## Package 'pcsstools'

March 23, 2021
Type Package
Title Tools for Regression Using Pre-Computed Summary Statistics
Version 0.1.1
Description Defines functions to describe regression models using only
pre-computed summary statistics (i.e. means, variances, and covariances) in place of individual participant data.
Possible models include linear models for linear combinations, products, and logical combinations of phenotypes.
Implements methods presented in
Wolf et al. (2021) [doi:10.1101/2021.03.08.433979](doi:10.1101/2021.03.08.433979)
Wolf et al. (2020) <doi: $10.1142 / 9789811215636 \_0063>$ and
Gasdaska et al. (2019) [doi:10.1142/9789813279827_0036](doi:10.1142/9789813279827_0036).
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anova.pcsslm ANOVA for linear models fit using PCSS

## Description

Compute an analysis of variance table for one or more linear model fitted using PCSS.

## Usage

```
## S3 method for class 'pcsslm'
anova(object, ...)
    ## S3 method for class 'pcsslmlist'
    anova(object, ..., scale = 0, test = "F")
```


## Arguments

object, ..
objects of class pcsslm.
scale numeric. An estimate of the noise variance $\sigma^{2}$. If zero this will be estimated from the largest model considered.
test a character string specifying the test statistic to be used. Can be one of "F", "Chisq" or "Cp", with partial matching allowed, or NULL for no test.

## Value

An object of class "anova" inheriting from class "data.frame".

## Author(s)

R Core Team and contributors worldwide. Modified by Jack Wolf

```
approx_and Approximate a linear model for a series of logical AND statements
```


## Description

approx_and approximates the linear model for the a conjunction of $m$ phenotypes as a function of a set of predictors.

## Usage

approx_and(
means,
covs,
n ,
predictors, add_intercept = TRUE, verbose = FALSE, response_assumption = "binary", )

## Arguments

means vector of predictor and response means with the last means being the means of $m$ binary responses to combine in a logical and statement.
covs a matrix of the covariance of all model predictors and the responses with the order of rows/columns corresponding to the order of means.
n
sample size.
predictors list of objects of class predictor corresponding to the order of the predictors in means.

```
add_intercept logical. Should the linear model add an intercept term?
verbose should output be printed to console?
response_assumption
    character. Either "binary" or "continuous". If "binary", specific calcula-
    tions will be done to estimate product means and variances.
... additional arguments
```


## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:
call the matched call
terms the terms object used
coefficients a $p x 4$ matrix with columns for the estimated coefficient, its standard error, t statistic and corresponding (two-sided) p-value.
sigma the square root of the estimated variance of the random error.
df degrees of freedom, a 3-vector $p, n-p, p *$, the first being the number of nonaliased coefficients, the last being the total number of coefficients.
fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom.
$r$. squared $\quad R^{2}$, the 'fraction of variance explained by the model'.
adj.r.squared the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$.
cov.unscaled a $p x p$ matrix of (unscaled) covariances of the $\operatorname{coe} f[j], j=1, \ldots p$.
Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares Error (SSE), and Sum of Squares Total (SST).

## References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10. 1101/2021.03.08.433979v1.

## Description

Approximate the mean of Y conditional on X

## Usage

approx_conditional(means, covs, response, n)

## Arguments

| means | Vector of the mean of X and the mean of $Y$ |
| :--- | :--- |
| covs | Matrix of covariances for X and Y |
| response | Character. If "binary" truncates means to interval [0, 1]. If "continuous" does <br> not restrict. |
| n | Sample size |

## Value

A list of length 2 consisting of 2 functions that give the estimated conditional mean and conditional variance of Y as a function of X

```
approx_mult_prod Approximate the covariance of a set of predictors and a product of
    responses
```


## Description

approx_mult_prod recursively estimates the covariances and means of a set of responses. Estimates are approximated using all unique response orderings and aggregated.

## Usage

approx_mult_prod(
means,
covs,
n,
response,
predictors,
responses, verbose = FALSE
)

## Arguments

| means | a vector of predictor and response means with all response means at the end of <br> the vector. |
| :--- | :--- |
| covs |  |
| covariance matrix of all predictors and responses with column and row order |  |
| corresponding to the order of means. |  |

## Value

A list containing the following elements:
means a vector of the (approximated) means of all predictors and the product of responses
covs a matrix of (approximated) covariances between all predictors and the product of responses

## References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10. 1101/2021.03.08.433979v1.

```
approx_or
```

Approximate a linear model for a series of logical OR statements

## Description

approx_or approximates the linear model for a disjunction of $m$ phenotypes as a function of a set of predictors.

## Usage

```
approx_or(
    means,
    covs,
    n,
    predictors,
    add_intercept = TRUE,
    verbose = FALSE,
    response_assumption = "binary",
)
```


## Arguments

```
means vector of predictor and response means with the last m means being the means of \(m\) binary responses to combine in a logical OR statement.
covs a matrix of the covariance of all model predictors and the responses with the order of rows/columns corresponding to the order of means.
n
sample size.
predictors list of objects of class predictor corresponding to the order of the predictors in means.
add_intercept logical. Should the linear model add an intercept term?
```

```
verbose should output be printed to console?
response_assumption
    character. Either "binary" or "continuous". If "binary", specific calcula-
    tions will be done to estimate product means and variances.
... additional arguments
```

Value
an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:
call the matched call
terms the terms object used
coefficients a $p x 4$ matrix with columns for the estimated coefficient, its standard error, t statistic and corresponding (two-sided) p-value.
sigma the square root of the estimated variance of the random error.
df degrees of freedom, a 3-vector $p, n-p, p *$, the first being the number of nonaliased coefficients, the last being the total number of coefficients.
fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom.
$r$.squared $\quad R^{2}$, the 'fraction of variance explained by the model'.
adj.r.squared the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$.
cov.unscaled a $p x p$ matrix of (unscaled) covariances of the $\operatorname{coef}[j], j=1, \ldots p$.
Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares Error (SSE), and Sum of Squares Total (SST).

## References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10. 1101/2021.03.08.433979v1.

$$
\begin{aligned}
& \text { approx_prod_stats } \quad \begin{array}{l}
\text { Approximate summary statistics for a product of phenotypes and a set } \\
\text { of predictors }
\end{array}
\end{aligned}
$$

## Description

Approximate summary statistics for a product of phenotypes and a set of predictors

## Usage

approx_prod_stats(means, covs, n, response, predictors)

## Arguments

| means | Vector of means of predictors and the two phenotypes to be multiplied |
| :--- | :--- |
| covs | Covariance matrix of all predictors and the two phenotypes |
| n | Sample size |
| response | character. Either "binary" or "continuous". |
| predictors | a list of elements of class predictor |

## Value

A list with the predicted covariance matrix of all predictors and the product and the means of all predictors and the product.

```
approx_response_cov_recursive
    Approximate the covariance of one response with an arbitrary product
    of responses.
```


## Description

Approximate the covariance of one response with an arbitrary product of responses.

## Usage

```
    approx_response_cov_recursive(
```

        ids,
        r_covs,
        r_means,
        n ,
        responses,
        response,
        verbose = FALSE
    )

## Arguments

| ids | Column ids of responses to use. First is taken alone while 2nd to last are to be <br> multiplied |
| :--- | :--- |
| $r_{-}$covs | Response covariance matrix |
| $r_{\text {_means }}$ | Response means (vector) |
| n | Sample size |
| responses | List of lists with elements of class predictor |
| response | Character, Either "binary" or "continuous" |
| verbose | logical |

## Value

A vector with the approximated covariance, and approximated mean and variance of the product

```
calculate_lm Calculate a linear model using PCSS
```


## Description

calculate_lm describes the linear model of the last listed variable in means and covs as a function of all other variables in means and covs.

## Usage

```
    calculate_lm(
        means,
        covs,
        n,
        add_intercept = FALSE,
        keep_pcss = FALSE,
        terms = NULL
    )
```


## Arguments

means a vector of means of all model predictors and the response with the last element the response mean.
covs a matrix of the covariance of all model predictors and the response with the order of rows/columns corresponding to the order of means.
n
sample size
add_intercept logical. If TRUE adds an intercept to the model.
keep_pcss logical. If TRUE, returns means and covs.
terms terms

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:
call the matched call
terms the terms object used
coefficients a $p x 4$ matrix with columns for the estimated coefficient, its standard error, tstatistic and corresponding (two-sided) p-value.
sigma the square root of the estimated variance of the random error.
df degrees of freedom, a 3 -vector $p, n-p, p *$, the first being the number of nonaliased coefficients, the last being the total number of coefficients.
fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom.
$r$.squared $\quad R^{2}$, the 'fraction of variance explained by the model'.
adj.r.squared the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$.
cov.unscaled a $p x p$ matrix of (unscaled) covariances of the $\operatorname{coef}[j], j=1, \ldots p$.
Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares Error (SSE), and Sum of Squares Total (SST).

## References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10. 1101/2021.03.08.433979v1.
Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." Pacific Symposium on Biocomputing, 25, 719-730. ISSN 23356928, doi: 10.1142/9789811215636_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

Gasdaska A, Friend D, Chen R, Westra J, Zawistowski M, Lindsey W, Tintle N (2019). "Leveraging summary statistics to make inferences about complex phenotypes in large biobanks." Pacific Symposium on Biocomputing, 24, 391-402. ISSN 2335-6928, doi: 10.1142/9789813279827_0036, https://pubmed.ncbi.nlm.nih.gov/30963077/.
calculate_lm_combo Calculate a linear model for a linear combination of responses

## Description

calculate_lm_combo describes the linear model for a linear combination of responses as a function of a set of predictors.

## Usage

calculate_lm_combo(means, covs, n, phi, m = length(phi), add_intercept, ...)

## Arguments

means a vector of means of all model predictors and the response with the last m elements the response means (with order corresponding to the order of weights in phi).
covs a matrix of the covariance of all model predictors and the responses with the order of rows/columns corresponding to the order of means.
n sample size.

| phi | vector of linear combination weights with one entry per response variable. |
| :--- | :--- |
| $m$ | number of responses to combine. Defaults to length(weighs). |
| add_intercept | logical. If TRUE adds an intercept to the model. |
| $\ldots$ | additional arguments |

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:

| call | the matched call |
| :--- | :--- |
| terms | the terms object used |
| coefficients | a $p x 4$ matrix with columns for the estimated coefficient, its standard error, t- <br> statistic and corresponding (two-sided) p-value. |
| sigma | the square root of the estimated variance of the random error. <br> degrees of freedom, a 3-vector $p, n-p, p *$, the first being the number of non- <br> aliased coefficients, the last being the total number of coefficients. |
| df | a 3-vector with the value of the F-statistic with its numerator and denominator <br> degrees of freedom. |
| r.squared | $R^{2}$, the 'fraction of variance explained by the model'. |
| adj.r.squared | the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$. |
| cov. unscaled | a $p x p$ matrix of (unscaled) covariances of the coef $[j], j=1, \ldots p$. |
| Sum Sq | a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares <br> Error (SSE), and Sum of Squares Total (SST). |

## References

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." Pacific Symposium on Biocomputing, 25, 719-730. ISSN 23356928, doi: 10.1142/9789811215636_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

Gasdaska A, Friend D, Chen R, Westra J, Zawistowski M, Lindsey W, Tintle N (2019). "Leveraging summary statistics to make inferences about complex phenotypes in large biobanks." Pacific Symposium on Biocomputing, 24, 391-402. ISSN 2335-6928, doi: 10.1142/9789813279827_0036, https://pubmed.ncbi.nlm.nih.gov/30963077/.
check_terms $\quad$ Check that independent and dependent variables are accounted for through PCSS

## Description

Check that independent and dependent variables are accounted for through PCSS

## Usage

check_terms(xterms, yterms, pcssterms, pcsstype)

## Arguments

xterms, yterms character vector of model's independent variables or variables combined to the dependent variable
pcssterms character vector of variables with provided PCSS
pcsstype character describing the PCSS being checked. Either "means", "covs", "predictors", or "responses".

## Value

No return value, called for side effects

```
extract_predictors Extract independent variables from a formula
```


## Description

Extract independent variables from a formula

## Usage

extract_predictors(formula $=$ formula())

## Arguments

formula an object of class formula.

## Value

A list with a character vector of all predictors and a logical value indicating whether the model includes an intercept term.
extract_response Extract dependent variables from a formula as a string

## Description

Extract dependent variables from a formula as a string

## Usage

extract_response(formula $=$ formula())

## Arguments

formula an object of class formula.

## Value

a character vector of all responses

$$
\text { get_pcor } \quad \text { Approximate the partial correlation of } Y \text { and } Z \text { given } X
$$

## Description

Approximate the partial correlation of Y and Z given X

## Usage

get_pcor(covs, cors = cov2cor(covs))

## Arguments

| covs | Covariance matrix of $\mathrm{X}, \mathrm{Y}$, and Z. |
| :--- | :--- |
| cors | Correlation matrix of $\mathrm{X}, \mathrm{Y}$, and Z. |

## Value

Approximated partial correlation of the later two terms given the first

Guess the function that is applied to a set of responses

## Description

guess_response takes a character vector of the dependent variable from a formula object and identifies which function separates the individual variables that make up the response. It then returns the model_* function to model the appropriate response using PCSS.

```
Usage
    guess_response(response = character())
```


## Arguments

response character. Output of extract_response.

## Value

A character. Either "model_combo", "model_product", "model_or", "model_and", or "model_singular".

```
make_permutations List all permutations of a sequence of integers
```


## Description

Lists all permutations of $1,2, \ldots, \mathrm{~m}$ unique up to the first two elements

## Usage

make_permutations(m)

## Arguments

m number of elements to permute

## Value

A list of vectors of permutations of $1,2, \ldots, m$.

```
model_and
Approximate a linear model for a series of logical AND statements using PCSS
```


## Description

model_and approximates the linear model for the conjunction of $m$ phenotypes as a function of a set of predictors.

## Usage

model_and(formula, n, means, covs, predictors, ...)

## Arguments

| formula | an object of class formula whose dependent variable is a combination of vari- <br> ables and logical \& operators. All model terms must be accounted for in means <br> and covs. |
| :--- | :--- |
| n | sample size. |
| means | named vector of predictor and response means. |
| covs | named matrix of the covariance of all model predictors and the responses. |
| predictors | named list of objects of class predictor. |
| $\ldots$ | additional arguments |

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:
call the matched call
terms the terms object used
coefficients a $p x 4$ matrix with columns for the estimated coefficient, its standard error, t statistic and corresponding (two-sided) p-value.
sigma the square root of the estimated variance of the random error.
df degrees of freedom, a 3-vector $p, n-p, p *$, the first being the number of nonaliased coefficients, the last being the total number of coefficients.
fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom.
$r$.squared $\quad R^{2}$, the 'fraction of variance explained by the model'.
adj.r.squared the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$.
cov. unscaled a $p x p$ matrix of (unscaled) covariances of the $\operatorname{coef}[j], j=1, \ldots p$.
Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares Error (SSE), and Sum of Squares Total (SST).

## References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10. 1101/2021.03.08.433979v1.

## Examples

```
    ex_data <- pcsstools_example[c("g1", "x1", "y4", "y5")]
    head(ex_data)
    means <- colMeans(ex_data)
    covs <- cov(ex_data)
    n <- nrow(ex_data)
    predictors <- list(
        g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
        x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1))
    )
    model_and(
        y4 & y5 ~ g1 + x1,
        means = means, covs = covs, n = n, predictors = predictors
)
summary(lm(y4 & y5 ~ g1 + x1, data = ex_data))
```

model_combo Model a linear combination of a set of phenotypes using PCSS

## Description

model_combo calculates the linear model for a linear combination of phenotypes as a function of a set of predictors.

## Usage

model_combo(formula, phi, n, means, covs, ...)

## Arguments

| formula | an object of class formula whose dependent variable is a series of variables <br> joined by + operators. model_combo will treat a principal component score of <br> those variables as the actual dependent variable. All model terms must be ac- <br> counted for in means and covs. |
| :--- | :--- |
| phi | named vector of linear weights for each variable in the dependent variable in <br> formula. |
| n | sample size. |
| means | named vector of predictor and response means. |
| covs | named matrix of the covariance of all model predictors and the responses. |
| $\ldots$ | additional arguments |

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:
call the matched call
terms the terms object used
coefficients a $p x 4$ matrix with columns for the estimated coefficient, its standard error, t statistic and corresponding (two-sided) p-value.
sigma the square root of the estimated variance of the random error.
$\mathrm{df} \quad$ degrees of freedom, a 3 -vector $p, n-p, p *$, the first being the number of nonaliased coefficients, the last being the total number of coefficients.
fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom.
$r$.squared $\quad R^{2}$, the 'fraction of variance explained by the model'.
adj.r.squared the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$.
cov. unscaled a $p x p$ matrix of (unscaled) covariances of the coef $[j], j=1, \ldots p$.
Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares Error (SSE), and Sum of Squares Total (SST).

## References

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." Pacific Symposium on Biocomputing, 25, 719-730. ISSN 23356928, doi: 10.1142/9789811215636_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.
Gasdaska A, Friend D, Chen R, Westra J, Zawistowski M, Lindsey W, Tintle N (2019). "Leveraging summary statistics to make inferences about complex phenotypes in large biobanks." Pacific Symposium on Biocomputing, 24, 391-402. ISSN 2335-6928, doi: 10.1142/9789813279827_0036, https://pubmed.ncbi.nlm.nih.gov/30963077/.

## Examples

```
ex_data <- pcsstools_example[c("g1", "x1", "x2", "x3", "y1", "y2", "y3")]
head(ex_data)
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)
phi <- c("y1" = 1, "y2" = -1, "y3" = 0.5)
model_combo(
    y1 + y2 + y3 ~ g1 + x1 + x2 + x3,
    phi = phi, n = n, means = means, covs = covs
)
summary(lm(y1 - y2 + 0.5 * y3 ~ g1 + x1 + x2 + x3, data = ex_data))
```

```
model_or
```

Approximate a linear model for a series of logical OR statements using
PCSS

## Description

model_or approximates the linear model for the a disjunction of $m$ phenotypes as a function of a set of predictors.

## Usage

model_or(formula, n , means, covs, predictors, ...)

## Arguments

| formula | an object of class formula whose dependent variable is a combination of vari- <br> ables and logical \| operators. All model terms must be accounted for in means <br> and covs. |
| :--- | :--- |
| n | sample size. |
| means | named vector of predictor and response means. |
| covs | named matrix of the covariance of all model predictors and the responses. |
| predictors | named list of objects of class predictor. |
| $\ldots$ | additional arguments |

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:

| call <br> terms <br> coefficients | the matched call <br> the terms object used <br> a $p x 4$ matrix with columns for the estimated coefficient, its standard error, t - <br> statistic and corresponding (two-sided) p-value. |
| :--- | :--- |
| sigma | the square root of the estimated variance of the random error. <br> df <br> aliased coefficients, the last being the total number of coefficients. |
| fstatistic | a 3-vector with the value of the F-statistic with its numerator and denominator <br> degrees of freedom. |
| r.squared | $R^{2}$, the 'fraction of variance explained by the model'. |
| adj.r.squared | the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$. |
| cov.unscaled | a $p x p$ matrix of (unscaled) covariances of the coef $[j], j=1, \ldots p$. <br> a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares |
|  | Error (SSE), and Sum of Squares Total (SST). |

## References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10. 1101/2021.03.08.433979v1.

## Examples

```
    ex_data <- pcsstools_example[c("g1", "x1", "y4", "y5")]
    head(ex_data)
    means <- colMeans(ex_data)
    covs <- cov(ex_data)
    n <- nrow(ex_data)
    predictors <- list(
        g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
        x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1))
)
    model_or(
        y4 | y5 ~ g1 + x1,
        means = means, covs = covs, n = n, predictors = predictors
)
summary(lm(y4 | y5 ~ g1 + x1, data = ex_data))
```

model_prcomp Model the principal component score of a set of phenotypes using
PCSS

## Description

model_prcomp calculates the linear model for the mth principal component score of a set of phenotypes as a function of a set of predictors.

## Usage

model_prcomp( formula, comp $=1$,
n , means, covs, center = FALSE, standardize = FALSE,
)

## Arguments

| formula | an object of class formula whose dependent variable is a series of variables <br> joined by + operators. model_prcomp will treat a principal component score <br> of those variables as the actual dependent variable. All model terms must be <br> accounted for in means and covs. <br> integer indicating which principal component score to analyze. Must be less <br> than or equal to the total number of phenotypes. |
| :--- | :--- |
| n | sample size. |
| means | named vector of predictor and response means. |
| covs | named matrix of the covariance of all model predictors and the responses. <br> center |
| logical. Should the dependent variables be centered before principal compo- |  |
| nents are calculated? |  |

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:
call the matched call
terms the terms object used
coefficients a $p x 4$ matrix with columns for the estimated coefficient, its standard error, tstatistic and corresponding (two-sided) p-value.
sigma the square root of the estimated variance of the random error.
df degrees of freedom, a 3-vector $p, n-p, p *$, the first being the number of nonaliased coefficients, the last being the total number of coefficients.
fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom.
$r$.squared $\quad R^{2}$, the 'fraction of variance explained by the model'.
adj.r.squared the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$.
cov. unscaled a $p x p$ matrix of (unscaled) covariances of the $\operatorname{coef}[j], j=1, \ldots p$.
Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares Error (SSE), and Sum of Squares Total (SST).

## References

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." Pacific Symposium on Biocomputing, 25, 719-730. ISSN 23356928, doi: 10.1142/9789811215636_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

## Examples

```
ex_data <- pcsstools_example[c("g1", "x1", "x2", "y1", "y2", "y3")]
head(ex_data)
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)
model_prcomp(
    y1 + y2 + y3 ~ g1 + x1 + x2,
    comp = 1, n = n, means = means, covs = covs
)
```

model_product

Approximate a linear model for a product using PCSS

## Description

model_product approximates the linear model for the product of $m$ phenotypes as a function of a set of predictors.

## Usage

```
model_product(
        formula,
        n,
        means,
        covs,
        predictors,
        responses = NULL,
        response = "continuous",
    )
```


## Arguments

formula an object of class formula whose dependent variable is a combination of variables and * operators. All model terms must be accounted for in means and covs.
$\mathrm{n} \quad$ sample size.
means named vector of predictor and response means.
covs named matrix of the covariance of all model predictors and the responses.
predictors named list of objects of class predictor
responses named list of objects of class predictor corresponding to all terms being multiplied in the response. Can be left NULL if only multiplying two terms
response character. Describe distribution of all product terms. Either "continuous" or "binary". If "binary" different approximations of product means and variances are used.
... additional arguments

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:

| call | the matched call |
| :---: | :---: |
| terms | the terms object used |
| coefficients | a $p x 4$ matrix with columns for the estimated coefficient, its standard error, t statistic and corresponding (two-sided) p-value. |
| sigma | the square root of the estimated variance of the random error. |
| df | degrees of freedom, a 3 -vector $p, n-p, p *$, the first being the number of nonaliased coefficients, the last being the total number of coefficients. |
| fstatistic | a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom. |
| $r$ r.squared | $R^{2}$, the 'fraction of variance explained by the model'. |
| adj.r.squared | the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$. |
| cov.unscaled | a $p x p$ matrix of (unscaled) covariances of the $\operatorname{coef}[j], j=1, \ldots p$. |
| Sum Sq | a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares Error (SSE), and Sum of Squares Total (SST). |

## References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10. 1101/2021.03.08.433979v1.

## Examples

```
ex_data <- pcsstools_example[c("g1", "g2", "g3", "x1", "y4", "y5", "y6")]
head(ex_data)
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)
predictors <- list(
    g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
    g2 = new_predictor_snp(maf = mean(ex_data$g2) / 2),
    g3 = new_predictor_snp(maf = mean(ex_data$g3) / 2),
    x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1))
)
responses <- lapply(means[c("y4", "y5", "y6")], new_predictor_binary)
model_product(
    y4 * y5 * y6 ~ g1 + g2 + g3 + x1,
    means = means, covs = covs, n = n,
    predictors = predictors, responses = responses, response = "binary"
)
summary(lm(y4 * y5 * y6 ~ g1 + g2 + g3 + x1, data = ex_data))
```

```
model_singular Model an individual phenotype using PCSS
```


## Description

model_singular calculates the linear model for a singular phenotype as a function of a set of predictors.

## Usage

model_singular(formula, n, means, covs, ...)

## Arguments

| formula | an object of class formula whose dependent variable is only variable. All model <br> terms must be accounted for in means and covs. |
| :--- | :--- |
| n | sample size. |
| means | named vector of predictor and response means. |
| covs | named matrix of the covariance of all model predictors and the responses. |
| $\ldots$ | additional arguments |

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:
call the matched call
terms the terms object used
coefficients a $p x 4$ matrix with columns for the estimated coefficient, its standard error, tstatistic and corresponding (two-sided) p-value.
sigma the square root of the estimated variance of the random error.
df degrees of freedom, a 3-vector $p, n-p, p *$, the first being the number of nonaliased coefficients, the last being the total number of coefficients.
fstatistic a 3-vector with the value of the F-statistic with its numerator and denominator degrees of freedom.
$r$. squared $\quad R^{2}$, the 'fraction of variance explained by the model'.
adj.r.squared the above $R^{2}$ statistic 'adjusted', penalizing for higher $p$.
cov.unscaled a $p x p$ matrix of (unscaled) covariances of the $\operatorname{coef}[j], j=1, \ldots p$.
Sum Sq a 3-vector with the model's Sum of Squares Regression (SSR), Sum of Squares Error (SSE), and Sum of Squares Total (SST).

## References

Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." Pacific Symposium on Biocomputing, 25, 719-730. ISSN 23356928, doi: 10.1142/9789811215636_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.

## Examples

```
ex_data <- pcsstools_example[c("g1", "x1", "y1")]
means <- colMeans(ex_data)
covs <- cov(ex_data)
n <- nrow(ex_data)
model_singular(
    y1 ~ g1 + x1,
    n = n, means = means, covs = covs
)
summary(lm(y1 ~ g1 + x1, data = ex_data))
```

new_predictor Create an object of class "predictor"

## Description

Create an object of class "predictor"

## Usage

new_predictor
f = function() \{ \},
predictor_type $=$ character () ,
lb,
ub,
support
)

## Arguments

$f \quad$ a function that gives the probability mass/distribution function of a random variable.
predictor_type a character describing the random variable. Either "discrete" or "continuous".
lb, ub if predictor_type == "continuous" double giving the lower/upper bound of the pdf $f$.
support if predictor_type=="discrete" vector of the support of the pmf for $f$.

## Value

an object of class "predictor".

## See Also

```
new_predictor_normal, new_predictor_snp and new_predictor_binary.
```


## Examples

```
new_predictor(
    f = function(x0) dnorm(x0, mean = 0, sd = 1),
    predictor_type = "continuous", lb = -Inf, ub = Inf
)
```

new_predictor_binary Shortcut to create a predictor object for a binary variable

## Description

new_predictor_binary calls new_predictor

## Usage

new_predictor_binary (p)

## Arguments

$\mathrm{p} \quad$ probability of success (predictor mean)

## Value

an object of class "predictor".

## Examples

new_predictor_binary(p = 0.75)
new_predictor_normal Shortcut to create a predictor object for a continuous variable

## Description

new_predictor_normal calls new_predictor

## Usage

new_predictor_normal(mean, sd)

## Arguments

| mean | predictor mean (double). |
| :--- | :--- |
| sd | predictor standard deviation (double) |

## Value

an object of class "predictor".

## Examples

```
new_predictor_normal(mean = 10, sd = 1)
```


## Description

```
new_predictor_snp calls new_predictor
```


## Usage

new_predictor_snp(maf)

## Arguments

> maf minor allele frequency

## Value

an object of class "predictor".

## Examples

```
new_predictor_snp(maf = 0.3)
```


## Description

pcsslm approximates a linear model of a combination of variables using precomputed summary statistics.

## Usage

pcsslm(formula, pcss = list(), ...)

## Arguments

formula an object of class formula whose dependent variable is a combination of variables and logical I operators. All model terms must have appropriate PCSS in pcss.
pcss a list of precomputed summary statistics. In all cases, this should include $n$ : the sample size, means: a named vector of predictor and response means, and covs: a named covariance matrix including all predictors and responses. See Details for more information.
.. . additional arguments. See Details for more information.

## Details

pcsslm parses the input formula's dependent variable for functions such as sums (+), products (*), or logical operators ( $\mid$ and \&). It then identifies models the combination of variables using one of model_combo, model_product, model_or, model_and, or model_prcomp.
Different precomputed summary statistics are needed inside pcss depending on the function that combines the dependent variable.

- For linear combinations (and principal component analysis), only n, means, and covs are required
- For products and logical combinations, the additional items predictors and responses are required. These are named lists of objects of class predictor generated by new_predictor, with a predictor object for each independent variable in predictors and each dependent variable in responses. However, if only modeling the product or logical combination of only two variables, responses can be NULL without consequence.

If modeling a principal component score of a set of variables, include the argument comp where comp is an integer indicating which principal component score to analyze. Optional logical arguments center and standardize determine if responses should be centered and standardized before principal components are calculated.
If modeling a linear combination, include the argument phi, a named vector of linear weights for each variable in the dependent variable in formula.
If modeling a product, include the argument response, a character equal to either "continuous" or "binary". If "binary", specialized approximations are performed to estimate means and variances.

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:


## References

Wolf JM, Westra J, Tintle N (2021). "Using summary statistics to evaluate the genetic architecture of multiplicative combinations of initially analyzed phenotypes with a flexible choice of covariates." bioRxiv. doi: 10.1101/2021.03.08.433979, https://www.biorxiv.org/content/10. 1101/2021.03.08.433979v1.
Wolf JM, Barnard M, Xia X, Ryder N, Westra J, Tintle N (2020). "Computationally efficient, exact, covariate-adjusted genetic principal component analysis by leveraging individual marker summary statistics from large biobanks." Pacific Symposium on Biocomputing, 25, 719-730. ISSN 23356928, doi: 10.1142/9789811215636_0063, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6907735/.
Gasdaska A, Friend D, Chen R, Westra J, Zawistowski M, Lindsey W, Tintle N (2019). "Leveraging summary statistics to make inferences about complex phenotypes in large biobanks." Pacific Symposium on Biocomputing, 24, 391-402. ISSN 2335-6928, doi: 10.1142/9789813279827_0036, https://pubmed.ncbi.nlm.nih.gov/30963077/.

## See Also

model_combo, model_product, model_or, model_and, and model_prcomp.

## Examples

```
## Principal Component Analysis
ex_data <- pcsstools_example[c("g1", "x1", "y1", "y2", "y3")]
pcss <- list(
    means = colMeans(ex_data),
    covs = cov(ex_data),
    n = nrow(ex_data)
```

```
)
pcsslm(y1 + y2 + y3 ~ g1 + x1, pcss = pcss, comp = 1)
## Linear combination of variables
ex_data <- pcsstools_example[c("g1", "g2", "y1", "y2")]
pcss <- list(
    means = colMeans(ex_data),
    covs = cov(ex_data),
    n = nrow(ex_data)
)
pcsslm(y1 + y2 ~ g1 + g2, pcss = pcss, phi = c(1, -1))
summary(lm(y1 - y2 ~ g1 + g2, data = ex_data))
## Product of variables
ex_data <- pcsstools_example[c("g1", "x1", "y4", "y5", "y6")]
pcss <- list(
    means = colMeans(ex_data),
    covs = cov(ex_data),
    n = nrow(ex_data),
    predictors = list(
        g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
        x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1))
    ),
    responses = lapply(
        colMeans(ex_data)[3:length(colMeans(ex_data))],
        new_predictor_binary
    )
)
pcsslm(y4 * y5 * y6 ~ g1 + x1, pcss = pcss, response = "binary")
summary(lm(y4 * y5 * y6 ~ g1 + x1, data = ex_data))
## Disjunct (OR statement) of variables
ex_data <- pcsstools_example[c("g1", "x1", "y4", "y5")]
pcss <- list(
    means = colMeans(ex_data),
    covs = cov(ex_data),
    n = nrow(ex_data),
    predictors = list(
        g1 = new_predictor_snp(maf = mean(ex_data$g1) / 2),
        x1 = new_predictor_normal(mean = mean(ex_data$x1), sd = sd(ex_data$x1))
    )
)
pcsslm(y4 | y5 ~ g1 + x1, pcss = pcss)
summary(lm(y4 | y5 ~ g1 + x1, data = ex_data))
```


## Description

The pcsstools package provides functions to describe various regression models using only precomputed summary statistics (PCSS) from genome-wide association studies (GWASs) and PCSS repositories.

## Details

The main function of interest is pcsslm, which can be used to approximate linear models for various combinations of phenotypes using PCSS.

```
pcsstools_example Simulated example data
```


## Description

A dataset containing simulated genetic data with 3 SNPs, 3 continuous covariates, and 6 continuous phenotypes.

## Usage

pcsstools_example

## Format

A data frame with 1000 rows and 12 columns:
g1,g2,g3 Minor allele counts at three sites
$\mathbf{x 1}, \mathbf{x} 2, \mathbf{x} 3$ Continuous covariates
$\mathbf{y} 1, \mathbf{y} 2, \mathbf{y} 3$ Continuous phenotypes
$\mathbf{y 4 , y 5 , y 6}$ Binary phenotypes

```
print.pcsslm Print an object of class pcsslm
```


## Description

Prints a linear model fit through pre-computed summary statistics

## Usage

```
## S3 method for class 'pcsslm'
print(
    x,
    digits = max(3L, getOption("digits") - 3L),
    symbolic.cor = x$symbolic.cor,
    signif.stars = getOption("show.signif.stars"),
    )
```


## Arguments

| x | an object of class "pcsslm" |
| :--- | :--- |
| digits | the number of significant digits to use when printing. |
| symbolic.cor | logical. If TRUE, print the correlations in a symbolic form (see symnum) rather <br> than as numbers. |
| signif.stars | logical. If TRUE, 'significance stars' are printed for each coefficient. <br> $\ldots$ |
| further arguments passed to or from other methods. |  |

## Value

an object of class "pcsslm".
An object of class "pcsslm" is a list containing at least the following components:

| call <br> terms <br> coefficients | the matched call <br> the terms object used <br> a $p x 4$ matrix with columns for the estimated coefficient, its standard error, t- <br> statistic and corresponding (two-sided) p-value. <br> the square root of the estimated variance of the random error. <br> degrees of freedom, a 3-vector $p, n-p, p *$, the first being the number of non- <br> aliased coefficients, the last being the total number of coefficients. <br> a 3-vector with the value of the F-statistic with its numerator and denominator <br> df |
| :--- | :--- |
| degrees of freedom. |  |

## Author(s)

R Core Team and contributors worldwide. Modified by Jack Wolf

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