degree with values decreasing exponentially. This prevents combinatorial explosions in the number of higher-degree basis functions generated. The default behavior of num_knots follows this logic - for smoothness_orders = 0, num_knots is set to 500/2 $2^{j-1}$, and for smoothness_orders = 1 or higher, num_knots is set to $200 / 2^{j-1}$, where $j$ is the interaction degree. We also include some other suitable settings for num_knots below, all of which are less complex than default num_knots and will thus result in a faster runtime:

- Some good settings for little to no cost in performance:
- If smoothness_orders $=0$ and max_degree $=3$, num_knots $=c(400,200,100)$.
- If smoothness_orders = $1+$ and max_degree $=3$, num_knots $=c(100,75,50)$.
- Recommended settings for fairly fast runtime:
- If smoothness_orders = 0 and max_degree $=3$, num_knots $=c(200,100,50)$.
- If smoothness_orders = $1+$ and max_degree $=3$, num_knots $=c(50,25,15)$.
- Recommended settings for fast runtime:
- If smoothness_orders $=0$ and max_degree $=3$, num_knots $=c(100,50,25)$.
- If smoothness_orders $=1+$ and max_degree $=3$, num_knots $=c(40,15,10)$.
- Recommended settings for very fast runtime:
- If smoothness_orders $=0$ and max_degree $=3$, num_knots $=c(50,25,10)$.
- If smoothness_orders = $1+$ and max_degree $=3$, num_knots $=c(25,10,5)$.


## Value

Object of class hal9001, containing a list of basis functions, a copy map, coefficients estimated for basis functions, and timing results (for assessing computational efficiency).

## Examples

```
n <- 100
p<- 3
x <- xmat <- matrix (rnorm(n * p), n, p)
y_prob <- plogis(3 * \(\sin (x[, 1])+\sin (x[, 2]))\)
y <- rbinom(n = n, size = 1, prob = y_prob)
hal_fit <- fit_hal(X = x, \(Y=y\), family = "binomial")
preds <- predict(hal_fit, new_data \(=x\) )
```

formula_hal

## Description

HAL Formula: Convert formula or string to formula_HAL object.

## Usage

formula_hal(formula, smoothness_orders, num_knots, X = NULL)

## Arguments

formula A formula_hal9001 object as outputted by h .
smoothness_orders
A default value for $s$ if not provided explicitly to the function $h$.
num_knots A default value for $k$ if not provided explicitly to the function $h$.
X
Controls inheritance of the variable $X$ from parent environment. When NULL (the default), such a variable is inherited.

```
formula_helpers Formula Helpers
```


## Description

Formula Helpers

## Usage

fill_dots_helper(var_names, .)
fill_dots(var_names, .)

## Arguments

var_names A character vector of variable names.
Specification of variables for use in the formula.
h

## Description

HAL Formula term: Generate a single term of the HAL basis

```
Usage
    h(
    ...,
    k = NULL,
    s = NULL,
    pf = 1,
    monotone = c("none", "i", "d"),
    . = NULL,
    dot_args_as_string = FALSE,
    X = NULL
)
```


## Arguments

k

S
pf

Variables for which to generate multivariate interaction basis function where the variables can be found in a matrix $X$ in a parent environment/frame. Note, just like standard formula objects, the variables should not be characters (e.g. do $\mathrm{h}(\mathrm{W} 1, \mathrm{~W} 2)$ not $\mathrm{h}($ "W1", "W2")) h(W1,W2,W3) will generate three-way HAL basis functions between W1, W2, and W3. It will not generate the lower dimensional basis functions.
monotone
The number of knots for each univariate basis function used to generate the tensor product basis functions. If a single value then this value is used for the univariate basis functions for each variable. Otherwise, this should be a variable named list that specifies for each variable how many knots points should be used. $h(W 1, W 2, W 3, k=\operatorname{list}(W 1=3, W 2=2, W 3=1))$ is equivalent to first binning the variables W1, W2 and W3 into 3,2 and 1 unique values and then calling $h(W 1, W 2, W 3)$. This coarsening of the data ensures that fewer basis functions are generated, which can lead to substantial computational speed-ups. If not provided and the variable num_knots is in the parent environment, then $s$ will be set to num_knots'.
The smoothness_orders for the basis functions. The possible values are 0 for piece-wise constant zero-order splines or 1 for piece-wise linear first-order splines. If not provided and the variable smoothness_orders is in the parent environment, then s will be set to smoothness_orders.
A penalty.factor value the generated basis functions that is used by glmnet in the LASSO penalization procedure. $\mathrm{pf}=1$ (default) is the standard penalization factor used by glmnet and $\mathrm{pf}=0$ means the generated basis functions are unpenalized.
Whether the basis functions should enforce monotonicity of the interaction term. If $\backslash \operatorname{code}\{\mathrm{s}\}=0$, this is monotonicity of the function, and, if $\backslash \operatorname{code}\{\mathrm{s}\}=1$, this is monotonicity of its derivative (e.g., enforcing a convex fit). Set "none" for no constraints, " i " for a monotone increasing constraint, and " d " for a monotone decreasing constraint. Using " $i$ " constrains the basis functions to have positive coefficients in the fit, and "d" constrains the basis functions to have negative coefficients.

Just like with formula, . as in $h($.$) or h(.,$.$) is treated as a wildcard variable$ that generates terms using all variables in the data. The argument . should be a character vector of variable names that . iterates over. Specifically, h(., k=1, . = $c(" W 1 ", " W 2 ", " W 3 "))$ is equivalent to $h(W 1, k=1)+h(W 2, k=1)+h(W 3, k=1)$, and $h(., ., k=1, .=c(" W 1 ", " W 2 ", " W 3 "))$ is equivalent to $h(W 1, W 2, k=1)+h(W 2, W 3, k=1)$ $+h(W 1, W 3, k=1)$
dot_args_as_string
Whether the arguments . . . are characters or character vectors and should thus be evaluated directly. When TRUE, the expression h("W1", "W2") can be used.
$x$
An optional design matrix where the variables given in . . . can be found. Otherwise, X is taken from the parent environment.
hal9000 HAL 9000 Quotes

## Description

Prints a quote from the HAL 9000 robot from 2001: A Space Odyssey
$\qquad$

## Description

Package for fitting the Highly Adaptive LASSO (HAL) estimator

```
hal_quotes
HAL9000 Quotes from "2001: A Space Odyssey"
```


## Description

Curated selection of quotes from the HAL9000 computer, from the critically acclaimed epic sciencefiction film "2001: A Space Odyssey" (1968).

## Usage

hal_quotes

## Format

A vector of quotes.

```
index_first_copy Find Copies of Columns
```


## Description

Index vector that, for each column in $X$, indicates the index of the first copy of that column

## Usage

index_first_copy(X)

## Arguments

$X \quad$ Sparse matrix containing columns of indicator functions.
lassi_fit_module Rcpp module: lassi_fit_module

## Description

Rcpp module: lassi_fit_module
lassi_origami Single Lasso estimation for cross-validation with Origami

## Description

Fits Lasso regression over a single fold of a cross-validated data set. This is meant to be called using cross_validate, which is done through cv_lasso. Note that this procedure is NOT meant to be invoked by itself. INTERNAL USE ONLY.

## Usage

lassi_origami(fold, data, lambdas, center = FALSE)

## Arguments

| fold | A fold object produced by a call to make_folds from the origami. |
| :--- | :--- |
| data | A dgCMatrix object containing the outcome values (Y) in its first column and <br> vectors corresponding to the basis functions of HAL in all other columns. Con- <br> sult the description of HAL regression for details. |
| lambdas | A numeric vector corresponding to a sequence of lambda values obtained by <br> fitting the Lasso on the full data. <br> binary. If TRUE, covariates are centered. This is much slower, but matches the <br> glmnet implementation. Default FALSE. |

## lassi_predict Prediction from a Lassi Model

## Description

Prediction from a Lassi Model

## Usage

lassi_predict(X, beta, intercept)

## Arguments

| $X$ | A sparse matrix of HAL basis functions. |
| :--- | :--- |
| beta | A vector of coefficient values for the HAL basis functions. |
| intercept | A numeric value giving the intercept of the HAL model. |

```
make_basis_list Sort Basis Functions
```


## Description

Build a sorted list of unique basis functions based on columns, where each basis function is a list

## Usage

make_basis_list(X_sub, cols, order_map)

## Arguments

X_sub A subset of the columns of $X$, the original design matrix.
cols An index of the columns that were reduced to by sub-setting.
order_map A vector with length the original unsubsetted matrix $X$ which specifies the smoothness of the function in each covariate.

## Details

Note that sorting of columns is performed such that the basis order equals cols.length() and each basis function is a list(cols, cutoffs).
make_copy_map

```
make_copy_map Build Copy Maps
```


## Description

Build Copy Maps

## Usage

make_copy_map(x_basis)

## Arguments

$$
\begin{array}{ll}
\text { x_basis } & \begin{array}{l}
\text { A design matrix consisting of basis (indicator) functions for covariates }(\mathrm{X}) \text { and } \\
\text { terms for interactions thereof. }
\end{array}
\end{array}
$$

## Value

A list of numeric vectors indicating indices of basis functions that are identical in the training set.

## Examples

```
gendata <- function(n) {
    W1 <- runif(n, -3, 3)
    W2 <- rnorm(n)
    W3 <- runif(n)
    W4 <- rnorm(n)
    g0 <- plogis(0.5 * (-0.8 * W1 + 0.39 * W2 + 0.08 * W3 - 0.12 * W4))
    A <- rbinom(n, 1, g0)
    Q0 <- plogis(0.15 * (2 * A + 2 * A * W1 + 6 * A * W3 * W4 - 3))
    Y <- rbinom(n, 1, Q0)
    data.frame(A, W1, W2, W3, W4, Y)
}
set.seed(1234)
data <- gendata(100)
covars <- setdiff(names(data), "Y")
X <- as.matrix(data[, covars, drop = FALSE])
basis_list <- enumerate_basis(X)
x_basis <- make_design_matrix(X, basis_list)
copy_map <- make_copy_map(x_basis)
```


## Description

Make a HAL design matrix based on original design matrix X and a list of basis functions in argument blist

## Usage

make_design_matrix(X, blist, p_reserve = 0.5)

## Arguments

X Matrix of covariates containing observed data in the columns.
blist List of basis functions with which to build HAL design matrix.
p_reserve Sparse matrix pre-allocation proportion. Default value is 0.5 . If one expects a dense HAL design matrix, it is useful to set p_reserve to a higher value.

## Value

A dgCMatrix sparse matrix of indicator basis functions corresponding to the design matrix in a zero-order highly adaptive lasso.

## Examples

```
gendata <- function(n) {
    W1 <- runif(n, -3, 3)
    W2 <- rnorm(n)
    W3 <- runif(n)
    W4 <- rnorm(n)
    g0 <- plogis(0.5 * (-0.8 * W1 + 0.39 * W2 + 0.08 * W3 - 0.12 * W4))
    A <- rbinom(n, 1, g0)
    Q0 <- plogis(0.15 * (2 * A + 2 * A * W1 + 6 * A * W3 * W4 - 3))
    Y <- rbinom(n, 1, Q0)
    data.frame(A, W1, W2, W3, W4, Y)
}
set.seed(1234)
data <- gendata(100)
covars <- setdiff(names(data), "Y")
X <- as.matrix(data[, covars, drop = FALSE])
basis_list <- enumerate_basis(X)
x_basis <- make_design_matrix(X, basis_list)
```

```
make_reduced_basis_map
```

Mass-based reduction of basis functions

## Description

A helper function that finds which basis functions to keep (and equivalently which to discard) based on the proportion of 1's (observations, i.e., "mass") included in a given basis function.

## Usage

make_reduced_basis_map(x_basis, reduce_basis_crit)

## Arguments

$$
\begin{aligned}
& \text { x_basis A matrix of basis functions with all redundant basis functions already removed. } \\
& \text { reduce_basis_crit } \\
& \text { A scalar numeric value bounded in the open interval (0,1) indicating the mini- } \\
& \text { mum proportion of 1's in a basis function column needed for the basis function } \\
& \text { to be included in the procedure to fit the Lasso. Any basis functions with a lower } \\
& \text { proportion of 1's than the specified cutoff will be removed. This argument de- } \\
& \text { faults to NULL, in which case all basis functions are used in the lasso-fitting stage } \\
& \text { of the HAL algorithm. }
\end{aligned}
$$

## Value

A binary numeric vector indicating which columns of the matrix of basis functions to keep (given a one) and which to discard (given a zero).

```
meets_basis Compute Values of Basis Functions
```


## Description

Computes and returns the indicator value for the basis described by cols and cutoffs for a given row of X

## Usage

meets_basis(X, row_num, cols, cutoffs, orders)

## Arguments

X
row_num Numeri for a row index over which to evaluate.
cols Numeric for the column indices of the basis function.
cutoffs Numeric providing thresholds.
orders Numeric providing smoothness orders

```
predict.hal9001 Prediction from HAL fits
```


## Description

Prediction from HAL fits

```
Usage
    ## S3 method for class 'hal9001'
    predict(
        object,
        new_data,
        new_X_unpenalized = NULL,
        offset = NULL,
        type = c("response", "link"),
        p_reserve = 0.75,
    )
```


## Arguments

object An object of class hal9001, containing the results of fitting the Highly Adaptive Lasso, as produced by fit_hal.
new_data A matrix or data.frame containing new data (i.e., observations not used for fitting the hal 9001 object that's passed in via the object argument) for which the hal9001 object will compute predicted values.
new_X_unpenalized
If the user supplied X_unpenalized during training, then user should also supply this matrix with the same number of observations as new_data.
offset A vector of offsets. Must be provided if provided at training.
type Either "response" for predictions of the response, or "link" for un-transformed predictions (on the scale of the link function).
p_reserve Sparse matrix pre-allocation proportion, which is the anticipated proportion of 1 's in the design matrix. Default value is recommended in most settings. If a dense design matrix is expected, it would be useful to set p_reserve to a higher value.
... Additional arguments passed to predict as necessary.

## Details

Method for computing and extracting predictions from fits of the Highly Adaptive Lasso estimator, returned as a single S3 objects of class hal9001.

## Value

A numeric vector of predictions from a hal9001 object.

Note
This prediction method does not function similarly to the equivalent method from glmnet. In particular, this procedure will not return a subset of lambdas originally specified in calling fit_hal nor result in re-fitting. Instead, it will return predictions for all of the lambdas specified in the call to fit_hal that constructs object, when fit_control's cv_select is set to FALSE. When fit_control's cv_select is set to TRUE, predictions will only be returned for the value of lambda selected by cross-validation.

```
predict.SL.hal9001 predict.SL.hal9001
```


## Description

Predict method for objects of class SL. hal9001

## Usage

\#\# S3 method for class 'SL.hal9001'
predict(object, newdata, ...)

## Arguments

| object | A fitted object of class hal9001. |
| :--- | :--- |
| newdata | A matrix of new observations on which to obtain predictions. |
| $\ldots$ | Not used. |

## Value

A numeric vector of predictions from a SL. hal9001 object based on the provide newdata.

```
print.formula_hal9001 Print formula_hal9001 object
```


## Description

Print formula_hal9001 object

## Usage

```
## S3 method for class 'formula_hal9001'
print(x, ...)
```


## Arguments

x A formula_hal9001 object.
$\ldots \quad$ Other arguments (ignored).
print. summary.hal9001 Print Method for Summary Class of HAL fits

## Description

Print Method for Summary Class of HAL fits

## Usage

\#\# S3 method for class 'summary.hal9001'
print(x, length = NULL, ...)

## Arguments

x
length The number of ranked coefficients to be summarized.
$\ldots \quad$ Other arguments (ignored).

## Description

Wrapper for SuperLearner for objects of class hal9001

## Usage

```
SL.hal9001(
    Y,
    X,
    newX = NULL,
    family = stats::gaussian(),
    obsWeights = rep(1, length(Y)),
    id = NULL,
    max_degree = ifelse(ncol(X) >= 20, 2, 3),
    smoothness_orders = 1,
    num_knots = ifelse(smoothness_orders >= 1, 25, 50),
    reduce_basis = 1/sqrt(length(Y)),
    lambda = NULL,
    ...
    )
```


## Arguments

| Y | A numeric vector of observations of the outcome variable. |
| :---: | :---: |
| X | An input matrix with dimensions number of observations -by- number of covariates that will be used to derive the design matrix of basis functions. |
| newX | A matrix of new observations on which to obtain predictions. The default of NULL computes predictions on training inputs $X$. |
| family | A family object (one that is supported by glmnet) specifying the error/link family for a generalized linear model. |
| obsWeights | A numeric vector of observational-level weights. |
| id | A numeric vector of IDs. |
| max_degree | The highest order of interaction terms for which basis functions ought to be generated. |
| smoothness_orders |  |
|  | An integer vector of length 1 or greater, specifying the smoothness of the basis functions. See the argument smoothness_orders of fit_hal for more information. |
| num_knots | An integer vector of length 1 or max_degree, specifying the maximum number of knot points (i.e., bins) for each covariate for generating basis functions. See num_knots argument in fit_hal for more information. |

reduce_basis A numeric value bounded in the open unit interval indicating the minimum proportion of 1 's in a basis function column needed for the basis function to be included in the procedure to fit the lasso. Any basis functions with a lower proportion of 1 's than the cutoff will be removed.
lambda A user-specified sequence of values of the regularization parameter for the lasso L1 regression. If NULL, the default sequence in cv.glmnet will be used. The cross-validated optimal value of this regularization parameter will be selected with cv.glmnet.
... Not used.

## Value

An object of class SL.hal9001 with a fitted hal9001 object and corresponding predictions based on the input data.

```
squash_hal_fit Squash HAL objects
```


## Description

Reduce footprint by dropping basis functions with coefficients of zero

## Usage

squash_hal_fit(object)

## Arguments

object An object of class hal9001, containing the results of fitting the Highly Adaptive LASSO, as produced by a call to fit_hal.

## Value

Object of class hal9001, similar to the input object but reduced such that coefficients belonging to bases with coefficients equal to zero removed.

## Examples

```
# generate simple test data
n <- 100
p<- 3
x <- matrix(rnorm(n * p), n, p)
y<- sin(x[, 1]) * sin(x[, 2]) + rnorm(n, mean = 0, sd = 0.2)
# fit HAL model and squash resulting object to reduce footprint
hal_fit <- fit_hal(X = x, Y = y, yolo = FALSE)
squashed <- squash_hal_fit(hal_fit)
```


## Description

Summary Method for HAL fit objects

## Usage

```
## S3 method for class 'hal9001'
    summary(
        object,
        lambda = NULL,
        only_nonzero_coefs = TRUE,
        include_redundant_terms = FALSE,
        round_cutoffs = 3,
    )
```


## Arguments

object An object of class hal9001, containing the results of fitting the Highly Adaptive Lasso, as produced by fit_hal.
lambda Optional numeric value of the lambda tuning parameter, for which corresponding coefficient values will be summarized. Defaults to fit_hal's optimal value, lambda_star, or the minimum value of lambda_star.
only_nonzero_coefs
A logical specifying whether the summary should include only terms with non-zero coefficients.
include_redundant_terms
A logical specifying whether the summary should remove so-called "redundant terms". We define a redundant term (say x1) as a term (1) with basis function corresponding to an existing basis function, a duplicate; and (2) the duplicate contains the x 1 term as part of its term, so that x 1 terms inclusion would be redundant. For example, say the same coefficient corresponds to these three terms: (1) "I(age >=50)*I(bmi >=18)", (2) "I(age >=50)", and (3) "I(education $>=16$ )". When include_redundant_terms is FALSE (default), the second basis function is omitted.
round_cutoffs An integer indicating the number of decimal places to be used for rounding cutoff values in the term. For example, if "bmi" was numeric that was rounded to the third decimal, in the example above we would have needed to specify round_cutoffs $=0$ in order to yield a term like "I(bmi $>=18)$ " opposed to something like " $\mathrm{I}(\mathrm{bmi}>=18.111)$ ". This rounding is intended to simplify the term-wise part of the output and only rounds the basis cutoffs, the hal9001 model's coefficients are not rounded.
... Additional arguments passed to summary, not supported.

## Details

Method for summarizing the coefficients of the Highly Adaptive Lasso estimator in terms of the basis functions corresponding to covariates and interactions of covariates, returned as a single S3 object of class hal9001.
Due to the nature of the basis function terms, the summary tables can be extremely wide. The R environment might not be the optimal location to view the summary. Tables can be exported from R to LaTeX with xtable package (or similar). Here's an example: print (xtable (summary (fit)\$table, type = "latex"),file = "dt.tex").

## Value

A list summarizing a hal9001 object's coefficients.

## Index

```
* datasets
    hal_quotes,16
+.formula_hal9001,3
apply_copy_map, 3
as_dgCMatrix,4
basis_list_cols, 5
basis_of_degree,6
cross_validate, 17
cv.glmnet, 11, 12, 26
cv_lasso, 7, 17
cv_lasso_early_stopping, 7
enumerate_basis, }
evaluate_basis,9
family, 11, 25
fill_dots (formula_helpers), 14
fill_dots_helper (formula_helpers), 14
fit_hal, 10, 22, 23, 25, 27
formula_hal, 10,13
formula_helpers,14
glmnet,11,12,25
h, 14
hal9000, 16
hal9001, 16
hal_quotes, 16
index_first_copy,17
lassi_fit_module, 17
lassi_origami,17
lassi_predict,18
make_basis_list,18
make_copy_map, 19
make_design_matrix, 20
```

