Package ‘nFactors’

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Type Package

Title Parallel Analysis and Other Non Graphical Solutions to the Cattell Scree Test

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Description
Indices, heuristics and strategies to help determine the number of factors/components to retain:
1. Acceleration factor (af with or without Parallel Analysis);
2. Optimal Coordinates (noc with or without Parallel Analysis);
3. Parallel analysis (components, factors and bootstrap);
4. lambda > mean(lambda) (Kaiser, CFA and related);
5. Cattell-Nelson-Gorsuch (CNG);
6. Zoski and Jurs multiple regression (b, t and p);
7. Zoski and Jurs standard error of the regression coeffcient (sescree);
8. Nelson R2;
9. Bartlett khi-2;
10. Anderson khi-2;
11. Lawley khi-2 and
12. Bentler-Yuan khi-2.

License GPL (>= 3.5.0)

Encoding UTF-8

LazyData true

Depends R (>= 3.5.0), lattice

Imports stats, MASS, psych

RoxygenNote 6.1.1

Suggests testthat

NeedsCompilation no

Repository CRAN

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R topics documented:

`bentlerParameters` *Bentler and Yuan's Computation of the LRT Index and the Linear Trend Coefficients*

Description

This function computes the Bentler and Yuan's (1996, 1998) *LRT* index for the linear trend in eigenvalues of a covariance matrix. The related χ^2 and *p*-value are also computed. This function is generally called from the `nBentler` function. But it could be of use for graphing the linear trend function and to study it's behavior.

Usage

```
bentlerParameters(x, N, nFactors, log = TRUE, cor = TRUE,
  minPar = c(min(lambda) - abs(min(lambda)) + 0.001, 0.001),
  maxPar = c(max(lambda), lm(lambda ~ I(length(lambda):1))$coef[2]),
  resParx = c(0.01, 2), resPary = c(0.01, 2), graphic = TRUE,
  resolution = 30, typePlot = "wireframe", ...)
```

Arguments

`x`	numeric: a `vector` of eigenvalues, a `matrix` of correlations or of covariances or a `data.frame` of data
`N`	numeric: number of subjects.
`nFactors`	numeric: number of components to test.
`log`	logical: if `TRUE` the minimization is applied on the log values.
`cor`	logical: if `TRUE` computes eigenvalues from a correlation matrix, else from a covariance matrix
`minPar`	numeric: minimums for the coefficient of the linear trend.
`maxPar`	numeric: maximums for the coefficient of the linear trend.
`resParx`	numeric: restriction on the α coefficient (x) to graph the function to minimize.
`resPary`	numeric: restriction on the β coefficient (y) to graph the function to minimize.
`graphic`	logical: if `TRUE` plots the minimized function `"wireframe"`, `"contourplot"` or `"levelplot"`.
`resolution`	numeric: resolution of the 3D graph (number of points from α and from β).
`typePlot`	character: plots the minimized function according to a 3D plot: `"wireframe"`, `"contourplot"` or `"levelplot"`.
`...`	variable: additionnal parameters from the `"wireframe"`, `"contourplot"` or `"levelplot"` `lattice` functions. Also additionnal parameters for the `eigenFrom` function.

Details

The implemented Bentler and Yuan's procedure must be used with care because the minimized function is not always stable. In many cases, constraints must applied to obtain a solution. The actual implementation did, but the user can modify these constraints.

The hypothesis tested (Bentler and Yuan, 1996, equation 10) is:

(1) $$H_k : \lambda_{k+i} = \alpha + \beta x_i, (i = 1, \ldots, q)$$

The solution of the following simultaneous equations is needed to find $(\alpha, \beta) \in$

(2) $$f(x) = \sum_{i=1}^{q} \frac{[\lambda_{k+j} - N\alpha + \beta x_j]x_j}{(\alpha + \beta x_j)^2} = 0$$

and $g(x) = \sum_{i=1}^{q} \frac{\lambda_{k+j} - N\alpha + \beta x_j x_j}{(\alpha + \beta x_j)^2} = 0$

The solution to this system of equations was implemented by minimizing the following equation:

(3) $(\alpha, \beta) \in \inf [h(x)] = \inf \log [f(x)^2 + g(x)^2]$

The likelihood ratio test LRT proposed by Bentler and Yuan (1996, equation 7) follows a χ^2 probability distribution with $q - 2$ degrees of freedom and is equal to:

(4) $$LRT = N(k-p)\left\{\ln\left(\frac{n}{N}\right) + 1\right\} - N \sum_{j=k+1}^{p} \ln\left\{\frac{\lambda_j}{\alpha + \beta x_j}\right\} + n \sum_{j=k+1}^{p} \left\{\frac{\lambda_j}{\alpha + \beta x_j}\right\}$$

With p beeing the number of eigenvalues, k the number of eigenvalues to test, q the $p-k$ remaining eigenvalues, N the sample size, and $n = N - 1$. Note that there is an error in the Bentler and Yuan equation, the variables N and n beeing inverted in the preceeding equation 4.

A better strategy proposed by Bentler an Yuan (1998) is to use a minimized χ^2 solution. This strategy will be implemented in a future version of the **nFactors** package.

Value

`nFactors` numeric: vector of the number of factors retained by the Bentler and Yuan's procedure.

`details` numeric: matrix of the details of the computation.

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References

Bentler, P. M. and Yuan, K.-H. (1996). Test of linear trend in eigenvalues of a covariance matrix with application to data analysis. *British Journal of Mathematical and Statistical Psychology, 49*, 299-312.

Bentler, P. M. and Yuan, K.-H. (1998). Test of linear trend in the smallest eigenvalues of the correlation matrix. *Psychometrika, 63*(2), 131-144.

See Also

`nBartlett`, `nBentler`

Examples

```
## ................................................
## SIMPLE EXAMPLE OF THE BENTLER AND YUAN PROCEDURE

# Bentler (1996, p. 309) Table 2 - Example 2 ............
n=649
bentler2<-c(5.785, 3.088, 1.505, 0.582, 0.424, 0.386, 0.360, 0.337, 0.303,
            0.281, 0.246, 0.238, 0.200, 0.160, 0.130)

results  <- nBentler(x=bentler2, N=n,  details=TRUE)
results

# Two different figures to verify the convergence problem identified with
# the 2th component
bentlerParameters(x=bentler2, N=n, nFactors= 2, graphic=TRUE,
                  typePlot="contourplot",
                  resParx=c(0,9), resPary=c(0,9), cor=FALSE)

bentlerParameters(x=bentler2, N=n, nFactors= 4, graphic=TRUE, drape=TRUE,
                  resParx=c(0,9), resPary=c(0,9),
                  scales = list(arrows = FALSE) )

plotuScree(x=bentler2, model="components",
  main=paste(results$nFactors,
  " factors retained by the Bentler and Yuan's procedure (1996, p. 309)",
  sep=""))
# ........................................................

# Bentler (1998, p. 140) Table 3 - Example 1 ............
n        <- 145
example1 <- c(8.135, 2.096, 1.693, 1.502, 1.025, 0.943, 0.901, 0.816,
              0.790,0.707, 0.639, 0.543,0.533, 0.509, 0.478, 0.390,
              0.382, 0.340, 0.334, 0.316, 0.297,0.268, 0.190, 0.173)

results  <- nBentler(x=example1, N=n,  details=TRUE)
results

# Two different figures to verify the convergence problem identified with
# the 10th component
bentlerParameters(x=example1, N=n, nFactors= 10, graphic=TRUE,
                  typePlot="contourplot",
                  resParx=c(0,0.4), resPary=c(0,0.4))

bentlerParameters(x=example1, N=n, nFactors= 10, graphic=TRUE, drape=TRUE,
                  resParx=c(0,0.4), resPary=c(0,0.4),
                  scales = list(arrows = FALSE) )

plotuScree(x=example1, model="components",
   main=paste(results$nFactors,
   " factors retained by the Bentler and Yuan's procedure (1998, p. 140)",
   sep=""))
# ........................................................
```

componentAxis *Principal Component Analysis With Only n First Components Retained*

Description

The `componentAxis` function returns a principal component analysis with the first *n* components retained.

Usage

```
componentAxis(R, nFactors = 2)
```

Arguments

`R`	numeric: correlation or covariance matrix
`nFactors`	numeric: number of components/factors to retain

Value

`values`	numeric: variance of each component/factor retained
`varExplained`	numeric: variance explained by each component/factor retained
`varExplained`	numeric: cumulative variance explained by each component/factor retained
`loadings`	numeric: loadings of each variable on each component/factor retained

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References

Kim, J.-O. and Mueller, C. W. (1978). *Introduction to factor analysis. What it is and how to do it*. Beverly Hills, CA: Sage.

Kim, J.-O. and Mueller, C. W. (1987). *Factor analysis. Statistical methods and practical issues*. Beverly Hills, CA: Sage.

See Also

`principalComponents`, `iterativePrincipalAxis`, `rRecovery`

Examples

```
# .......................................................
# Example from Kim and Mueller (1978, p. 10)
# Simulated sample: lower diagonal
 R <- matrix(c( 1.000, 0.560, 0.480, 0.224, 0.192, 0.16,
                0.560, 1.000, 0.420, 0.196, 0.168, 0.14,
                0.480, 0.420, 1.000, 0.168, 0.144, 0.12,
                0.224, 0.196, 0.168, 1.000, 0.420, 0.35,
                0.192, 0.168, 0.144, 0.420, 1.000, 0.30,
                0.160, 0.140, 0.120, 0.350, 0.300, 1.00),
                nrow=6, byrow=TRUE)

# Factor analysis: Selected principal components - Kim and Mueller
# (1978, p. 20)
 componentAxis(R, nFactors=2)

# .......................................................
```

---

corFA — *Insert Communalities in the Diagonal of a Correlation or a Covariance Matrix*

Description

This function inserts communalities in the diagonal of a correlation/covariance matrix.

Usage

```
corFA(R, method = "ginv")
```

Arguments

R	An integer matrix or a data.frame of correlations
method	A character vector: inversion method

Value

A correlation matrix with coerced variables with communalities in the diagonal.

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See Also

plotuScree, nScree, plotnScree, plotParallel

Examples

```
## LOWER CORRELATION MATRIX WITH ZEROS ON UPPER PART
## From Gorsuch (table 1.3.1)
 gorsuch <- c(
 1,0,0,0,0,0,0,0,0,0,
 .6283, 1,0,0,0,0,0,0,0,0,
 .5631, .7353, 1,0,0,0,0,0,0,0,
 .8689, .7055, .8444, 1,0,0,0,0,0,0,
 .9030, .8626, .6890, .8874, 1,0,0,0,0,0,
 .6908, .9028, .9155, .8841, .8816, 1,0,0,0,0,
 .8633, .7495, .7378, .9164, .9109, .8572, 1,0,0,0,
 .7694, .7902, .7872, .8857, .8835, .8884, .7872, 1,0,0,
 .8945, .7929, .7656, .9494, .9546, .8942, .9434, .9000, 1,0,
 .5615, .6850, .8153, .7004, .6583, .7720, .6201, .6141, .6378, 1)

## UPPER CORRELATION MATRIX FILLED WITH UPPER CORRELATION MATRIX
 gorsuch <- makeCor(gorsuch)

## REPLACE DIAGONAL WITH COMMUNALITIES
 gorsuchCfa <- corFA(gorsuch)
 gorsuchCfa
```

---

dFactors *Eigenvalues from classical studies*

Description

Classical examples of eigenvalues vectors used to study the number of factors to retain in the litterature. These examples generally give the number of subjects use to obtain these eigenvalues. The number of subjects is used with the parallel analysis.

Usage

```
dFactors
```

Format

A list of examples. For each example, a list is also used to give the eigenvalues vector and the number of subjects.

Bentler $eigenvalues and $nsubjects

Buja $eigenvalues and $nsubjects

Cliff1 $eigenvalues and $nsubjects

Cliff2 $eigenvalues and $nsubjects

Cliff3 $eigenvalues and $nsubjects

Hand $eigenvalues and $nsubjects

Harman $eigenvalues and $nsubjects

Lawley $eigenvalues and $nsubjects

Raiche $eigenvalues and $nsubjects

Tucker1 $eigenvalues and $nsubjects

Tucker2 $eigenvalues and $nsubjects

Details

Other datasets will be added in future versions of the package.

Source

Lawley and Hand dataset: Bartholomew *et al*. (2002, p. 123, 126)

Bentler dataset: Bentler and Yuan (1998, p. 139-140)

Buja datasets: Buja and Eyuboglu (1992, p. 516, 519) < Number of subjects not specified by Buja and Eyuboglu >

Cliff datasets: Cliff (1970, p. 165)

Raiche dataset: Raiche, Langevin, Riopel and Mauffette (2006)

Raiche dataset: Raiche, Riopel and Blais (2006, p. 9)

Tucker datasets: Tucker *et al*. (1969, p. 442)

References

Bartholomew, D. J., Steele, F., Moustaki, I. and Galbraith, J. I. (2002). *The analysis and interpretation of multivariate data for social scientists*. Boca Raton, FL: Chapman and Hall.

Bentler, P. M. and Yuan, K.-H. (1998). Tests for linear trend in the smallest eigenvalues of the correlation matrix. *Psychometrika, 63*(2), 131-144.

Buja, A. and Eyuboglu, N. (1992). Remarks on parallel analysis. *Multivariate Behavioral Research, 27*(4), 509-540.

Cliff, N. (1970). The relation between sample and population characteristic vectors. *Psychometrika, 35*(2), 163-178.

Hand, D. J., Daly, F., Lunn, A. D., McConway, K. J. and Ostrowski, E. (1994). *A handbook of small data sets*. Boca Raton, FL: Chapman and Hall.

Lawley, D. N. and Maxwell, A. E. (1971). *Factor analysis as a statistical method* (2nd edition). London: Butterworth.

Raiche, G., Langevin, L., Riopel, M. and Mauffette, Y. (2006). Etude exploratoire de la dimensionnalite et des facteurs expliques par une traduction francaise de l'Inventaire des approches d'enseignement de Trigwell et Prosser dans trois universite quebecoises. *Mesure et Evaluation en Education, 29*(2), 41-61.

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

Tucker, L. D., Koopman, R. F. and Linn, R. L. (1969). Evaluation of factor analytic research procedures by mean of simulated correlation matrices. *Psychometrika, 34*(4), 421-459.

Zoski, K. and Jurs, S. (1993). Using multiple regression to determine the number of factors to retain in factor analysis. *Multiple Linear Regression Viewpoint, 20*(1), 5-9.

Examples

```
# EXAMPLES FROM DATASET
 data(dFactors)

# COMMAND TO VISUALIZE THE CONTENT AND ATTRIBUTES OF THE DATASETS
 names(dFactors)
 attributes(dFactors)
 dFactors$Cliff1$eigenvalues
 dFactors$Cliff1$nsubjects

# SCREE PLOT OF THE Cliff1 DATASET
 plotuScree(dFactors$Cliff1$eigenvalues)
```

`diagReplace` *Replacing Upper or Lower Diagonal of a Correlation or Covariance Matrix*

Description

The `diagReplace` function returns a modified correlation or covariance matrix by replacing upper diagonal with lower diagonal, or lower diagonal with upper diagonal.

Usage

```
diagReplace(R, upper = TRUE)
```

Arguments

`R`	numeric: correlation or covariance matrix
`upper`	logical: if `TRUE` upper diagonal is replaced with lower diagonal. If `FALSE`, lower diagonal is replaced with upper diagonal.

Value

`R`	numeric: correlation or covariance matrix

Author(s)

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Examples

```
# ......................................................
# Example from Kim and Mueller (1978, p. 10)
# Population: upper diagonal
# Simulated sample: lower diagnonal
 R <- matrix(c( 1.000, .6008, .4984, .1920, .1959, .3466,
               .5600, 1.000, .4749, .2196, .1912, .2979,
               .4800, .4200, 1.000, .2079, .2010, .2445,
               .2240, .1960, .1680, 1.000, .4334, .3197,
               .1920, .1680, .1440, .4200, 1.000, .4207,
               .1600, .1400, .1200, .3500, .3000, 1.000),
               nrow=6, byrow=TRUE)

# Replace upper diagonal with lower diagonal
 RU <- diagReplace(R, upper=TRUE)

# Replace lower diagonal with upper diagonal
 RL <- diagReplace(R, upper=FALSE)
# ......................................................
```

`eigenBootParallel` *Bootstrapping of the Eigenvalues From a Data Frame*

Description

The `eigenBootParallel` function samples observations from a `data.frame` to produce correlation or covariance matrices from which eigenvalues are computed. The function returns statistics about these bootstrapped eigenvalues. Their means or their quantile could be used later to replace the eigenvalues inputted to a parallel analysis. The `eigenBootParallel` can also compute random eigenvalues from empirical data by column permutation (Buja and Eyuboglu, 1992).

Usage

```
eigenBootParallel(x, quantile = 0.95, nboot = 30,
  option = "permutation", cor = TRUE, model = "components", ...)
```

Arguments

`x`	data.frame: data from which a correlation matrix will be obtained
`quantile`	numeric: eigenvalues quantile to be reported
`nboot`	numeric: number of bootstrap samples
`option`	character: `"permutation"` or `"bootstrap"`
`cor`	logical: if `TRUE` computes eigenvalues from a correlation matrix, else from a covariance matrix (`eigenComputes`)
`model`	character: bootstraps from a principal component analysis (`"components"`) or from a factor analysis (`"factors"`)
`...`	variable: additionnal parameters to give to the `cor` or `cov` functions

Value

values — data.frame: mean, median, quantile, standard deviation, minimum and maximum of bootstrapped eigenvalues

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References

Buja, A. and Eyuboglu, N. (1992). Remarks on parallel analysis. *Multivariate Behavioral Research, 27*(4), 509-540.

Zwick, W. R. and Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological bulletin, 99*, 432-442.

See Also

principalComponents, iterativePrincipalAxis, rRecovery

Examples

```
# .......................................................
# Example from the iris data
 eigenvalues <- eigenComputes(x=iris[,-5])

# Permutation parallel analysis distribution
 aparallel   <- eigenBootParallel(x=iris[,-5], quantile=0.95)$quantile

# Number of components to retain
 results     <- nScree(x = eigenvalues, aparallel = aparallel)
 results$Components
 plotnScree(results)
# ......................................................

# ......................................................
# Bootstrap distributions study of the eigenvalues from iris data
# with different correlation methods
 eigenBootParallel(x=iris[,-5],quantile=0.05,
                   option="bootstrap",method="pearson")
 eigenBootParallel(x=iris[,-5],quantile=0.05,
                   option="bootstrap",method="spearman")
 eigenBootParallel(x=iris[,-5],quantile=0.05,
                   option="bootstrap",method="kendall")
```

`eigenComputes` *Computes Eigenvalues According to the Data Type*

Description

The `eigenComputes` function computes eigenvalues from the identified data type. It is used internally in many fonctions of the **nFactors** package in order to apply these to a vector of eigenvalues, a matrix of correlations or covariance or a data frame.

Usage

```
eigenComputes(x, cor = TRUE, model = "components", ...)
```

Arguments

`x`	numeric: a `vector` of eigenvalues, a `matrix` of correlations or of covariances or a `data.frame` of data
`cor`	logical: if `TRUE` computes eigenvalues from a correlation matrix, else from a covariance matrix
`model`	character: `"components"` or `"factors"`
`...`	variable: additionnal parameters to give to the `cor` or `cov` functions

Value

numeric: return a vector of eigenvalues

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Examples

```
# .......................................................
# Different data types
# Vector of eigenvalues
data(dFactors)
x1 <- dFactors$Cliff1$eigenvalues
eigenComputes(x1)
```

```
# Data from a data.frame
x2 <- data.frame(matrix(20*rnorm(100), ncol=5))
eigenComputes(x2, cor=TRUE,  use="everything")
eigenComputes(x2, cor=FALSE, use="everything")
eigenComputes(x2, cor=TRUE,  use="everything", method="spearman")
eigenComputes(x2, cor=TRUE,  use="everything", method="kendall")

x3 <- cov(x2)
eigenComputes(x3, cor=TRUE,  use="everything")
eigenComputes(x3, cor=FALSE, use="everything")

x4 <- cor(x2)
eigenComputes(x4, use="everything")
# ......................................................
```

---

eigenFrom *Identify the Data Type to Obtain the Eigenvalues*

Description

The `eigenFrom` function identifies the data type from which to obtain the eigenvalues. The function is used internally in many functions of the **nFactors** package to be able to apply these to a vector of eigenvalues, a matrix of correlations or covariance or a `data.frame`.

Usage

```
eigenFrom(x)
```

Arguments

`x`	numeric: a `vector` of eigenvalues, a `matrix` of correlations or of covariances or a `data.frame` of data

Value

character: return the data type to obtain the eigenvalues: `"eigenvalues"`, `"correlation"` or `"data"`

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Examples

```
# ........................................................
# Different data types
# Examples of adequate data sources
# Vector of eigenvalues
data(dFactors)
x1 <- dFactors$Cliff1$eigenvalues
eigenFrom(x1)

# Data from a data.frame
x2 <- data.frame(matrix(20*rnorm(100), ncol=5))
eigenFrom(x2)

# From a covariance matrix
x3 <- cov(x2)
eigenFrom(x3)

# From a correlation matrix
x4 <- cor(x2)
eigenFrom(x4)

# Examples of inadequate data sources: not run because of errors generated
# x0 <- c(2,1)             # Error: not enough eigenvalues
# eigenFrom(x0)
# x2 <- matrix(x1, ncol=5) # Error: non a symetric covariance matrix
# eigenFrom(x2)
# eigenFrom(x3[,(1:2)])    # Error: not enough variables
# x6 <- table(x5)          # Error: not a valid data class
# eigenFrom(x6)
# ........................................................
```

---

generateStructure — *Generate a Factor Structure Matrix*

Description

The generateStructure function returns a *mjc* factor structure matrix. The number of variables per major factor *pmjc* is equal for each factor. The argument *pmjc* must be divisible by *nVar*. The arguments are strongly inspired from Zick and Velicer (1986, p. 435-436) methodology.

Usage

```
generateStructure(var, mjc, pmjc, loadings, unique)
```

Arguments

var	numeric: number of variables
mjc	numeric: number of major factors (factors with practical significance)
pmjc	numeric: number of variables that load significantly on each major factor

`loadings`	numeric: loadings on the significant variables on each major factor
`unique`	numeric: loadings on the non significant variables on each major factor

Value

values numeric matrix: factor structure

Author(s)

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References

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

Zwick, W. R. and Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin, 99*, 432-442.

See Also

`principalComponents`, `iterativePrincipalAxis`, `rRecovery`

Examples

```
# ......................................................
# Example inspired from Zwick and Velicer (1986, table 2, p. 437)
## ...................................................................
unique=0.2; loadings=0.5
zwick1 <- generateStructure(var=36, mjc=6, pmjc= 6, loadings=loadings,
                            unique=unique)
zwick2 <- generateStructure(var=36, mjc=3, pmjc=12, loadings=loadings,
                            unique=unique)
zwick3 <- generateStructure(var=72, mjc=9, pmjc= 8, loadings=loadings,
                            unique=unique)
zwick4 <- generateStructure(var=72, mjc=6, pmjc=12, loadings=loadings,
                            unique=unique)
sat=0.8
## ...................................................................
zwick5 <- generateStructure(var=36, mjc=6, pmjc= 6, loadings=loadings,
                            unique=unique)
zwick6 <- generateStructure(var=36, mjc=3, pmjc=12, loadings=loadings,
                            unique=unique)
zwick7 <- generateStructure(var=72, mjc=9, pmjc= 8, loadings=loadings,
```

```
                           unique=unique)
zwick8 <- generateStructure(var=72, mjc=6, pmjc=12, loadings=loadings,
                          unique=unique)
## ..................................................................

# nsubjects <- c(72, 144, 180, 360)
# require(psych)
# Produce an usual correlation matrix from a congeneric model
nsubjects <- 72
mzwick5   <- psych::sim.structure(fx=as.matrix(zwick5), n=nsubjects)
mzwick5$r

# Factor analysis: recovery of the factor structure
iterativePrincipalAxis(mzwick5$model, nFactors=6,
                      communalities="ginv")$loadings
iterativePrincipalAxis(mzwick5$r    , nFactors=6,
                      communalities="ginv")$loadings
factanal(covmat=mzwick5$model,         factors=6)
factanal(covmat=mzwick5$r    ,         factors=6)

# Number of components to retain
eigenvalues  <- eigen(mzwick5$r)$values
aparallel    <- parallel(var      = length(eigenvalues),
                        subject  = nsubjects,
                        rep      = 30,
                        quantile = 0.95,
                        model="components")$eigen$qevpea
results <- nScree(x         = eigenvalues,
                 aparallel = aparallel)
results$Components
plotnScree(results)

# Number of factors to retain
eigenvalues.fa  <- eigen(corFA(mzwick5$r))$values
aparallel.fa    <- parallel(var      = length(eigenvalues.fa),
                           subject  = nsubjects,
                           rep      = 30,
                           quantile = 0.95,
                           model="factors")$eigen$qevpea
results.fa <- nScree(x      = eigenvalues.fa,
                    aparallel = aparallel.fa,
                    model     ="factors")
results.fa$Components
plotnScree(results.fa)
# .......................................................
```

`is.nFactors` *Utility Functions for nFactors Class Objects*

Description

Utility functions for `nFactors` class objects.

Usage

```
is.nFactors(x)

## S3 method for class 'nFactors'
print(x, ...)

## S3 method for class 'nFactors'
summary(object, ...)
```

Arguments

`x`	nFactors: an object of the class nFactors
`...`	variable: additionnal parameters to give to the `print` function with `print.nFactors` or to the `summary` function with `summary.nFactors`
`object`	nFactors: an object of the class nFactors

Value

Generic functions for the nFactors class:

`is.nFactors`	logical: is the object of the class nFactors?
`print.nFactors`	numeric: vector of the number of components/factors to retain: same as the `nFactors` vector from the `nFactors` object
`summary.nFactors`	data.frame: details of the results from a nFactors object: same as the `details` data.frame from the `nFactors` object, but with easier control of the number of decimals with the `digits` parameter

Author(s)

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References

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

See Also

`nBentler`, `nBartlett`, `nCng`, `nMreg`, `nSeScree`

Examples

```
## SIMPLE EXAMPLE
 data(dFactors)
 eig      <- dFactors$Raiche$eigenvalues
 N        <- dFactors$Raiche$nsubjects

 res <- nBartlett(eig,N); res; is.nFactors(res); summary(res, digits=2)
 res <- nBentler(eig,N);  res; is.nFactors(res); summary(res, digits=2)
 res <- nCng(eig);        res; is.nFactors(res); summary(res, digits=2)
 res <- nMreg(eig);       res; is.nFactors(res); summary(res, digits=2)
 res <- nSeScree(eig);    res; is.nFactors(res); summary(res, digits=2)

## SIMILAR RESULTS, BUT NOT A nFactors OBJECT
 res <- nScree(eig);      res; is.nFactors(res); summary(res, digits=2)
```

---

iterativePrincipalAxis   *Iterative Principal Axis Analysis*

Description

The `iterativePrincipalAxis` function returns a principal axis analysis with iterated communality estimates. Four different choices of initial communality estimates are given: maximum correlation, multiple correlation (usual and generalized inverse) or estimates based on the sum of the squared principal component analysis loadings. Generally, statistical packages initialize the communalities at the multiple correlation value. Unfortunately, this strategy cannot always deal with singular correlation or covariance matrices. If a generalized inverse, the maximum correlation or the estimated communalities based on the sum of loadings are used instead, then a solution can be computed.

Usage

```
iterativePrincipalAxis(R, nFactors = 2, communalities = "component",
  iterations = 20, tolerance = 0.001)
```

Arguments

`R`	numeric: correlation or covariance matrix
`nFactors`	numeric: number of factors to retain
`communalities`	character: initial values for communalities (`"component"`,`"maxr"`,`"ginv"` or `"multiple"`)
`iterations`	numeric: maximum number of iterations to obtain a solution
`tolerance`	numeric: minimal difference in the estimated communalities after a given iteration

Value

values numeric: variance of each component

varExplained numeric: variance explained by each component

varExplained numeric: cumulative variance explained by each component

loadings numeric: loadings of each variable on each component

iterations numeric: maximum number of iterations to obtain a solution

tolerance numeric: minimal difference in the estimated communalities after a given iteration

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References

Kim, J.-O. and Mueller, C. W. (1978). *Introduction to factor analysis. What it is and how to do it*. Beverly Hills, CA: Sage.

Kim, J.-O. and Mueller, C. W. (1987). *Factor analysis. Statistical methods and practical issues*. Beverly Hills, CA: Sage.

See Also

componentAxis, principalAxis, rRecovery

Examples

```
## ................................................
# Example from Kim and Mueller (1978, p. 10)
# Population: upper diagonal
# Simulated sample: lower diagnonal
R <- matrix(c( 1.000, .6008, .4984, .1920, .1959, .3466,
               .5600, 1.000, .4749, .2196, .1912, .2979,
               .4800, .4200, 1.000, .2079, .2010, .2445,
               .2240, .1960, .1680, 1.000, .4334, .3197,
               .1920, .1680, .1440, .4200, 1.000, .4207,
               .1600, .1400, .1200, .3500, .3000, 1.000),
            nrow=6, byrow=TRUE)

# Factor analysis: Principal axis factoring with iterated communalities
# Kim and Mueller (1978, p. 23)
# Replace upper diagonal with lower diagonal
```

```
RU         <- diagReplace(R, upper=TRUE)
nFactors   <- 2
fComponent <- iterativePrincipalAxis(RU, nFactors=nFactors,
                                     communalities="component")
fComponent
rRecovery(RU,fComponent$loadings, diagCommunalities=FALSE)

fMaxr      <- iterativePrincipalAxis(RU, nFactors=nFactors,
                                     communalities="maxr")
fMaxr
rRecovery(RU,fMaxr$loadings, diagCommunalities=FALSE)

fMultiple  <- iterativePrincipalAxis(RU, nFactors=nFactors,
                                     communalities="multiple")
fMultiple
rRecovery(RU,fMultiple$loadings, diagCommunalities=FALSE)
# ......................................................
```

---

makeCor *Create a Full Correlation/Covariance Matrix from a Matrix With Lower Part Filled and Upper Part With Zeros*

Description

This function creates a full correlation/covariance matrix from a matrix with lower part filled and upper part with zeros.

Usage

```
makeCor(x)
```

Arguments

x	numeric: matrix

Value

numeric: full correlation matrix

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See Also

plotuScree, nScree, plotnScree, plotParallel

Examples

```
## ................................................
## LOWER CORRELATION MATRIX WITH ZEROS ON UPPER PART
## From Gorsuch (table 1.3.1)
gorsuch <- c(
 1,0,0,0,0,0,0,0,0,0,
 .6283, 1,0,0,0,0,0,0,0,0,
 .5631, .7353, 1,0,0,0,0,0,0,0,
 .8689, .7055, .8444, 1,0,0,0,0,0,0,
 .9030, .8626, .6890, .8874, 1,0,0,0,0,0,
 .6908, .9028, .9155, .8841, .8816, 1,0,0,0,0,
.8633, .7495, .7378, .9164, .9109, .8572, 1,0,0,0,
 .7694, .7902, .7872, .8857, .8835, .8884, .7872, 1,0,0,
 .8945, .7929, .7656, .9494, .9546, .8942, .9434, .9000, 1,0,
 .5615, .6850, .8153, .7004, .6583, .7720, .6201, .6141, .6378, 1)

## UPPER CORRELATION MATRIX FILLED WITH UPPER CORRELATION MATRIX
gorsuch <- makeCor(gorsuch)
gorsuch
```

---

moreStats *Statistical Summary of a Data Frame*

Description

This function produces another summary of a `data.frame`. This function was proposed in order to apply some functions globally on a `data.frame`: `quantile`, `median`, `min` and `max`. The usual *R* version cannot do so.

Usage

```
moreStats(x, quantile = 0.95, show = FALSE)
```

Arguments

`x`	numeric: matrix or `data.frame`
`quantile`	numeric: quantile of the distribution
`show`	logical: if `TRUE` prints the quantile choosen

Value

numeric: `data.frame` of statistics: mean, median, quantile, standard deviation, minimum and maximum

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See Also

plotuScree, nScree, plotnScree, plotParallel

Examples

```
## ................................................
## GENERATION OF A MATRIX OF 100 OBSERVATIONS AND 10 VARIABLES
x   <- matrix(rnorm(1000),ncol=10)

## STATISTICS
res <- moreStats(x, quantile=0.05, show=TRUE)
res
```

---

nBartlett — *Bartlett, Anderson and Lawley Procedures to Determine the Number of Components/Factors*

Description

This function computes the Bartlett, Anderson and Lawley indices for determining the number of components/factors to retain.

Usage

```
nBartlett(x, N, alpha = 0.05, cor = TRUE, details = TRUE,
  correction = TRUE, ...)
```

Arguments

x	numeric: a vector of eigenvalues, a matrix of correlations or of covariances or a data.frame of data (eigenFrom)
N	numeric: number of subjects
alpha	numeric: statistical significance level
cor	logical: if TRUE computes eigenvalues from a correlation matrix, else from a covariance matrix
details	logical: if TRUE also returns detains about the computation for each eigenvalue
correction	logical: if TRUE uses a correction for the degree of freedom after the first eigenvalue
...	variable: additionnal parameters to give to the cor or cov functions

Details

Note: the latex formulas are available only in the pdf version of this help file.

The hypothesis tested is:

(1) $$H_k : \lambda_{k+1} = \ldots = \lambda_p$$

This hypothesis is verified by the application of different version of a χ^2 test with different values for the degrees of freedom. Each of these tests shares the compution of a V_k value:

(2) $$V_k = \prod_{i=k+1}^{p} \left\{ \frac{\lambda_i}{{}^1\!/_q \sum_{i=k+1}^{p} \lambda_i} \right\}$$

p is the number of eigenvalues, k the number of eigenvalues to test, and q the $p - k$ remaining eigenvalues. n is equal to the sample size minus 1 ($n = N - 1$).

The Anderson statistic is distributed as a χ^2 with $(q+2)(q-1)/2$ degrees of freedom and is equal to:

(3) $$-n \log(V_k) \sim \chi^2_{(q+2)(q-1)/2}$$

An improvement of this statistic from Bartlett (Bentler, and Yuan, 1996, p. 300; Horn and Engstrom, 1979, equation 8) is distributed as a χ^2 with $(q)(q-1)/2$ degrees of freedom and is equal to:

(4) $$-\left[n - k - \frac{2q^2q+2}{6q}\right] \log(V_k) \sim \chi^2_{(q+2)(q-1)/2}$$

Finally, Anderson (1956) and James (1969) proposed another statistic.

(5) $$-\left[n - k - \frac{2q^2q+2}{6q} + \sum_{i=1}^{k} \frac{\bar{\lambda}_q^2}{\left(\lambda_i - \bar{\lambda}_q\right)^2}\right] \log(V_k) \sim \chi^2_{(q+2)(q-1)/2}$$

Bartlett (1950, 1951) proposed a correction to the degrees of freedom of these χ^2 after the first significant test: $(q + 2)(q - 1)/2$.

Value

`nFactors`	numeric: vector of the number of factors retained by the Bartlett, Anderson and Lawley procedures.
`details`	numeric: matrix of the details for each index.

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References

Anderson, T. W. (1963). Asymptotic theory for principal component analysis. *Annals of Mathematical Statistics, 34*, 122-148.

Bartlett, M. S. (1950). Tests of significance in factor analysis. *British Journal of Psychology, 3*, 77-85.

Bartlett, M. S. (1951). A further note on tests of significance. *British Journal of Psychology, 4*, 1-2.

Bentler, P. M. and Yuan, K.-H. (1996). Test of linear trend in eigenvalues of a covariance matrix with application to data analysis. *British Journal of Mathematical and Statistical Psychology, 49*, 299-312.

Bentler, P. M. and Yuan, K.-H. (1998). Test of linear trend in the smallest eigenvalues of the correlation matrix. *Psychometrika, 63*(2), 131-144.

Horn, J. L. and Engstrom, R. (1979). Cattell's scree test in relation to Bartlett's chi-square test and other observations on the number of factors problem. *Multivariate Behavioral Reasearch, 14*(3), 283-300.

James, A. T. (1969). Test of equality of the latent roots of the covariance matrix. *In* P. K. Krishna (Eds): *Multivariate analysis, volume 2*.New-York, NJ: Academic Press.

Lawley, D. N. (1956). Tests of significance for the latent roots of covarianceand correlation matrix. *Biometrika, 43*(1/2), 128-136.

See Also

plotuScree, nScree, plotnScree, plotParallel

Examples

```
## ................................................
## SIMPLE EXAMPLE OF A BARTLETT PROCEDURE

data(dFactors)
eig      <- dFactors$Raiche$eigenvalues

results  <- nBartlett(x=eig, N= 100, alpha=0.05, details=TRUE)
results

plotuScree(eig, main=paste(results$nFactors[1], ", ",
                           results$nFactors[2], " or ",
                           results$nFactors[3],
                           " factors retained by the LRT procedures",
                           sep=""))
```

`nBentler` *Bentler and Yuan's Procedure to Determine the Number of Components/Factors*

Description

This function computes the Bentler and Yuan's indices for determining the number of components/factors to retain.

Usage

```
nBentler(x, N, log = TRUE, alpha = 0.05, cor = TRUE,
  details = TRUE, minPar = c(min(lambda) - abs(min(lambda)) + 0.001,
  0.001), maxPar = c(max(lambda), lm(lambda ~
  I(length(lambda):1))$coef[2]), ...)
```

Arguments

`x`	numeric: a `vector` of eigenvalues, a `matrix` of correlations or of covariances or a `data.frame` of data
`N`	numeric: number of subjects.
`log`	logical: if `TRUE` does the maximization on the log values.
`alpha`	numeric: statistical significance level.
`cor`	logical: if `TRUE` computes eigenvalues from a correlation matrix, else from a covariance matrix
`details`	logical: if `TRUE` also returns detains about the computation for each eigenvalue.
`minPar`	numeric: minimums for the coefficient of the linear trend to maximize.
`maxPar`	numeric: maximums for the coefficient of the linear trend to maximize.
`...`	variable: additionnal parameters to give to the `cor` or `cov` functions

Details

The implemented Bentler and Yuan's procedure must be used with care because the minimized function is not always stable, as Bentler and Yan (1996, 1998) already noted. In many cases, constraints must applied to obtain a solution, as the actual implementation did, but the user can modify these constraints.

The hypothesis tested (Bentler and Yuan, 1996, equation 10) is:

(1) $$H_k : \lambda_{k+i} = \alpha + \beta x_i, (i = 1, \ldots, q)$$

The solution of the following simultaneous equations is needed to find $(\alpha, \beta) \in$

(2) $f(x) = \sum_{i=1}^{q} \frac{[\lambda_{k+j} - N\alpha + \beta x_j] x_j}{(\alpha + \beta x_j)^2} = 0$

and $g(x) = \sum_{i=1}^{q} \frac{\lambda_{k+j} - N\alpha + \beta x_j x_j}{(\alpha + \beta x_j)^2} = 0$

The solution to this system of equations was implemented by minimizing the following equation:

(3) $(\alpha, \beta) \in \inf\left[h(x)\right] = \inf \log\left[f(x)^2 + g(x)^2\right]$

The likelihood ratio test LRT proposed by Bentler and Yuan (1996, equation 7) follows a χ^2 probability distribution with $q - 2$ degrees of freedom and is equal to:

(4) $LRT = N(k-p)\left\{\ln\left(\frac{n}{N}\right) + 1\right\} - N \sum_{j=k+1}^{p} \ln\left\{\frac{\lambda_j}{\alpha + \beta x_j}\right\} + n \sum_{j=k+1}^{p} \left\{\frac{\lambda_j}{\alpha + \beta x_j}\right\}$

With p beeing the number of eigenvalues, k the number of eigenvalues to test, q the $p - k$ remaining eigenvalues, N the sample size, and $n = N - 1$. Note that there is an error in the Bentler and Yuan equation, the variables N and n beeing inverted in the preceeding equation 4.

A better strategy proposed by Bentler an Yuan (1998) is to used a minimized χ^2 solution. This strategy will be implemented in a future version of the **nFactors** package.

Value

`nFactors`	numeric: vector of the number of factors retained by the Bentler and Yuan's procedure.
`details`	numeric: matrix of the details of the computation.

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References

Bentler, P. M. and Yuan, K.-H. (1996). Test of linear trend in eigenvalues of a covariance matrix with application to data analysis. *British Journal of Mathematical and Statistical Psychology, 49*, 299-312.

Bentler, P. M. and Yuan, K.-H. (1998). Test of linear trend in the smallest eigenvalues of the correlation matrix. *Psychometrika, 63*(2), 131-144.

See Also

nBartlett, bentlerParameters

Examples

```
## ................................................
## SIMPLE EXAMPLE OF THE BENTLER AND YUAN PROCEDURE

# Bentler (1996, p. 309) Table 2 - Example 2 ............
n=649
bentler2<-c(5.785, 3.088, 1.505, 0.582, 0.424, 0.386, 0.360, 0.337, 0.303,
            0.281, 0.246, 0.238, 0.200, 0.160, 0.130)

results  <- nBentler(x=bentler2, N=n)
results

plotuScree(x=bentler2, model="components",
    main=paste(results$nFactors,
    " factors retained by the Bentler and Yuan's procedure (1996, p. 309)",
    sep=""))
# ........................................................

# Bentler (1998, p. 140) Table 3 - Example 1 ............
n        <- 145
example1 <- c(8.135, 2.096, 1.693, 1.502, 1.025, 0.943, 0.901, 0.816, 0.790,
              0.707, 0.639, 0.543,
              0.533, 0.509, 0.478, 0.390, 0.382, 0.340, 0.334, 0.316, 0.297,
              0.268, 0.190, 0.173)

results  <- nBentler(x=example1, N=n)
results

plotuScree(x=example1, model="components",
   main=paste(results$nFactors,
   " factors retained by the Bentler and Yuan's procedure (1998, p. 140)",
   sep=""))
# ........................................................
```

---

nCng — *Cattell-Nelson-Gorsuch CNG Indices*

Description

This function computes the *CNG* indices for the eigenvalues of a correlation/covariance matrix (Gorsuch and Nelson, 1981; Nasser, 2002, p. 400; Zoski and Jurs, 1993, p. 6).

Usage

```
nCng(x, cor = TRUE, model = "components", details = TRUE, ...)
```

Arguments

x	numeric: a `vector` of eigenvalues, a `matrix` of correlations or of covariances or a `data.frame` of data
cor	logical: if `TRUE` computes eigenvalues from a correlation matrix, else from a covariance matrix
model	character: `"components"` or `"factors"`
details	logical: if `TRUE` also returns detains about the computation for each eigenvalue.
...	variable: additionnal parameters to give to the `eigenComputes` function

Details

Note that the `nCng` function is only valid when more than six eigenvalues are used and that these are obtained in the context of a principal component analysis. For a factor analysis, some eigenvalues could be negative and the function will stop and give an error message.

The slope of all possible sets of three adjacent eigenvalues are compared, so *CNG* indices can be applied only when more than six eigenvalues are used. The eigenvalue at which the greatest difference between two successive slopes occurs is the indicator of the number of components/factors to retain.

Value

nFactors	numeric: number of factors retained by the CNG procedure.
details	numeric: matrix of the details for each index.

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References

Gorsuch, R. L. and Nelson, J. (1981). *CNG scree test: an objective procedure for determining the number of factors*. Presented at the annual meeting of the Society for multivariate experimental psychology.

Nasser, F. (2002). The performance of regression-based variations of the visual scree for determining the number of common factors. *Educational and Psychological Measurement, 62(3)*, 397-419.

Zoski, K. and Jurs, S. (1993). Using multiple regression to determine the number of factors to retain in factor analysis. *Multiple Linear Regression Viewpoints, 20*(1), 5-9.

See Also

plotuScree, nScree, plotnScree, plotParallel

Examples

```
## SIMPLE EXAMPLE OF A CNG ANALYSIS

 data(dFactors)
 eig      <- dFactors$Raiche$eigenvalues

 results  <- nCng(eig, details=TRUE)
 results

 plotuScree(eig, main=paste(results$nFactors,
                            " factors retained by the CNG procedure",
                            sep=""))
```

---

nFactors *nFactors: Number of factor or components to retain in a factor analysis*

Description

A package for determining the number of factor or components to retain in a factor analysis. The methods are all based on eigenvalues.

Author(s)

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References

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

nMreg	*Multiple Regression Procedure to Determine the Number of Components/Factors*

Description

This function computes the β indices, like their associated Student *t* and probability (Zoski and Jurs, 1993, 1996, p. 445). These three values can be used as three different indices for determining the number of components/factors to retain.

Usage

```
nMreg(x, cor = TRUE, model = "components", details = TRUE, ...)
```

Arguments

x	numeric: a `vector` of eigenvalues, a `matrix` of correlations or of covariances or a `data.frame` of data (eigenFrom)
cor	logical: if `TRUE` computes eigenvalues from a correlation matrix, else from a covariance matrix
model	character: `"components"` or `"factors"`
details	logical: if `TRUE` also returns details about the computation for each eigenvalue.
...	variable: additionnal parameters to give to the `eigenComputes` and `cor` or `cov` functions

Details

When the associated Student *t* test is applied, the following hypothesis is considered:

(1) $$H_k : \beta(\lambda_1 \ldots \lambda_k) - \beta(\lambda_{k+1} \ldots \lambda_p), (k = 3, \ldots, p-3) = 0$$

Value

nFactors	numeric: number of components/factors retained by the *MREG* procedures.
details	numeric: matrix of the details for each indices.

Author(s)

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References

Zoski, K. and Jurs, S. (1993). Using multiple regression to determine the number of factors to retain in factor analysis. *Multiple Linear Regression Viewpoints, 20*(1), 5-9.

Zoski, K. and Jurs, S. (1996). An objective counterpart to the visual scree test for factor analysis: the standard error scree test. *Educational and Psychological Measurement, 56*(3), 443-451.

See Also

plotuScree, nScree, plotnScree, plotParallel

Examples

```
## SIMPLE EXAMPLE OF A MREG ANALYSIS

 data(dFactors)
 eig      <- dFactors$Raiche$eigenvalues

 results  <- nMreg(eig)
 results

 plotuScree(eig, main=paste(results$nFactors[1], ", ",
                           results$nFactors[2], " or ",
                           results$nFactors[3],
                           " factors retained by the MREG procedures",
                           sep=""))
```

---

nScree — *Non Graphical Cattel's Scree Test*

Description

The nScree function returns an analysis of the number of component or factors to retain in an exploratory principal component or factor analysis. The function also returns information about the number of components/factors to retain with the Kaiser rule and the parallel analysis.

Usage

```
nScree(eig = NULL, x = eig, aparallel = NULL, cor = TRUE,
  model = "components", criteria = NULL, ...)
```

Arguments

eig	depreciated parameter (use x instead): eigenvalues to analyse
x	numeric: a vector of eigenvalues, a matrix of correlations or of covariances or a data.frame of data

aparallel	numeric: results of a parallel analysis. Defaults eigenvalues fixed at $\lambda >= \bar{\lambda}$ (Kaiser and related rule) or $\lambda >= 0$ (CFA analysis)
cor	logical: if TRUE computes eigenvalues from a correlation matrix, else from a covariance matrix
model	character: "components" or "factors"
criteria	numeric: by default fixed at $\bar{\lambda}$. When the λs are computed from a principal component analysis on a correlation matrix, it corresponds to the usual Kaiser $\lambda >= 1$ rule. On a covariance matrix or from a factor analysis, it is simply the mean. To apply $\lambda >= 0$, sometimes used with factor analysis, fix the criteria to 0.
...	variabe: additionnal parameters to give to the cor or cov functions

Details

The nScree function returns an analysis of the number of components/factors to retain in an exploratory principal component or factor analysis. Different solutions are given. The classical ones are the Kaiser rule, the parallel analysis, and the usual scree test (plotuScree). Non graphical solutions to the Cattell subjective scree test are also proposed: an acceleration factor (*af*) and the optimal coordinates index oc. The acceleration factor indicates where the elbow of the scree plot appears. It corresponds to the acceleration of the curve, i.e. the second derivative. The optimal coordinates are the extrapolated coordinates of the previous eigenvalue that allow the observed eigenvalue to go beyond this extrapolation. The extrapolation is made by a linear regression using the last eigenvalue coordinates and the $k+1$ eigenvalue coordinates. There are $k-2$ regression lines like this. The Kaiser rule or a parallel analysis criterion (parallel) must also be simultaneously satisfied to retain the components/factors, whether for the acceleration factor, or for the optimal coordinates.

If λ_i is the i^{th} eigenvalue, and LS_i is a location statistics like the mean or a centile (generally the followings: $1^{st},\ 5^{th},\ 95^{th},\ or\ 99^{th}$).

The Kaiser rule is computed as:

$$n_{Kaiser} = \sum_i (\lambda_i \geq \bar{\lambda}).$$

Note that $\bar{\lambda}$ is equal to 1 when a correlation matrix is used.

The parallel analysis is computed as:

$$n_{parallel} = \sum_i (\lambda_i \geq LS_i).$$

The acceleration factor (AF) corresponds to a numerical solution to the elbow of the scree plot:

$$n_{AF} \equiv If\ \left[(\lambda_i \geq LS_i)\ and\ max(AF_i)\right].$$

The optimal coordinates (OC) corresponds to an extrapolation of the preceeding eigenvalue by a regression line between the eigenvalue coordinates and the last eigenvalue coordinates:

$$n_{OC} = \sum_i \left[(\lambda_i \geq LS_i) \cap (\lambda_i \geq (\lambda_{i\ predicted})\right].$$

Value

`Components` Data frame for the number of components/factors according to different rules

`Components$noc` Number of components/factors to retain according to optimal coordinates *oc*

`Components$naf` Number of components/factors to retain according to the acceleration factor *af*

`Components$npar.analysis` Number of components/factors to retain according to parallel analysis

`Components$nkaiser` Number of components/factors to retain according to the Kaiser rule

`Analysis` Data frame of vectors linked to the different rules

`Analysis$Eigenvalues` Eigenvalues

`Analysis$Prop` Proportion of variance accounted by eigenvalues

`Analysis$Cumu` Cumulative proportion of variance accounted by eigenvalues

`Analysis$Par.Analysis` Centiles of the random eigenvalues generated by the parallel analysis.

`Analysis$Pred.eig` Predicted eigenvalues by each optimal coordinate regression line

`Analysis$OC` Critical optimal coordinates *oc*

`Analysis$Acc.factor` Acceleration factor *af*

`Analysis$AF` Critical acceleration factor *af*

Otherwise, returns a summary of the analysis.

Author(s)

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References

Cattell, R. B. (1966). The scree test for the number of factors. *Multivariate Behavioral Research, 1*, 245-276.

Dinno, A. (2009). *Gently clarifying the application of Horn's parallel analysis to principal component analysis versus factor analysis*. Portland, Oregon: Portland Sate University.

Guttman, L. (1954). Some necessary conditions for common factor analysis. *Psychometrika, 19, 149-162*.

Horn, J. L. (1965). A rationale for the number of factors in factor analysis. *Psychometrika, 30*, 179-185.

Kaiser, H. F. (1960). The application of electronic computer to factor analysis. *Educational and Psychological Measurement, 20*, 141-151.

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

See Also

plotuScree, plotnScree, parallel, plotParallel,

Examples

```
## INITIALISATION
 data(dFactors)                      # Load the nFactors dataset
 attach(dFactors)
 vect         <- Raiche              # Uses the example from Raiche
 eigenvalues  <- vect$eigenvalues    # Extracts the observed eigenvalues
 nsubjects    <- vect$nsubjects      # Extracts the number of subjects
 variables    <- length(eigenvalues) # Computes the number of variables
 rep          <- 100                 # Number of replications for PA analysis
 cent         <- 0.95                # Centile value of PA analysis

## PARALLEL ANALYSIS (qevpea for the centile criterion, mevpea for the
## mean criterion)
 aparallel    <- parallel(var     = variables,
                          subject = nsubjects,
                          rep     = rep,
                          cent    = cent
                          )$eigen$qevpea  # The 95 centile

## NUMBER OF FACTORS RETAINED ACCORDING TO DIFFERENT RULES
 results      <- nScree(x=eigenvalues, aparallel=aparallel)
 results
 summary(results)

## PLOT ACCORDING TO THE nScree CLASS
 plotnScree(results)
```

---

nSeScree — *Standard Error Scree and Coefficient of Determination Procedures to Determine the Number of Components/Factors*

Description

This function computes the *seScree* ($S_{Y \bullet X}$) indices (Zoski and Jurs, 1996) and the coefficient of determination indices of Nelson (2005) R^2 for determining the number of components/factors to retain.

Usage

```
nSeScree(x, cor = TRUE, model = "components", details = TRUE,
  r2limen = 0.75, ...)
```

Arguments

x	numeric: eigenvalues.
cor	logical: if TRUE computes eigenvalues from a correlation matrix, else from a covariance matrix
model	character: "components" or "factors"
details	logical: if TRUE also returns details about the computation for each eigenvalue.
r2limen	numeric: criterion value retained for the coefficient of determination indices.
...	variable: additionnal parameters to give to the eigenComputes and cor or cov functions

Details

The Zoski and Jurs $S_{Y \bullet X}$ index is the standard error of the estimate (predicted) eigenvalues by the regression from the $(k+1,\ldots,p)$ subsequent ranks of the eigenvalues. The standard error is computed as:

(1) $$S_{Y \bullet X} = \sqrt{\frac{(\lambda_k - \hat{\lambda}_k)^2}{p-2}}$$

A value of $1/p$ is choosen as the criteria to determine the number of components or factors to retain, *p* corresponding to the number of variables.

The Nelson R^2 index is simply the multiple regresion coefficient of determination for the $k+1,\ldots,p$ eigenvalues. Note that Nelson didn't give formal prescriptions for the criteria for this index. He only suggested that a value of 0.75 or more must be considered. More is to be done to explore adequate values.

Value

nFactors	numeric: number of components/factors retained by the seScree procedure.
details	numeric: matrix of the details for each index.

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References

Nasser, F. (2002). The performance of regression-based variations of the visual scree for determining the number of common factors. *Educational and Psychological Measurement, 62(3)*, 397-419.

Nelson, L. R. (2005). Some observations on the scree test, and on coefficient alpha. *Thai Journal of Educational Research and Measurement, 3(1)*, 1-17.

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

Zoski, K. and Jurs, S. (1993). Using multiple regression to determine the number of factors to retain in factor analysis. *Multiple Linear Regression Viewpoints, 20*(1), 5-9.

Zoski, K. and Jurs, S. (1996). An objective counterpart to the visuel scree test for factor analysis: the standard error scree. *Educational and Psychological Measurement, 56*(3), 443-451.

See Also

plotuScree, nScree, plotnScree, plotParallel

Examples

```
## SIMPLE EXAMPLE OF SESCREE AND R2 ANALYSIS

 data(dFactors)
 eig      <- dFactors$Raiche$eigenvalues

 results  <- nSeScree(eig)
 results

 plotuScree(eig, main=paste(results$nFactors[1], " or ", results$nFactors[2],
                            " factors retained by the sescree and R2 procedures",
                            sep=""))
```

---

parallel — *Parallel Analysis of a Correlation or Covariance Matrix*

Description

This function gives the distribution of the eigenvalues of correlation or a covariance matrices of random uncorrelated standardized normal variables. The mean and a selected quantile of this distribution are returned.

Usage

```
parallel(subject = 100, var = 10, rep = 100, cent = 0.05,
  quantile = cent, model = "components", sd = diag(1, var), ...)
```

Arguments

subject	numeric: nmber of subjects (default is 100)
var	numeric: number of variables (default is 10)
rep	numeric: number of replications of the correlation matrix (default is 100)
cent	depreciated numeric (use quantile instead): quantile of the distribution on which the decision is made (default is 0.05)

`quantile`	numeric: quantile of the distribution on which the decision is made (default is 0.05)
`model`	character: `"components"` or `"factors"`
`sd`	numeric: vector of standard deviations of the simulated variables (for a parallel analysis on a covariance matrix)
`...`	variable: other parameters for the `"mvrnorm"`, `corr` or `cov` functions

Details

Note that if the decision is based on a quantile value rather than on the mean, care must be taken with the number of replications (`rep`). In fact, the smaller the quantile (`cent`), the bigger the number of necessary replications.

Value

`eigen`	Data frame consisting of the mean and the quantile of the eigenvalues distribution
`eigen$mevpea`	Mean of the eigenvalues distribution
`eigen$sevpea`	Standard deviation of the eigenvalues distribution
`eigen$qevpea`	quantile of the eigenvalues distribution
`eigen$sqevpea`	Standard error of the quantile of the eigenvalues distribution
`subject`	Number of subjects
`variables`	Number of variables
`centile`	Selected quantile

Otherwise, returns a summary of the parallel analysis.

Author(s)

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References

Drasgow, F. and Lissak, R. (1983) Modified parallel analysis: a procedure for examining the latent dimensionality of dichotomously scored item responses. *Journal of Applied Psychology, 68*(3), 363-373.

Hoyle, R. H. and Duvall, J. L. (2004). Determining the number of factors in exploratory and confirmatory factor analysis. In D. Kaplan (Ed.): *The Sage handbook of quantitative methodology for the social sciences*. Thousand Oaks, CA: Sage.

Horn, J. L. (1965). A rationale and test of the number of factors in factor analysis. *Psychometrika, 30*, 179-185.

See Also

plotuScree, nScree, plotnScree, plotParallel

Examples

```
## SIMPLE EXAMPLE OF A PARALLEL ANALYSIS
## OF A CORRELATION MATRIX WITH ITS PLOT
 data(dFactors)
 eig      <- dFactors$Raiche$eigenvalues
 subject  <- dFactors$Raiche$nsubjects
 var      <- length(eig)
 rep      <- 100
 quantile <- 0.95
 results  <- parallel(subject, var, rep, quantile)

 results

## IF THE DECISION IS BASED ON THE CENTILE USE qevpea INSTEAD
## OF mevpea ON THE FIRST LINE OF THE FOLLOWING CALL
 plotuScree(x    = eig,
            main = "Parallel Analysis"
            )

 lines(1:var,
       results$eigen$qevpea,
       type="b",
       col="green"
       )


## ANOTHER SOLUTION IS SIMPLY TO
 plotParallel(results)
```

---

plotnScree *Scree Plot According to a nScree Object Class*

Description

Plot a scree plot adding information about a non graphical nScree analysis.

Usage

```
plotnScree(nScree, legend = TRUE, ylab = "Eigenvalues",
  xlab = "Components", main = "Non Graphical Solutions to Scree Test")
```

Arguments

nScree	Results of a previous nScree analysis
legend	Logical indicator of the presence or not of a legend
ylab	Label of the y axis (default to "Eigenvalue")
xlab	Label of the x axis (default to "Component")
main	Main title (default to "Non Graphical Solutions to the Scree Test")

Value

Nothing returned.

Author(s)

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References

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

See Also

plotuScree, nScree, plotParallel, parallel

Examples

```
## INITIALISATION
 data(dFactors)                      # Load the nFactors dataset
 attach(dFactors)
 vect         <- Raiche              # Use the second example from Buja and Eyuboglu
                                     # (1992, p. 519, nsubjects not specified by them)
 eigenvalues  <- vect$eigenvalues    # Extract the observed eigenvalues
 nsubjects    <- vect$nsubjects      # Extract the number of subjects
 variables    <- length(eigenvalues) # Compute the number of variables
 rep          <- 100                 # Number of replications for the parallel analysis
 cent         <- 0.95                # Centile value of the parallel analysis

## PARALLEL ANALYSIS (qevpea for the centile criterion, mevpea for the mean criterion)
 aparallel    <- parallel(var     = variables,
                          subject = nsubjects,
                          rep     = rep,
                          cent    = cent)$eigen$qevpea  # The 95 centile

## NOMBER OF FACTORS RETAINED ACCORDING TO DIFFERENT RULES
 results <- nScree(eig       = eigenvalues,
                   aparallel = aparallel
```

```
                    )

 results

## PLOT ACCORDING TO THE nScree CLASS
 plotnScree(results)
```

`plotParallel` *Plot a Parallel Analysis Class Object*

Description

Plot a scree plot adding information about a parallel analysis.

Usage

```
plotParallel(parallel, eig = NA, x = eig, model = "components",
  legend = TRUE, ylab = "Eigenvalues", xlab = "Components",
  main = "Parallel Analysis", ...)
```

Arguments

`parallel`	numeric: vector of the results of a previous parallel analysis
`eig`	depreciated parameter: eigenvalues to analyse (not used if x is used, recommended)
`x`	numeric: a `vector` of eigenvalues, a `matrix` of correlations or of covariances or a `data.frame` of data
`model`	character: `"components"` or `"factors"`
`legend`	logical: indicator of the presence or not of a legend
`ylab`	character: label of the y axis
`xlab`	character: label of the x axis
`main`	character: title of the plot
`...`	variable: additionnal parameters to give to the `cor` or `cov` functions

Details

If `eig` is `FALSE` the plot shows only the parallel analysis without eigenvalues.

Value

Nothing returned.

Author(s)

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References

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

See Also

plotuScree, nScree, plotnScree, parallel

Examples

```
## SIMPLE EXAMPLE OF A PARALLEL ANALYSIS
## OF A CORRELATION MATRIX WITH ITS PLOT
 data(dFactors)
 eig      <- dFactors$Raiche$eigenvalues
 subject  <- dFactors$Raiche$nsubjects
 var      <- length(eig)
 rep      <- 100
 cent     <- 0.95
 results  <- parallel(subject,var,rep,cent)

 results


## PARALLEL ANALYSIS SCREE PLOT
 plotParallel(results, x=eig)
 plotParallel(results)
```

---

plotuScree *Plot of the Usual Cattell's Scree Test*

Description

uScree plot a usual scree test of the eigenvalues of a correlation matrix.

Usage

```
plotuScree(Eigenvalue, x = Eigenvalue, model = "components",
  ylab = "Eigenvalues", xlab = "Components", main = "Scree Plot",
  ...)
```

Arguments

`Eigenvalue`	depreciated parameter: eigenvalues to analyse (not used if x is used, recommended)
`x`	numeric: a `vector` of eigenvalues, a `matrix` of correlations or of covariances or a `data.frame` of data
`model`	character: `"components"` or `"factors"`
`ylab`	character: label of the y axis (default is `Eigenvalue`)
`xlab`	character: label of the x axis (default is `Component`)
`main`	character: title of the plot (default is `Scree Plot`)
`...`	variable: additionnal parameters to give to the `eigenComputes` function

Value

Nothing returned with this function.

Author(s)

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References

Cattell, R. B. (1966). The scree test for the number of factors. *Multivariate Behavioral Research, 1*, 245-276.

See Also

`nScree`, `parallel`

Examples

```
## SCREE PLOT
 data(dFactors)
 attach(dFactors)
 eig = Cliff1$eigenvalues
 plotuScree(x=eig)
```

`principalAxis` *Principal Axis Analysis*

Description

The `PrincipalAxis` function returns a principal axis analysis without iterated communalities estimates. Three different choices of communalities estimates are given: maximum corelation, multiple correlation or estimates based on the sum of the squared principal component analysis loadings. Generally statistical packages initialize the the communalities at the multiple correlation value (usual inverse or generalized inverse). Unfortunately, this strategy cannot deal with singular correlation or covariance matrices. If a generalized inverse, the maximum correlation or the estimated communalities based on the sum of loading are used instead, then a solution can be computed.

Usage

```
principalAxis(R, nFactors = 2, communalities = "component")
```

Arguments

`R`	numeric: correlation or covariance matrix
`nFactors`	numeric: number of factors to retain
`communalities`	character: initial values for communalities (`"component"`,`"maxr"`,`"ginv"` or `"multiple"`)

Value

`values`	numeric: variance of each component/factor
`varExplained`	numeric: variance explained by each component/factor
`varExplained`	numeric: cumulative variance explained by each component/factor
`loadings`	numeric: loadings of each variable on each component/factor

Author(s)

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References

Kim, J.-O. and Mueller, C. W. (1978). *Introduction to factor analysis. What it is and how to do it*. Beverly Hills, CA: Sage.

Kim, J.-O. and Mueller, C. W. (1987). *Factor analysis. Statistical methods and practical issues*. Beverly Hills, CA: Sage.

See Also

componentAxis, iterativePrincipalAxis, rRecovery

Examples

```
# ......................................................
# Example from Kim and Mueller (1978, p. 10)
# Population: upper diagonal
# Simulated sample: lower diagnonal
 R <- matrix(c( 1.000, .6008, .4984, .1920, .1959, .3466,
                .5600, 1.000, .4749, .2196, .1912, .2979,
                .4800, .4200, 1.000, .2079, .2010, .2445,
                .2240, .1960, .1680, 1.000, .4334, .3197,
                .1920, .1680, .1440, .4200, 1.000, .4207,
                .1600, .1400, .1200, .3500, .3000, 1.000),
                nrow=6, byrow=TRUE)

# Factor analysis: Principal axis factoring
# without iterated communalities -
# Kim and Mueller (1978, p. 21)
# Replace upper diagonal with lower diagonal
 RU <- diagReplace(R, upper=TRUE)
 principalAxis(RU, nFactors=2, communalities="component")
 principalAxis(RU, nFactors=2, communalities="maxr")
 principalAxis(RU, nFactors=2, communalities="multiple")
# Replace lower diagonal with upper diagonal
 RL <- diagReplace(R, upper=FALSE)
 principalAxis(RL, nFactors=2, communalities="component")
 principalAxis(RL, nFactors=2, communalities="maxr")
 principalAxis(RL, nFactors=2, communalities="multiple")
# ......................................................
```

---

principalComponents *Principal Component Analysis*

Description

The principalComponents function returns a principal component analysis. Other R functions give the same results, but principalComponents is customized mainly for the other factor analysis functions available in the **nfactors** package. In order to retain only a small number of components the componentAxis function has to be used.

Usage

```
principalComponents(R)
```

Arguments

R	numeric: correlation or covariance matrix

Value

values	numeric: variance of each component
varExplained	numeric: variance explained by each component
varExplained	numeric: cumulative variance explained by each component
loadings	numeric: loadings of each variable on each component

Author(s)

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References

Joliffe, I. T. (2002). *Principal components analysis* (2th Edition). New York, NJ: Springer-Verlag.

Kim, J.-O. and Mueller, C. W. (1978). *Introduction to factor analysis. What it is and how to do it*. Beverly Hills, CA: Sage.

Kim, J.-O. and Mueller, C. W. (1987). *Factor analysis. Statistical methods and practical issues*. Beverly Hills, CA: Sage.

See Also

componentAxis, iterativePrincipalAxis, rRecovery

Examples

```
# .......................................................
# Example from Kim and Mueller (1978, p. 10)
# Population: upper diagonal
# Simulated sample: lower diagnonal
 R <- matrix(c( 1.000, .6008, .4984, .1920, .1959, .3466,
               .5600, 1.000, .4749, .2196, .1912, .2979,
               .4800, .4200, 1.000, .2079, .2010, .2445,
               .2240, .1960, .1680, 1.000, .4334, .3197,
               .1920, .1680, .1440, .4200, 1.000, .4207,
               .1600, .1400, .1200, .3500, .3000, 1.000),
               nrow=6, byrow=TRUE)

# Factor analysis: Principal component -
# Kim et Mueller (1978, p. 21)
# Replace upper diagonal with lower diagonal
 RU <- diagReplace(R, upper=TRUE)
 principalComponents(RU)
```

```
# Replace lower diagonal with upper diagonal
 RL <- diagReplace(R, upper=FALSE)
 principalComponents(RL)
# ....................................................
```

---

rRecovery — *Test of Recovery of a Correlation or a Covariance matrix from a Factor Analysis Solution*

Description

The `rRecovery` function returns a verification of the quality of the recovery of the initial correlation or covariance matrix by the factor solution.

Usage

```
rRecovery(R, loadings, diagCommunalities = FALSE)
```

Arguments

`R` numeric: initial correlation or covariance matrix

`loadings` numeric: loadings from a factor analysis solution

`diagCommunalities`
logical: if `TRUE`, the correlation between the initial solution and the estimated one will use a correlation of one in the diagonal. If `FALSE` (default) the diagonal is not used in the computation of this correlation.

Value

`R` numeric: initial correlation or covariance matrix

`recoveredR` numeric: recovered estimated correlation or covariance matrix

`difference` numeric: difference between initial and recovered estimated correlation or covariance matrix

`cor` numeric: Pearson correlation between initial and recovered estimated correlation or covariance matrix. Computations depend on the logical value of the `communalities` argument.

Author(s)

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See Also

componentAxis, iterativePrincipalAxis, principalAxis

Examples

```
# ......................................................
# Example from Kim and Mueller (1978, p. 10)
# Population: upper diagonal
# Simulated sample: lower diagnonal
 R <- matrix(c( 1.000, .6008, .4984, .1920, .1959, .3466,
                .5600, 1.000, .4749, .2196, .1912, .2979,
                .4800, .4200, 1.000, .2079, .2010, .2445,
                .2240, .1960, .1680, 1.000, .4334, .3197,
                .1920, .1680, .1440, .4200, 1.000, .4207,
                .1600, .1400, .1200, .3500, .3000, 1.000),
                nrow=6, byrow=TRUE)


# Replace upper diagonal with lower diagonal
 RU         <- diagReplace(R, upper=TRUE)
 nFactors   <- 2
 loadings   <- principalAxis(RU, nFactors=nFactors,
                              communalities="component")$loadings
 rComponent <- rRecovery(RU,loadings, diagCommunalities=FALSE)$cor

 loadings   <- principalAxis(RU, nFactors=nFactors,
                              communalities="maxr")$loadings
 rMaxr      <- rRecovery(RU,loadings, diagCommunalities=FALSE)$cor

 loadings   <- principalAxis(RU, nFactors=nFactors,
                              communalities="multiple")$loadings
 rMultiple  <- rRecovery(RU,loadings, diagCommunalities=FALSE)$cor

 round(c(rComponent = rComponent,
         rmaxr      = rMaxr,
         rMultiple  = rMultiple), 3)
# ......................................................
```

---

structureSim *Population or Simulated Sample Correlation Matrix from a Given Factor Structure Matrix*

Description

The structureSim function returns a population and a sample correlation matrices from a predefined congeneric factor structure.

Usage

```
structureSim(fload, reppar = 30, repsim = 100, N, quantile = 0.95,
  model = "components", adequacy = FALSE, details = TRUE,
  r2limen = 0.75, all = FALSE)
```

Arguments

fload	matrix: loadings of the factor structure
reppar	numeric: number of replications for the parallel analysis
repsim	numeric: number of replications of the matrix correlation simulation
N	numeric: number of subjects
quantile	numeric: quantile for the parallel analysis
model	character: "components" or "factors"
adequacy	logical: if TRUE prints the recovered population matrix from the factor structure
details	logical: if TRUE outputs details of the repsim simulations
r2limen	numeric: R2 limen value for the R2 Nelson index
all	logical: if TRUE computes the Bentler and Yuan index (very long computing time to consider)

Value

values	the output depends of the logical value of details. If FALSE, returns only statistics about the eigenvalues: mean, median, quantile, standard deviation, minimum and maximum. If TRUE, returns also details about the repsim simulations. If adequacy = TRUE returns the recovered factor structure

Author(s)

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References

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

Zwick, W. R. and Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin, 99*, 432-442.

See Also

principalComponents, iterativePrincipalAxis, rRecovery

Examples

```
## Not run:
# ....................................................
# Example inspired from Zwick and Velicer (1986, table 2, p. 437)
## ...................................................................
 nFactors  <- 3
 unique    <- 0.2
 loadings  <- 0.5
 nsubjects <- 180
 repsim    <- 30
 zwick     <- generateStructure(var=36, mjc=nFactors, pmjc=12,
                                loadings=loadings,
                                unique=unique)
## ...................................................................

# Produce statistics about a replication of a parallel analysis on
# 30 sampled correlation matrices

 mzwick.fa <-  structureSim(fload=as.matrix(zwick), reppar=30,
                            repsim=repsim, N=nsubjects, quantile=0.5,
                            model="factors")

 mzwick    <-  structureSim(fload=as.matrix(zwick), reppar=30,
                            repsim=repsim, N=nsubjects, quantile=0.5, all=TRUE)

# Very long execution time that could be used only with model="components"
# mzwick    <-  structureSim(fload=as.matrix(zwick), reppar=30,
#                            repsim=repsim, N=nsubjects, quantile=0.5, all=TRUE)

 par(mfrow=c(2,1))
 plot(x=mzwick,    nFactors=nFactors, index=c(1:14), cex.axis=0.7, col="red")
 plot(x=mzwick.fa, nFactors=nFactors, index=c(1:11), cex.axis=0.7, col="red")
 par(mfrow=c(1,1))

 par(mfrow=c(2,1))
 boxplot(x=mzwick,    nFactors=3, cex.axis=0.8, vLine="blue", col="red")
 boxplot(x=mzwick.fa, nFactors=3, cex.axis=0.8, vLine="blue", col="red",
         xlab="Components")
 par(mfrow=c(1,1))
# ....................................................

## End(Not run)
```

---

studySim *Simulation Study from Given Factor Structure Matrices and Conditions*

Description

The `structureSim` function returns statistical results from simulations from predefined congeneric factor structures. The main ideas come from the methodology applied by Zwick and Velicer (1986).

Usage

```
studySim(var, nFactors, pmjc, loadings, unique, N, repsim, reppar,
  stats = 1, quantile = 0.5, model = "components", r2limen = 0.75,
  all = FALSE, dir = NA, trace = TRUE)
```

Arguments

`var`	numeric: vector of the number of variables
`nFactors`	numeric: vector of the number of components/factors
`pmjc`	numeric: vector of the number of major loadings on each component/factor
`loadings`	numeric: vector of the major loadings on each component/factor
`unique`	numeric: vector of the unique loadings on each component/factor
`N`	numeric: vector of the number of subjects/observations
`repsim`	numeric: number of replications of the matrix correlation simulation
`reppar`	numeric: number of replications for the parallel and permutation analysis
`stats`	numeric: vector of the statistics to return: mean(1), median(2), sd(3), quantile(4), min(5), max(6)
`quantile`	numeric: quantile for the parallel and permutation analysis
`model`	character: `"components"` or `"factors"`
`r2limen`	numeric: R2 limen value for the R2 Nelson index
`all`	logical: if `TRUE` computes the Bentler and Yuan index (very long computing time to consider)
`dir`	character: directory where to save output. Default to NA
`trace`	logical: if `TRUE` outputs details of the status of the simulations

Value

`values`	Returns selected statistics about the number of components/factors to retain: mean, median, quantile, standard deviation, minimum and maximum.

Author(s)

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References

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

Zwick, W. R. and Velicer, W. F. (1986). Comparison of five rules for determining the number of components to retain. *Psychological Bulletin, 99*, 432-442.

See Also

generateStructure, structureSim

Examples

```
## Not run:
# ....................................................................
# Example inspired from Zwick and Velicer (1986)
# Very long computimg time
# ...................................................................

# 1. Initialisation
# reppar    <- 30
# repsim    <- 5
# quantile  <- 0.50

# 2. Simulations
# X         <- studySim(var=36,nFactors=3, pmjc=c(6,12), loadings=c(0.5,0.8),
#                        unique=c(0,0.2), quantile=quantile,
#                        N=c(72,180), repsim=repsim, reppar=reppar,
#                        stats=c(1:6))

# 3. Results (first 10 results)
# print(X[1:10,1:14],2)
# names(X)

# 4. Study of the error done in the determination of the number
#    of components/factors. A positive value is associated to over
#    determination.
# results   <- X[X$stats=="mean",]
# residuals <- results[,c(11:25)] - X$nfactors
# BY        <- c("nsubjects","var","loadings")
# round(aggregate(residuals, by=results[BY], mean),0)

## End(Not run)
```

---

summary.nScree *Utility Functions for nScree Class Objects*

Description

Utility functions for nScree class objects. Some of these functions are already implemented in the nFactors package, but are easier to use with generic functions like these.

Usage

```
## S3 method for class 'nScree'
summary(object, ...)

## S3 method for class 'nScree'
print(x, ...)

## S3 method for class 'nScree'
plot(x, ...)

is.nScree(object)
```

Arguments

object	nScree: an object of the class nScree
...	variable: additionnal parameters to give to the print function with print.nScree, the plotnScree with plot.nScree or to the summary function with summary.nScree
x	Results of a previous nScree analysis

Value

Generic functions for the nScree class:

is.nScree	logical: is the object of the class nScree?
plot.nScree	graphic: plots a figure according to the plotnScree function
print.nScree	numeric: vector of the number of components/factors to retain: same as the Components vector from the nScree object
summary.nScree	data.frame: details of the results from a nScree analysis: same as the Analysis data.frame from the nScree object, but with easier control of the number of decimals with the digits parameter

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References

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Examples

```
## INITIALISATION
 data(dFactors)                      # Load the nFactors dataset
 attach(dFactors)
 vect         <- Raiche              # Use the example from Raiche
 eigenvalues  <- vect$eigenvalues    # Extract the observed eigenvalues
 nsubjects    <- vect$nsubjects      # Extract the number of subjects
 variables    <- length(eigenvalues) # Compute the number of variables
 rep          <- 100                 # Number of replications for the parallel analysis
 cent         <- 0.95                # Centile value of the parallel analysis

## PARALLEL ANALYSIS (qevpea for the centile criterion, mevpea for the mean criterion)
 aparallel    <- parallel(var     = variables,
                          subject = nsubjects,
                          rep     = rep,
                          cent    = cent
                          )$eigen$qevpea  # The 95 centile

## NOMBER OF FACTORS RETAINED ACCORDING TO DIFFERENT RULES
 results      <- nScree(x=eigenvalues, aparallel=aparallel)

 is.nScree(results)
 results
 summary(results)

## PLOT ACCORDING TO THE nScree CLASS
 plot(results)
```

`summary.structureSim` *Utility Functions for nScree Class Objects*

Description

Utility functions for `structureSim` class objects. Note that with the `plot.structureSim` a dotted black vertical line shows the median number of factors retained by all the different indices.

Usage

```
## S3 method for class 'structureSim'
summary(object, index = c(1:15),
  eigenSelect = NULL, ...)

## S3 method for class 'structureSim'
print(x, index = NULL, ...)

## S3 method for class 'structureSim'
```

```
boxplot(x, nFactors = NULL, eigenSelect = NULL,
  vLine = "green", xlab = "Factors", ylab = "Eigenvalues",
  main = "Eigen Box Plot", ...)

## S3 method for class 'structureSim'
plot(x, nFactors = NULL, index = NULL,
  main = "Index Acuracy Plot", ...)

is.structureSim(object)
```

Arguments

`object`	structureSim: an object of the class `structureSim`
`index`	numeric: vector of the index of the selected indices
`eigenSelect`	numeric: vector of the index of the selected eigenvalues
`...`	variable: additionnal parameters to give to the `boxplot`, `plot`, `print` and `summary` `functions.`
`x`	structureSim: an object of the class `structureSim`
`nFactors`	numeric: if known, number of factors
`vLine`	character: color of the vertical indicator line of the initial number of factors in the eigen boxplot
`xlab`	character: x axis label
`ylab`	character: y axis label
`main`	character: main title

Value

Generic functions for the `structureSim` class:

`boxplot.structureSim`
: graphic: plots an eigen boxplot

`is.structureSim`
: logical: is the object of the class `structureSim`?

`plot.structureSim`
: graphic: plots an index acuracy plot

`print.structureSim`
: numeric: data.frame of statistics about the number of components/factors to retain according to different indices following a `structureSim` simulation

`summary.structureSim`
: list: two data.frame, the first with the details of the simulated eigenvalues, the second with the details of the simulated indices

Author(s)

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References

Raiche, G., Walls, T. A., Magis, D., Riopel, M. and Blais, J.-G. (2013). Non-graphical solutions for Cattell's scree test. Methodology, 9(1), 23-29.

See Also

nFactors-package

Examples

```
## Not run:
## INITIALISATION
 library(xtable)
 library(nFactors)
 nFactors  <- 3
 unique    <- 0.2
 loadings  <- 0.5
 nsubjects <- 180
 repsim    <- 10
 var       <- 36
 pmjc      <- 12
 reppar    <- 10
 zwick     <- generateStructure(var=var, mjc=nFactors, pmjc=pmjc,
                                 loadings=loadings,
                                 unique=unique)

## SIMULATIONS
mzwick    <-  structureSim(fload=as.matrix(zwick), reppar=reppar,
                           repsim=repsim, details=TRUE,
                           N=nsubjects, quantile=0.5)

## TEST OF structureSim METHODS
 is(mzwick)
 summary(mzwick, index=1:5, eigenSelect=1:10, digits=3)
 print(mzwick, index=1:10)
 plot(x=mzwick, index=c(1:10), cex.axis=0.7, col="red")
 boxplot(x=mzwick, nFactors=3, vLine="blue", col="red")

## End(Not run)
```

Index