# Package 'nvetr' 

October 28, 2020
Type Package
Title The n-vector Approach to Geographical Position Calculations using an Ellipsoidal Model of Earth
Version 0.1.4
Maintainer Enrico Spinielli [enrico.spinielli@eurocontrol.int](mailto:enrico.spinielli@eurocontrol.int)
Description The n -vector framework uses the normal vector to the Earth ellipsoid (called n-vector) as a non-singular position representation that turns out to be very convenient for practical position calculations. The n -vector is simple to use and gives exact answers for all global positions, and all distances, for both ellipsoidal and spherical Earth models. This package is a translation of the 'Matlab' library from FFI, the Norwegian Defence Research Establishment, as described in Gade (2010) [doi:10.1017/S0373463309990415](doi:10.1017/S0373463309990415).
License MIT + file LICENSE
URL https://github.com/euctrl-pru/nvctr
BugReports https://github.com/euctrl-pru/nvctr/issues
Imports magrittr, pracma
Suggests bookdown, covr, geosphere, knitr, png, rmarkdown, spelling, testthat
VignetteBuilder knitr
Encoding UTF-8
Language en-US
LazyData true
RoxygenNote 7.1.1
NeedsCompilation no
Author Enrico Spinielli [aut, cre] ([https://orcid.org/0000-0001-8584-9131](https://orcid.org/0000-0001-8584-9131)), EUROCONTROL [cph, fnd]

## Repository CRAN

Date/Publication 2020-10-28 14:00:02 UTC

## $R$ topics documented:

along_track_distance ..... 2
altitude_azimuth_distance ..... 3
cross_track_distance ..... 4
cross_track_intersection ..... 5
deg ..... 6
lat_lon2n_E ..... 7
nvetr ..... 7
n_E2lat_lon ..... 8
n_E2R_EN ..... 9
n_EA_E_and_n_EB_E2p_AB_E ..... 9
n_EA_E_and_p_AB_E2n_EB_E ..... 11
n_EB_E2p_EB_E ..... 12
n_E_and_wa2R_EL ..... 13
p_EB_E2n_EB_E ..... 14
R2xyz ..... 15
R2zyx ..... 16
rad ..... 17
R_Ee ..... 17
R_EL2n_E ..... 18
R_EN2n_E ..... 19
unit ..... 20
xyz2R ..... 21
zyx2R ..... 22
Index ..... 23
along_track_distance Compute the along-track distance from a great circle arc

## Description

Compute the along-track distances of a body, ' $b$ ' (for example a ground level projection position of an aircraft), from two geographical coordinates, 'a1' and 'a2' (for example an airport's runway thresholds), of a great circle arc.

## Usage

along_track_distance(b, a1, a2)

## Arguments

b the geographical coordinates (WGS84) of a body: a vector of longitude, latitude (in decimal degrees) and eventually altitude (in meters)
a1 the geographical coordinates (WGS84) of one end of a great circle arc: a vector of longitude, latitude (in decimal degrees) and eventually altitude (in meters)

$$
\begin{aligned}
& \text { a2 the geographical coordinates (WGS84) of the other end of a great circle arc: } \\
& \text { a vector of longitude, latitude (in decimal degrees) and eventually altitude (in } \\
& \text { meters) }
\end{aligned}
$$

## Value

the surface along-track distances from 'b's cross-track intersection to 'a1' - 'a2'

## See Also

Other utilities: altitude_azimuth_distance(), cross_track_distance(), cross_track_intersection()

## Examples

```
## Not run:
b <- c(8.086135, 49.973942, 6401)
# EDDF: 07R (longitude, latitude, altitude)
a1 <- c(8.53417, 50.0275, 328)
# EDDF: 25L
a2 <- c(8.58653, 50.0401, 362)
along_track_distance(b, a1, a2)
## End(Not run)
```

```
altitude_azimuth_distance
```

Calculate the altitude, azimuth and distance of $B$ from $A$

## Description

The altitude (elevation from the horizon), azimuth and distance of a point B from A are the coordinates of the Topocentric Coordinate System as typically used in astronomy to aim your telescope to a heavenly body. It can be also of use to know where an airplane is in the sky with respect to an observer on Earth.

## Usage

altitude_azimuth_distance(a, b)

## Arguments

a
the observer position: a vector of longitude, latitude (in decimal degrees) and altitude (in meters) in WGS84
b
the observed position: a vector of longitude, latitude (in decimal degrees) and altitude (in meters) in WGS84

## Value

the coordinates in North-East-Up of the observed, B, with respect to the observer A. A vector of altitude (elevation from the horizon) in decimal degrees, azimuth) in decimal degrees and distance in meters.

## See Also

Other utilities: along_track_distance(), cross_track_distance(), cross_track_intersection()

## Examples

```
## Not run:
# sensor (longitude, latitude, altitude)
a <- c(49.47, 7.697, 274)
# aircraft (longitude, latitude, altitude)
b <- c(49.52, 7.803, 6401)
altitude_azimuth_distance(a, b)
## End(Not run)
```

cross_track_distance Compute the cross-track distance from a great circle arc

## Description

Compute the cross-track distance of a body, ' $b$ ' (for example a ground level projection position of an aircraft), from a great circle arc determined by two geographical coordinates, 'a1' and 'a2' (for example an airport's runway thresholds).

## Usage

cross_track_distance(b, a1, a2)

## Arguments

b
the geographical coordinates (WGS84) of a body: a vector of longitude, latitude (in decimal degrees) and eventually altitude (in meters)
a1 the geographical coordinates (WGS84) of one end of a great circle arc: a vector of longitude, latitude (in decimal degrees) and eventually altitude (in meters)
a2 the geographical coordinates (WGS84) of the other end of a great circle arc: a vector of longitude, latitude (in decimal degrees) and eventually altitude (in meters)

## Value

the surface cross-track distance from 'b' to the arc 'a1' - ' $a$ '

## See Also

Other utilities: along_track_distance(), altitude_azimuth_distance(), cross_track_intersection()

## Examples

```
    ## Not run:
    b <- c(8.086135, 49.973942, 6401)
    # EDDF: 07R (longitude, latitude, altitude)
    a1 <- c(8.53417, 50.0275, 328)
    # EDDF: 25L
    a2 <- c(8.58653, 50.0401, 362)
    cross_track_distance(b, a1, a2)
    ## End(Not run)
```

cross_track_intersection
Calculate cross-track intersection

## Description

Calculate the cross-track intersection between the position of a body (i.e. an aircraft) and a great circle arc as defined by two points (i.e. the runway's thresholds).

## Usage

cross_track_intersection(b, a1, a2)

## Arguments

b
coordinates of the body, b : a vector of longitude, latitude (in decimal degrees) and altitude (in meters) in WGS84
a1 first coordinate of a great circle arc: a vector of longitude, latitude (in decimal degrees) and elevation (in meters) in WGS84
a2 second coordinate of a great circle arc: a vector of longitude, latitude (in decimal degrees) and elevation (in meters) in WGS84

## Details

The cross-track intersection between the position of a body, B, (i.e. an aircraft) and a great circle arc as defined by two points, A1 and A2, (i.e. the runway's thresholds) is the intersection, X , of the above arc with the great circle arc passing through the ground projection of $\mathrm{B}, \mathrm{G}$, and perpendicular to A1-A2.


## Value

a WGS84 vector with longitude and latitude (decimal degrees)

## See Also

Other utilities: along_track_distance(), altitude_azimuth_distance(), cross_track_distance()

## Examples

```
## Not run:
# aircraft (longitude, latitude, altitude)
b <- c(8.086135, 49.973942, 6401)
# EDDF: 07R (longitude, latitude, altitude)
a1 <- c(8.53417, 50.0275, 328)
# EDDF: 25L
a2 <- c(8.58653, 50.0401, 362)
cross_track_intersection(b, a1, a2)
## End(Not run)
```

deg

## Description

Convert angle in radians to degrees

## Usage

```
deg(radians)
```


## Arguments

radians angle in radians.

## Value

angle in degrees.

## See Also

rad.
Other helpers: $\operatorname{rad}()$, unit ()

## Examples

```
deg(pi/2)
```

lat_lon2n_E Convert (geodetic) latitude and longitude to n-vector

## Description

Convert (geodetic) latitude and longitude to n-vector

## Usage

lat_lon2n_E(latitude, longitude)

## Arguments

| latitude | Geodetic latitude (rad) |
| :--- | :--- |
| longitude | Geodetic longitude (rad) |

## Value

n-vector decomposed in E (3x1 vector) (no unit)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

n_E2lat_lon.

## Examples

lat_lon2n_E(rad(1), rad(2))
nvctr $\quad n v c t r:$ non-singular geographical position calculations

## Description

nvctr provides functions to calculate geographical positions for both the ellipsoidal and spherical Earth models.

## Author(s)

Maintainer: Enrico Spinielli [enrico.spinielli@eurocontrol.int](mailto:enrico.spinielli@eurocontrol.int) (ORCID)
Other contributors:

- EUROCONTROL [copyright holder, funder]


## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

See Also
Useful links:

- https://github.com/euctrl-pru/nvctr
- Report bugs at https://github.com/euctrl-pru/nvctr/issues


## Description

Convert n -vector to latitude and longitude

## Usage

n_E2lat_lon(n_E)

## Arguments

n_E $\quad n$-vector decomposed in $E(3 x 1$ vector) (no unit)

## Value

A vector of geodetic latitude and longitude (rad)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

lat_lon2n_E.

## Examples

```
n_E2lat_lon(c(1, 0, 0))
```

n_E2R_EN Find the rotation matrix $R_{-} E N$ from $n$-vector

## Description

Find the rotation matrix R_EN from n-vector

## Usage

n_E2R_EN(n_E)

## Arguments

n_E
n -vector decomposed in E (3x1 vector) (no unit)

## Value

The resulting rotation matrix (direction cosine matrix) (no unit)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

R_EN2n_E, n_E_and_wa2R_EL and R_EL2n_E.

## Examples

$$
\mathrm{n} \_E 2 R \_\operatorname{EN}(\mathrm{c}(1,0,0))
$$

```
n_EA_E_and_n_EB_E2p_AB_E
```

Find the delta position from two positions $A$ and $B$

## Description

Given the n-vectors for positions $A\left(n \_E A \_E\right)$ and $B\left(n \_E B \_E\right)$, the output is the delta vector from $A$ to $B$ ( $p \_A B \_E$ ).

## Usage

```
n_EA_E_and_n_EB_E2p_AB_E(
    n_EA_E,
    n_EB_E,
    \(z_{-} E A=0\),
    \(z_{\text {_ }} \mathrm{EB}=0\),
    \(a=6378137\),
    \(f=1 / 298.257223563\)
)
```


## Arguments

| n_EA_E | n-vector of position A, decomposed in E (3x1 vector) (no unit) |
| :--- | :--- |
| n_EB_E | n-vector of position B, decomposed in E $(3 \times 1$ vector) (no unit) |
| z_EA | Depth of system A, relative to the ellipsoid (z_EA = -height) (m, default 0) |
| z_EB | Depth of system B, relative to the ellipsoid (z_EB = -height) (m, default 0$)$ |
| a | Semi-major axis of the Earth ellipsoid (m, default [WGS-84] 6378137) |
| $f$ | Flattening of the Earth ellipsoid (no unit, default [WGS-84] 1/298.257223563) |

## Details

The calculation is exact, taking the ellipticity of the Earth into account. It is also nonsingular as both n-vector and p-vector are nonsingular (except for the center of the Earth). The default ellipsoid model used is WGS-84, but other ellipsoids (or spheres) might be specified via the optional parameters a and f.

## Value

Position vector from $A$ to $B$, decomposed in $E$ ( $3 \times 1$ vector)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

n_EA_E_and_p_AB_E2n_EB_E, p_EB_E2n_EB_E and n_EB_E2p_EB_E

## Examples

```
lat_EA <- rad(1); lon_EA <- rad(2); z_EA <- 3
lat_EB <- rad(4); lon_EB <- rad(5); z_EB <- 6
n_EA_E <- lat_lon2n_E(lat_EA, lon_EA)
n_EB_E <- lat_lon2n_E(lat_EB, lon_EB)
n_EA_E_and_n_EB_E2p_AB_E(n_EA_E, n_EB_E, z_EA, z_EB)
```

```
n_EA_E_and_p_AB_E2n_EB_E
```

Find position B from position $A$ and delta

## Description

Given the $n$-vector for position $A$ ( $n \_E A \_E$ ) and the position-vector from position $A$ to position $B$ ( $p \_A B \_E$ ), the output is the n-vector of position $B\left(n_{\_} E B \_E\right)$ and depth of $B\left(z_{-} E B\right)$.

## Usage

```
n_EA_E_and_p_AB_E2n_EB_E(
        n_EA_E,
        p_AB_E,
        z_EA = 0,
        a = 6378137,
        f = 1/298.257223563
)
```


## Arguments

n_EA_E $\quad$-vector of position A, decomposed in E ( $3 \times 1$ vector) (no unit)
p_AB_E Position vector from A to $B$, decomposed in E (3x1 vector) (m)
z_EA Depth of system A, relative to the ellipsoid (z_EA = -height) (m, default 0)
a Semi-major axis of the Earth ellipsoid (m, default [WGS-84] 6378137)
$f \quad$ Flattening of the Earth ellipsoid (no unit, default [WGS-84] 1/298.257223563)

## Details

The calculation is exact, taking the ellipticity of the Earth into account.
It is also nonsingular as both $n$-vector and p -vector are nonsingular (except for the center of the Earth). The default ellipsoid model used is WGS-84, but other ellipsoids (or spheres) might be specified.

## Value

a list with n-vector of position $B$, decomposed in $E$ ( $3 \times 1$ vector) (no unit) and the depth of system $B$, relative to the ellipsoid ( $z_{-} E B=-$ height $)$

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

n_EA_E_and_n_EB_E2p_AB_E, p_EB_E2n_EB_E and n_EB_E2p_EB_E

## Examples

```
p_BC_B <- c(3000, 2000, 100)
# Position and orientation of B is given:
n_EB_E <- unit(c(1,2,3)) # unit to get unit length of vector
z_EB <- -400
R_NB <- zyx2R(rad(10), rad(20), rad(30)) # yaw, pitch, and roll
R_EN <- n_E2R_EN(n_EB_E)
R_EB <- R_EN %*% R_NB
# Decompose the delta vector in E:
p_BC_E <- (R_EB %*% p_BC_B) %>% as.vector() # no transpose of R_EB, since the vector is in B
# Find the position of C, using the functions that goes from one
# position and a delta, to a new position:
(n_EB_E <- n_EA_E_and_p_AB_E2n_EB_E(n_EB_E, p_BC_E, z_EB))
```

n_EB_E2p_EB_E Convert n-vector to cartesian position vector in meters

## Description

The function converts the position of $B$ (typically body) relative to $E$ (typically Earth), the n-vector n_EB_E to cartesian position vector ("ECEF-vector"), p_EB_E, in meters.

## Usage

n_EB_E2p_EB_E(n_EB_E, z_EB = 0, a = 6378137, f = 1/298.257223563)

## Arguments

n_EB_E $\quad n$-vector of position B, decomposed in E ( $3 \times 1$ vector) (no unit)
z_EB Depth of system B, relative to the ellipsoid (z_EB = -height) (m, default 0)
a Semi-major axis of the Earth ellipsoid (m, default [WGS-84] 6378137)
$f \quad$ Flattening of the Earth ellipsoid (no unit, default [WGS-84] 1/298.257223563)

## Details

The calculation is exact, taking the ellipticity of the Earth into account.
It is also nonsingular as both $n$-vector and p-vector are nonsingular (except for the center of the Earth). The default ellipsoid model used is WGS-84, but other ellipsoids (or spheres) might be specified via the optional parameters a and $f$.

## Value

Cartesian position vector from $E$ to $B$, decomposed in $E$ (3x1 vector) (m)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

p_EB_E2n_EB_E, n_EA_E_and_p_AB_E2n_EB_E and n_EA_E_and_n_EB_E2p_AB_E.

## Examples

```
n_EB_E <- lat_lon2n_E(rad(1), rad(2))
n_EB_E2p_EB_E(n_EB_E)
```

    n_E_and_wa2R_EL Find R_EL from n-vector and wander azimuth angle
    
## Description

Calculate the rotation matrix (direction cosine matrix) R_EL using n-vector ( $n \_E$ ) and the wander azimuth angle. When wander_azimuth $=0$, we have that $\mathrm{N}=\mathrm{L}$ (See Table 2 in Gade (2010) for details)

## Usage

n_E_and_wa2R_EL(n_E, wander_azimuth)

## Arguments

n_E $\quad n$-vector decomposed in E ( $3 \times 1$ vector) (no unit)
wander_azimuth The angle between L's x-axis and north, positive about L's z-axis (rad)

## Value

The resulting rotation matrix (3x3) (no unit)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

R_EL2n_E, R_EN2n_E and n_E2R_EN.

## Examples

\# Calculates the rotation matrix (direction cosine matrix) R_EL
\# using n-vector ( $\mathrm{n} \_\mathrm{E}$ ) and the wander azimuth angle.
n_E <- c(1, 0, 0)
(R_EL <- n_E_and_wa2R_EL(n_E, wander_azimuth = pi / 2))
p_EB_E2n_EB_E Convert cartesian position vector in meters to $n$-vector

## Description

The position of $B$ (typically body) relative to $E$ (typically Earth) is given as cartesian position vector p_EB_E, in meters ("ECEF-vector").

## Usage

p_EB_E2n_EB_E(p_EB_E, $a=6378137, f=1 / 298.257223563$ )

## Arguments

$$
\begin{array}{ll}
\text { P_EB_E } & \text { Cartesian position vector from } \mathrm{E} \text { to } \mathrm{B}, \text { decomposed in E }(3 \times 1 \text { vector) (m) } \\
\mathrm{a} & \text { Semi-major axis of the Earth ellipsoid (m, default [WGS-84] 6378137) } \\
\mathrm{f} & \text { Flattening of the Earth ellipsoid (no unit, default [WGS-84] 1/298.257223563) }
\end{array}
$$

## Details

The function converts to n-vector, n_EB_E and its depth, $z_{-}$EB.
The calculation is exact, taking the ellipticity of the Earth into account. It is also nonsingular as both n-vector and p-vector are nonsingular (except for the center of the Earth). The default ellipsoid model used is WGS-84, but other ellipsoids (or spheres) might be specified.

## Value

n -vector representation of position B , decomposed in E ( $3 \times 1$ vector) (no unit) and depth of system $B$ relative to the ellipsoid ( $\mathrm{z} \_\mathrm{EB}=-$ height )

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

$n \_E B \_E 2 p \_E B \_E, n_{-} E A \_E \_a n d \_p \_A B \_E 2 n \_E B \_E$ and $n \_E A \_E \_a n d \_n \_E B \_E 2 p \_A B \_E$

## Examples

```
p_EB_E <- 6371e3 * c(0.9, -1, 1.1)
(n_EB_E <- p_EB_E2n_EB_E(p_EB_E))
```

R2xyz Find the three rotation angles about new axes in the xyz order from a rotation matrix

## Description

The angles (called Euler angles or Tait-Bryan angles) are defined by the following procedure of successive rotations: Given two arbitrary coordinate frames A and B, consider a temporary frame $T$ that initially coincides with $A$. In order to make $T$ align with $B$, we first rotate $T$ an angle $x$ about its x -axis (common axis for both A and T). Secondly, T is rotated an angle y about the NEW y-axis of T. Finally, T is rotated an angle z about its NEWEST z-axis. The final orientation of T now coincides with the orientation of B . The signs of the angles are given by the directions of the axes and the right hand rule.

## Usage

R2xyz(R_AB)

## Arguments

R_AB a $3 \times 3$ rotation matrix (direction cosine matrix) such that the relation between a vector v decomposed in A and B is given by: $\mathrm{v} \_A=R_{\_} A B * \mathrm{v}_{-} B$

## Value

$x, y, z$ Angles of rotation about new axes (rad)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

$x y z 2 R, R 2 z y x$ and $z y x 2 R$.

## Examples

```
    R_AB <- matrix(
        c( 0.9980212 , 0.05230407, -0.0348995,
            -0.05293623, 0.99844556, -0.01744177,
            0.03393297, 0.01925471, 0.99923861),
        nrow = 3, ncol = 3, byrow = TRUE)
R2xyz(R_AB)
```

R2zyx Find the three angles about new axes in the zyx order from a rotation matrix

## Description

The 3 angles $z, y, x$ about new axes (intrinsic) in the order $z-y-x$ are found from the rotation matrix R_AB. The angles (called Euler angles or Tait-Bryan angles) are defined by the following procedure of successive rotations:

1. Given two arbitrary coordinate frames A and B , consider a temporary frame T that initially coincides with $A$. In order to make $T$ align with $B$, we first rotate $T$ an angle $z$ about its $z$-axis (common axis for both A and T).
2. Secondly, T is rotated an angle $y$ about the NEW $y$-axis of T. Finally, T is rotated an angle $x$ about its NEWEST x-axis.
3. The final orientation of $T$ now coincides with the orientation of $B$.

The signs of the angles are given by the directions of the axes and the right hand rule.

## Usage

R2zyx (R_AB)

## Arguments

R_AB a $3 x 3$ rotation matrix (direction cosine matrix) such that the relation between a vector $v$ decomposed in $A$ and $B$ is given by: $v_{-} A=R_{-} A B * v_{-} B$

## Details

Note that if A is a north-east-down frame and B is a body frame, we have that $z=y a w, y=p i t c h$ and $\mathrm{x}=$ roll.

## Value

$\mathrm{z}, \mathrm{y}, \mathrm{x}$ angles of rotation about new axes (rad)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

$z y x 2 R, x y z 2 R$ and R2xyz.

## Examples

```
zyx2R(rad(1), rad(-2), rad(-3))
```

rad Convert angle in degrees to radians.

## Description

Convert angle in degrees to radians.

## Usage

rad(degrees)

## Arguments

degrees angle in degrees.

## Value

angle in radians

## See Also

deg.
Other helpers: deg(), unit ()

## Examples

rad(30)
R_Ee Select the axes of the coordinate frame E

## Description

This function returns the axes of the coordinate frame E (Earth-Centered, Earth-Fixed, ECEF).

## Usage

R_Ee(axes = "e")

## Arguments

- 'e': z-axis points to the North Pole along the Earth's rotation axis, x-axis points towards the point where latitude $=$ longitude $=0$. This choice is very common in many fields.
- 'E': x-axis points to the North Pole along the Earth's rotation axis, y-axis points towards longitude +90 deg (east) and latitude $=0$. (the yz-plane coincides with the equatorial plane). This choice of axis ensures that at zero latitude and longitude, frame N (North-East-Down) has the same orientation as frame E. If roll/pitch/yaw are zero, also frame B (forward-starboarddown) has this orientation. In this manner, the axes of frame $E$ is chosen to correspond with the axes of frame N and B . The functions in this library originally used this option.


## Details

There are two choices of E-axes that are described in Table 2 in Gade (2010):

- e: z-axis points to the North Pole and x -axis points to the point where latitude $=$ longitude $=$ 0 . This choice is very common in many fields.
- E: x-axis points to the North Pole, y-axis points towards longitude +90 deg (east) and latitude $=0$. This choice of axis directions ensures that at zero latitude and longitude, N (North-East-Down) has the same orientation as E. If roll/pitch/yaw are zero, also B (Body, forward, starboard, down) has this orientation. In this manner, the axes of $E$ is chosen to correspond with the axes of N and B .


## Value

rotation matrix defining the axes of the coordinate frame E as described in Table 2 in Gade (2010)

## References

Kenneth Gade (2010) A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## Examples

R_Ee()

## Description

Find n-vector from the rotation matrix (direction cosine matrix) R_EL

## Usage

R_EL2n_E(R_EL)

## Arguments

R_EL Rotation matrix (direction cosine matrix) (no unit)

## Value

n-vector decomposed in E (3x1 vector) (no unit)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

n_E2R_EN, R_EL2n_E and n_E_and_wa2R_EL.

## Examples

```
R_EL <- matrix(
        c(-1, 0, 0,
            0, 1, 0,
            0, 0, -1),
    nrow = 3, ncol = 3, byrow = TRUE)
R_EL2n_E(R_EL)
```

R_EN2n_E Find n-vector from $R_{-} E$

## Description

Find n-vector from R_E

## Usage

R_EN2n_E(R_EN)

## Arguments

R_EN Rotation matrix (direction cosine matrix) (no unit)

## Value

n-vector decomposed in E (3x1 vector) (no unit)

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

n_E2R_EN, R_EL2n_E and n_E_and_wa2R_EL.

## Examples

```
R_EN <- matrix(
    c(-1, 0, 0,
            0, 1, 0,
            0, 0, -1),
    nrow = 3, ncol = 3, byrow = TRUE)
R_EL2n_E(R_EN)
```

unit
Make input vector unit length, i.e. norm $==1$

## Description

Make input vector unit length, i.e. norm $==1$

## Usage

```
    unit(vector)
```


## Arguments

vector
a vector

## Value

a unit length vector

## See Also

Other helpers: $\operatorname{deg}(), \operatorname{rad}()$

## Examples

unit( $c(1,2,3))$
$x y z 2 R$
Create a rotation matrix from 3 angles about new axes in the xyz order.

## Description

The rotation matrix $R \_A B$ is created based on 3 angles $x, y$ and $z$ about new axes (intrinsic) in the order x-y-z. The angles (called Euler angles or Tait-Bryan angles) are defined by the following procedure of successive rotations:

1. Given two arbitrary coordinate frames $A$ and $B$, consider a temporary frame $T$ that initially coincides with $A$. In order to make $T$ align with $B$, we first rotate $T$ an angle $x$ about its $x$-axis (common axis for both $A$ and $T$ ).
2. Secondly, $T$ is rotated an angle $y$ about the NEW $y$-axis of $T$.
3. Finally, codeT is rotated an angle $z$ about its NEWEST $z$-axis. The final orientation of $T$ now coincides with the orientation of $B$.

The signs of the angles are given by the directions of the axes and the right hand rule.

## Usage

$x y z 2 R(x, y, z)$

## Arguments

| $x$ | Angle of rotation about new $x$ axis (rad) |
| :--- | :--- |
| $y$ | Angle of rotation about new $y$ axis (rad) |
| $z$ | Angle of rotation about new $z$ axis (rad) |

## Value

$3 \times 3$ rotation matrix (direction cosine matrix) such that the relation between a vector v decomposed in $A$ and $B$ is given by: $v_{-} A=R_{-} A B * v_{-} B$

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

R2xyz, zyx2R and R2zyx.

## Examples

```
xyz2R(rad(10), rad(20), rad(30))
```


## Description

The rotation matrix $R_{-} A B$ is created based on 3 angles $z, y$ and $x$ about new axes (intrinsic) in the order z-y-x. The angles (called Euler angles or Tait-Bryan angles) are defined by the following procedure of successive rotations:

1. Given two arbitrary coordinate frames A and B , consider a temporary frame T that initially coincides with $A$. In order to make $T$ align with $B$, we first rotate $T$ an angle $z$ about its $z$-axis (common axis for both A and T ).
2. Secondly, T is rotated an angle $y$ about the NEW $y$-axis of T.
3. Finally, $T$ is rotated an angle $x$ about its NEWEST $x$-axis. The final orientation of $T$ now coincides with the orientation of B.

The signs of the angles are given by the directions of the axes and the right hand rule. Note that if $A$ is a north-east-down frame and $B$ is a body frame, we have that $z=y a w, y=p i t c h$ and $x=$ roll.

## Usage

$z y x 2 R(z, y, x)$

## Arguments

| $z$ | Angle of rotation about new $z$ axis |
| :--- | :--- |
| $y$ | Angle of rotation about new $y$ axis |
| $x$ | Angle of rotation about new $x$ axis |

## Value

$3 x 3$ rotation matrix $R \_A B$ (direction cosine matrix) such that the relation between a vector $v$ decomposed in A and B is given by: $\mathrm{v} \_A=\mathrm{R}_{-} A B * \mathrm{v}_{-} B$

## References

Kenneth Gade A Nonsingular Horizontal Position Representation. The Journal of Navigation, Volume 63, Issue 03, pp 395-417, July 2010.

## See Also

R2zyx, xyz2R and R2xyz.

## Examples

$$
\operatorname{zyx} 2 R(\operatorname{rad}(30), \operatorname{rad}(20), \operatorname{rad}(10))
$$

## Index

```
* helpers
    deg, }
    rad, 17
    unit,20
* utilities
    along_track_distance, 2
    altitude_azimuth_distance, 3
    cross_track_distance, 4
    cross_track_intersection, 5
along_track_distance, 2, 4-6
altitude_azimuth_distance, 3, 3, 5, 6
cross_track_distance, 3, 4, 4, 6
cross_track_intersection, 3-5,5
deg, 6, 17, 20
lat_lon2n_E, 7, 8
n_E2lat_lon, 7, 8
n_E2R_EN, 9, 13, 19, 20
n_E_and_wa2R_EL, 9, 13, 19, 20
n_EA_E_and_n_EB_E2p_AB_E, 9, 11, 13, 14
n_EA_E_and_p_AB_E2n_EB_E, 10,11, 13, 14
n_EB_E2p_EB_E, 10, 11, 12, 14
nvctr,7
nvctr-package (nvctr), 7
p_EB_E2n_EB_E, 10, 11, 13,14
R2xyz, 15, 16, 21, 22
R2zyx, 15, 16, 21, 22
R_Ee,17
R_EL2n_E, 9, 13,18,19, 20
R_EN2n_E, 9, 13,19
rad, 6, 17, 20
unit, 6, 17, 20
xyz2R, 15, 16, 21, 22
zyx2R, 15, 16, 21, 22
```

