# Package 'philentropy' 

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Description Computes 46 optimized distance and similarity measures for comparing probability functions (Drost (2018) [doi:10.21105/joss.00765](doi:10.21105/joss.00765)). These comparisons between probability functions have their foundations in a broad range of scientific disciplines from mathematics to ecology. The aim of this package is to provide a core framework for clustering, classification, statistical inference, goodness-of-fit, non-parametric statistics, information theory, and machine learning tasks that are based on comparing univariate or multivariate probability functions.

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## Description

The lowlevel function for computing the additive_symm_chi_sq distance.

## Usage <br> additive_symm_chi_sq(P, Q, testNA)

## Arguments

| $P$ | a numeric vector storing the first distribution. |
| :--- | :--- |
| Q | a numeric vector storing the second distribution. |
| testNA | a logical value indicating whether or not distributions shall be checked for NA <br> values. |

## Author(s)

Hajk-Georg Drost

## Examples

```
additive_symm_chi_sq(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```


## Description

The lowlevel function for computing the avg distance.

## Usage

$\operatorname{avg}(P, Q, t e s t N A)$

## Arguments

P a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

$\operatorname{avg}(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 /$ sum(20:29), testNA $=$ FALSE $)$
bhattacharyya Bhattacharyya distance (lowlevel function)

## Description

The lowlevel function for computing the bhattacharyya distance.

## Usage

bhattacharyya(P, Q, testNA, unit, epsilon)

## Arguments

P
Q

## testNA

unit type of log function. Option are
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.

## Author(s)

Hajk-Georg Drost

## Examples

```
bhattacharyya(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE,
    unit = "log2", epsilon = 0.00001)
```


## Description

This function implements an interface to the kernel density estimation functions provided by the KernSmooth package.

## Usage

binned.kernel.est( data, kernel = "normal", bandwidth = NULL, canonical = FALSE, scalest = "minim", level = 2L,

```
        gridsize = 401L,
        range.data = range(data),
        truncate = TRUE
    )
```


## Arguments

| data | a numeric vector containing the sample on which the kernel density estimate is to be constructed. |
| :---: | :---: |
| kernel | character string specifying the smoothing kernel |
| bandwidth | the kernel bandwidth smoothing parameter. |
| canonical scalest | a logical value indicating whether canonically scaled kernels should be used estimate of scale. |
|  | - "stdev" - standard deviation is used. <br> - "iqr" - inter-quartile range divided by 1.349 is used. <br> - "minim" - minimum of "stdev" and "iqr" is used. |
| level | number of levels of functional estimation used in the plug-in rule. |
| gridsize | the number of equally-spaced points over which binning is performed to obtain kernel functional approximation. |
| range.data | vector containing the minimum and maximum values of data at which to compute the estimate. The default is the minimum and maximum data values. |
| truncate | logical value indicating whether data with x values outside the range specified by range. data should be ignored. |

## Author(s)

Hajk-Georg Drost

## References

Matt Wand (2015). KernSmooth: Functions for Kernel Smoothing Supporting Wand \& Jones (1995). R package version 2.23-14.

Henry Deng and Hadley Wickham (2011). Density estimation in R. http://vita.had.co.nz/ papers/density-estimation.pdf.
canberra Canberra distance (lowlevel function)

## Description

The lowlevel function for computing the canberra distance.

## Usage

canberra(P, Q, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA
a numeric vector storing the first distribution.
a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
canberra(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

CE Shannon's Conditional-Entropy $H(X \mid Y)$

## Description

Compute Shannon's Conditional-Entropy based on the chain rule $H(X \mid Y)=H(X, Y)-H(Y)$ based on a given joint-probability vector $P(X, Y)$ and probability vector $P(Y)$.

## Usage

$C E(x y, y, u n i t=" \log 2 ")$

## Arguments

xy a numeric joint-probability vector $P(X, Y)$ for which Shannon's Joint-Entropy $H(X, Y)$ shall be computed.
y a numeric probability vector $P(Y)$ for which Shannon's Entropy $H(Y)$ (as part of the chain rule) shall be computed. It is important to note that this probability vector must be the probability distribution of random variable $\mathrm{Y}(\mathrm{P}(\mathrm{Y})$ for which $\mathrm{H}(\mathrm{Y})$ is computed).
unit a character string specifying the logarithm unit that shall be used to compute distances that depend on log computations.

## Details

This function might be useful to fastly compute Shannon's Conditional-Entropy for any given jointprobability vector and probability vector.

## Value

Shannon's Conditional-Entropy in bit.

## Note

Note that the probability vector $\mathrm{P}(\mathrm{Y})$ must be the probability distribution of random variable Y ( $\mathrm{P}(\mathrm{Y})$ for which $\mathrm{H}(\mathrm{Y})$ is computed ) and furthermore used for the chain rule computation of $H(X \mid Y)=H(X, Y)-H(Y)$.

## Author(s)

Hajk-Georg Drost

## References

Shannon, Claude E. 1948. "A Mathematical Theory of Communication". Bell System Technical Journal 27 (3): 379-423.

## See Also

H, JE

## Examples

$\operatorname{CE}(1: 10 / \operatorname{sum}(1: 10), 1: 10 / \operatorname{sum}(1: 10))$

$$
\text { chebyshev } \quad \text { Chebyshev distance (lowlevel function) }
$$

## Description

The lowlevel function for computing the chebyshev distance.

## Usage

chebyshev( $\mathrm{P}, \mathrm{Q}$, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

$$
\text { chebyshev }(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 / \operatorname{sum}(20: 29) \text {, testNA }=\text { FALSE })
$$

clark_sq Clark squared distance (lowlevel function)

## Description

The lowlevel function for computing the clark_sq distance.

## Usage

clark_sq(P, Q, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
clark_sq(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

```
cosine_dist Cosine distance (lowlevel function)
```


## Description

The lowlevel function for computing the cosine_dist distance.

## Usage

cosine_dist(P, Q, testNA)

## Arguments

| P | a numeric vector storing the first distribution. |
| :--- | :--- |
| Q | a numeric vector storing the second distribution. |
| testNA | a logical value indicating whether or not distributions shall be checked for NA <br> values. |

## Author(s)

Hajk-Georg Drost

## Examples

cosine_dist( $P=1: 10 /$ sum ( $1: 10$ ), $Q=20: 29 / \operatorname{sum}(20: 29)$, testNA $=$ FALSE $)$
czekanowski Czekanowski distance (lowlevel function)

## Description

The lowlevel function for computing the czekanowski distance.

## Usage

czekanowski(P, Q, testNA)

## Arguments

P
Q
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

czekanowski( $\mathrm{P}=1: 10 /$ sum(1:10), $\mathrm{Q}=20: 29 /$ sum(20:29), testNA $=$ FALSE)

```
dice_dist Dice distance (lowlevel function)
```


## Description

The lowlevel function for computing the dice_dist distance.

## Usage

```
dice_dist(P, Q, testNA)
```


## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

dice_dist $(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 / \operatorname{sum}(20: 29)$, testNA $=$ FALSE $)$
dist.diversity Distance Diversity between Probability Density Functions

## Description

This function computes all distance values between two probability density functions that are available in getDistMethods and returns a vector storing the corresponding distance measures. This vector is named distance diversity vector.

## Usage

dist.diversity(x, p, test.na = FALSE, unit = "log2")

## Arguments

x
p
test.na a boolean value indicating whether input vectors should be tested for NA values. Faster computations if test. na = FALSE.
unit a character string specifying the logarithm unit that should be used to compute distances that depend on log computations. Options are:

- unit $=$ "log"
- unit $=" \log 2 "$
- unit $=" \log 10 "$


## Author(s)

Hajk-Georg Drost

## Examples

```
dist.diversity(rbind(1:10/sum(1:10), 20:29/sum(20:29)), p = 2, unit = "log2")
```

distance Distances and Similarities between Probability Density Functions

## Description

This functions computes the distance/dissimilarity between two probability density functions.

## Usage

```
    distance(
        x,
        method = "euclidean",
        p = NULL,
        test.na = TRUE,
        unit = "log",
        epsilon = 1e-05,
        est.prob = NULL,
        use.row.names = FALSE,
        as.dist.obj = FALSE,
        diag = FALSE,
        upper = FALSE,
        mute.message = FALSE
    )
```


## Arguments

$x \quad a \quad$ numeric data.frame or matrix (storing probability vectors) or a numeric data.frame or matrix storing counts (if est.prob is specified).
method a character string indicating whether the distance measure that should be computed.
p power of the Minkowski distance.
test.na a boolean value indicating whether input vectors should be tested for NA values. Faster computations if test. na = FALSE.
unit a character string specifying the logarithm unit that should be used to compute distances that depend on log computations.
epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0
values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.
est.prob method to estimate probabilities from input count vectors such as non-probability vectors. Default: est. prob = NULL. Options are:

- est. prob = "empirical": The relative frequencies of each vector are computed internally. For example an input matrix $\operatorname{rbind}(1: 10,11: 20)$ will be transformed to a probability vector rbind(1:10/sum(1:10), 11:20/ $\operatorname{sum}(11: 20))$
use.row. names a logical value indicating whether or not row names from the input matrix shall be used as rownames and colnames of the output distance matrix. Default value is use. row. names = FALSE.
as.dist.obj shall the return value or matrix be an object of class link[stats]\{dist\}? Default is as.dist. obj = FALSE.
diag if as.dist.obj = TRUE, then this value indicates whether the diagonal of the distance matrix should be printed. Default
upper if as.dist.obj = TRUE, then this value indicates whether the upper triangle of the distance matrix should be printed.
mute.message a logical value indicating whether or not messages printed by distance shall be muted. Default is mute. message $=$ FALSE.


## Details

Here a distance is defined as a quantitative degree of how far two mathematical objects are apart from eachother (Cha, 2007).
This function implements the following distance/similarity measures to quantify the distance between probability density functions:

- L_p Minkowski family
- Euclidean : $d=\operatorname{sqrt}\left(\sum\left|P_{i}-Q_{i}\right|^{2}\right)$
- Manhattan : $d=\sum\left|P_{i}-Q_{i}\right|$
- Minkowski : $d=\left(\sum\left|P_{i}-Q_{i}\right|^{p}\right)^{1} / p$
- Chebyshev : $d=\max \left|P_{i}-Q_{i}\right|$
- L_1 family
- Sorensen : $d=\sum\left|P_{i}-Q_{i}\right| / \sum\left(P_{i}+Q_{i}\right)$
- Gower : $d=1 / d * \sum\left|P_{i}-Q_{i}\right|$
- Soergel : $d=\sum\left|P_{i}-Q_{i}\right| / \sum \max \left(P_{i}, Q_{i}\right)$
- Kulczynski d : $d=\sum\left|P_{i}-Q_{i}\right| / \sum \min \left(P_{i}, Q_{i}\right)$
- Canberra : $d=\sum\left|P_{i}-Q_{i}\right| /\left(P_{i}+Q_{i}\right)$
- Lorentzian : $d=\sum \ln \left(1+\left|P_{i}-Q_{i}\right|\right)$
- Intersection family
- Intersection : $s=\sum \min \left(P_{i}, Q_{i}\right)$
- Non-Intersection : $d=1-\sum \min \left(P_{i}, Q_{i}\right)$
- Wave Hedges : $d=\sum\left|P_{i}-Q_{i}\right| / \max \left(P_{i}, Q_{i}\right)$
- Czekanowski : $d=\sum\left|P_{i}-Q_{i}\right| / \sum\left|P_{i}+Q_{i}\right|$
- Motyka : $d=\sum \min \left(P_{i}, Q_{i}\right) /\left(P_{i}+Q_{i}\right)$
- Kulczynski s : $d=1 / \sum\left|P_{i}-Q_{i}\right| / \sum \min \left(P_{i}, Q_{i}\right)$
- Tanimoto : $d=\sum\left(\max \left(P_{i}, Q_{i}\right)-\min \left(P_{i}, Q_{i}\right)\right) / \sum \max \left(P_{i}, Q_{i}\right)$; equivalent to Soergel
- Ruzicka : $s=\sum \min \left(P_{i}, Q_{i}\right) / \sum \max \left(P_{i}, Q_{i}\right)$; equivalent to 1 - Tanimoto $=1$ - Soergel
- Inner Product family
- Inner Product : $s=\sum P_{i} * Q_{i}$
- Harmonic mean : $s=2 * \sum\left(P_{i} * Q_{i}\right) /\left(P_{i}+Q_{i}\right)$
- Cosine : $s=\sum\left(P_{i} * Q_{i}\right) / \operatorname{sqrt}\left(\sum P_{i}^{2}\right) * \operatorname{sqrt}\left(\sum Q_{i}^{2}\right)$
- Kumar-Hassebrook (PCE) : $s=\sum\left(P_{i} * Q_{i}\right) /\left(\sum P_{i}^{2}+\sum Q_{i}^{2}-\sum\left(P_{i} * Q_{i}\right)\right)$
- Jaccard : $d=1-\sum\left(P_{i} * Q_{i}\right) /\left(\sum P_{i}^{2}+\sum Q_{i}^{2}-\sum\left(P_{i} * Q_{i}\right)\right)$; equivalent to 1 -Kumar-Hassebrook
- Dice : $d=\sum\left(P_{i}-Q_{i}\right)^{2} /\left(\sum P_{i}^{2}+\sum Q_{i}^{2}\right)$
- Squared-chord family
- Fidelity : $s=\sum \operatorname{sqrt}\left(P_{i} * Q_{i}\right)$
- Bhattacharyya: $d=-\ln \sum \operatorname{sqrt}\left(P_{i} * Q_{i}\right)$
- Hellinger : $d=2 * \operatorname{sqrt}\left(1-\sum \operatorname{sqrt}\left(P_{i} * Q_{i}\right)\right)$
- Matusita : $d=\operatorname{sqrt}\left(2-2 * \sum \operatorname{sqrt}\left(P_{i} * Q_{i}\right)\right)$
- Squared-chord : $d=\sum\left(\operatorname{sqrt}\left(P_{i}\right)-\operatorname{sqrt}\left(Q_{i}\right)\right)^{2}$
- Squared L_2 family ( $X^{\wedge} 2$ squared family)
- Squared Euclidean : $d=\sum\left(P_{i}-Q_{i}\right)^{2}$
- Pearson $X^{\wedge} 2: d=\sum\left(\left(P_{i}-Q_{i}\right)^{2} / Q_{i}\right)$
- Neyman $X^{\wedge} 2: d=\sum\left(\left(P_{i}-Q_{i}\right)^{2} / P_{i}\right)$
- Squared $X^{\wedge} 2: d=\sum\left(\left(P_{i}-Q_{i}\right)^{2} /\left(P_{i}+Q_{i}\right)\right)$
- Probabilistic Symmetric $X^{\wedge} 2: d=2 * \sum\left(\left(P_{i}-Q_{i}\right)^{2} /\left(P_{i}+Q_{i}\right)\right)$
- Divergence : $X^{\wedge} 2: d=2 * \sum\left(\left(P_{i}-Q_{i}\right)^{2} /\left(P_{i}+Q_{i}\right)^{2}\right)$
- Clark : $d=\operatorname{sqrt}\left(\sum\left(\left|P_{i}-Q_{i}\right| /\left(P_{i}+Q_{i}\right)\right)^{2}\right)$
- Additive Symmetric $X^{\wedge} 2: d=\sum\left(\left(\left(P_{i}-Q_{i}\right)^{2} *\left(P_{i}+Q_{i}\right)\right) /\left(P_{i} * Q_{i}\right)\right)$
- Shannon's entropy family
- Kullback-Leibler : $d=\sum P_{i} * \log \left(P_{i} / Q_{i}\right)$
- Jeffreys : $d=\sum\left(P_{i}-Q_{i}\right) * \log \left(P_{i} / Q_{i}\right)$
- K divergence : $d=\sum P_{i} * \log \left(2 * P_{i} / P_{i}+Q_{i}\right)$
- Topsoe : $d=\sum\left(P_{i} * \log \left(2 * P_{i} / P_{i}+Q_{i}\right)\right)+\left(Q_{i} * \log \left(2 * Q_{i} / P_{i}+Q_{i}\right)\right)$
- Jensen-Shannon : $d=0.5 *\left(\sum P_{i} * \log \left(2 * P_{i} / P_{i}+Q_{i}\right)+\sum Q_{i} * \log \left(2 * Q_{i} / P_{i}+Q_{i}\right)\right)$
- Jensen difference : $d=\sum\left(\left(P_{i} * \log \left(P_{i}\right)+Q_{i} * \log \left(Q_{i}\right) / 2\right)-\left(P_{i}+Q_{i} / 2\right) * \log \left(P_{i}+Q_{i} / 2\right)\right)$
- Combinations
- Taneja : $d=\sum\left(P_{i}+Q_{i} / 2\right) * \log \left(P_{i}+Q_{i} /\left(2 * \operatorname{sqrt}\left(P_{i} * Q_{i}\right)\right)\right)$
- Kumar-Johnson : $d=\sum\left(P_{i}^{2}-Q_{i}^{2}\right)^{2} / 2 *\left(P_{i} * Q_{i}\right)^{1} .5$
$-\operatorname{Avg}\left(\mathrm{L} \_1, \mathrm{~L} \_\mathrm{n}\right): d=\sum\left|P_{i}-Q_{i}\right|+\max \left|P_{i}-Q_{i}\right| / 2$
In cases where $x$ specifies a count matrix, the argument est.prob can be selected to first estimate probability vectors from input count vectors and second compute the corresponding distance measure based on the estimated probability vectors.
The following probability estimation methods are implemented in this function:
- est.prob = "empirical" : relative frequencies of counts.


## Value

The following results are returned depending on the dimension of x :

- in case $\operatorname{nrow}(x)=2$ : a single distance value.
- in case $\operatorname{nrow}(x)>2$ : a distance matrix storing distance values for all pairwise probability vector comparisons.


## Note

According to the reference in some distance measure computations invalid computations can occur when dealing with 0 probabilities.
In these cases the convention is treated as follows:

- division by zero - case $0 / 0$ : when the divisor and dividend become zero, $0 / 0$ is treated as 0 .
- division by zero - case $n / 0$ : when only the divisor becomes 0 , the corresponsning 0 is replaced by a small $\epsilon=0.00001$.
- $\log$ of zero - case $0 * \log (0)$ : is treated as 0 .
- $\log$ of zero - case $\log (0):$ zero is replaced by a small $\epsilon=0.00001$.


## Author(s)

Hajk-Georg Drost

## References

Sung-Hyuk Cha. (2007). Comprehensive Survey on Distance/Similarity Measures between Probability Density Functions. International Journal of Mathematical Models and Methods in Applied Sciences 4: 1.

## See Also

```
getDistMethods, estimate.probability, dist.diversity
```


## Examples

```
# Simple Examples
# receive a list of implemented probability distance measures
getDistMethods()
## compute the euclidean distance between two probability vectors
distance(rbind(1:10/sum(1:10), 20:29/sum(20:29)), method = "euclidean")
## compute the euclidean distance between all pairwise comparisons of probability vectors
ProbMatrix <- rbind(1:10/sum(1:10), 20:29/sum(20:29),30:39/sum(30:39))
distance(ProbMatrix, method = "euclidean")
# compute distance matrix without testing for NA values in the input matrix
distance(ProbMatrix, method = "euclidean", test.na = FALSE)
# alternatively use the colnames of the input data for the rownames and colnames
# of the output distance matrix
ProbMatrix <- rbind(1:10/sum(1:10), 20:29/sum(20:29),30:39/sum(30:39))
rownames(ProbMatrix) <- paste0("Example", 1:3)
distance(ProbMatrix, method = "euclidean", use.row.names = TRUE)
# Specialized Examples
CountMatrix <- rbind(1:10, 20:29, 30:39)
## estimate probabilities from a count matrix
distance(CountMatrix, method = "euclidean", est.prob = "empirical")
## compute the euclidean distance for count data
## NOTE: some distance measures are only defined for probability values,
distance(CountMatrix, method = "euclidean")
## compute the Kullback-Leibler Divergence with different logarithm bases:
### case: unit = log (Default)
distance(ProbMatrix, method = "kullback-leibler", unit = "log")
### case: unit = log2
distance(ProbMatrix, method = "kullback-leibler", unit = "log2")
### case: unit = log10
distance(ProbMatrix, method = "kullback-leibler", unit = "log10")
```


## Description

This functions computes the distance/dissimilarity between two sets of probability density functions.

## Usage

```
    dist_many_many(
```

        dists1,
        dists2,
        method,
        p = NA_real_,
        testNA = TRUE,
        unit = "log",
        epsilon \(=1 \mathrm{e}-05\)
    )
    
## Arguments

dists1
dists2 a numeric matrix storing distributions in its rows.
method a character string indicating whether the distance measure that should be computed.
p power of the Minkowski distance.
testNA a logical value indicating whether or not distributions shall be checked for NA values.
unit type of $\log$ function. Option are

- unit = "log"
- unit = "log2"
- unit = " $\log 10 "$
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon $=0.01$ ). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.


## Value

A matrix of distance values

## Examples

```
set.seed(2020-08-20)
M1 <- t(replicate(10, sample(1:10, size = 10) / 55))
M2 <- t(replicate(10, sample(1:10, size = 10) / 55))
result <- dist_many_many(M1, M2, method = "euclidean", testNA = FALSE)
```

dist_one_many Distances and Similarities between One and Many Probability Density Functions

## Description

This functions computes the distance/dissimilarity between one probability density functions and a set of probability density functions.

## Usage

```
    dist_one_many(
```

        P,
        dists,
        method,
        p = NA_real_,
        testNA = TRUE,
        unit = "log",
        epsilon \(=1 \mathrm{e}-05\)
    )
    
## Arguments

P
dists
method
p
testNA
unit
nit type of log function. Option are

- unit = "log"
- unit $=" \log 2 "$
- unit $=" \log 10 "$
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x / 0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0
values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.


## Value

A vector of distance values

## Examples

set. seed (2020-08-20)
$P<-1: 10 / \operatorname{sum}(1: 10)$
$M<-\mathrm{t}($ replicate $(100$, sample(1:10, size $=10) / 55)$ )
dist_one_many (P, M, method = "euclidean", testNA = FALSE)
dist_one_one Distances and Similarities between Two Probability Density Functions

## Description

This functions computes the distance/dissimilarity between two probability density functions.

## Usage

```
dist_one_one(
    P,
    Q,
    method,
    p = NA_real_,
    testNA = TRUE,
    unit = "log",
    epsilon = 1e-05
    )
```


## Arguments

P
Q a numeric vector storing the second distribution.
method a character string indicating whether the distance measure that should be computed.
p power of the Minkowski distance.

```
testNA a logical value indicating whether or not distributions shall be checked for NA
                values.
unit type of log function. Option are
    - unit = "log"
    - unit \(=" \log 2 "\)
    - unit = " \(\log 10 "\)
```

epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon=0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.

## Value

A single distance value

## Examples

P <- 1:10/ sum(1:10)
Q <- 20:29 / sum(20:29)
dist_one_one( $\mathrm{P}, \mathrm{Q}$, method = "euclidean", testNA $=$ FALSE)

```
divergence_sq Divergence squared distance (lowlevel function)
```


## Description

The lowlevel function for computing the divergence_sq distance.

## Usage

divergence_sq(P, Q, testNA)

## Arguments

P a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
divergence_sq(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

estimate.probability Estimate Probability Vectors From Count Vectors

## Description

This function takes a numeric count vector and returns estimated probabilities of the corresponding counts.
The following probability estimation methods are implemented in this function:

- method = "empirical" : generates the relative frequency of the data $x / \operatorname{sum}(x)$.
- 
- 


## Usage

estimate.probability(x, method = "empirical")

## Arguments

$x \quad a \quad$ numeric vector storing count values.
method a character string specifying the estimation method tht should be used to estimate probabilities from input counts.

## Value

a numeric probability vector.

## Author(s)

Hajk-Georg Drost

## Examples

```
# generate a count vector
x <- runif(100)
# generate a probability vector from corresponding counts
# method = "empirical"
x.prob <- estimate.probability(x, method = "empirical")
```


## Description

The lowlevel function for computing the euclidean distance.

## Usage

euclidean(P, Q, testNA)

## Arguments

| P | a numeric vector storing the first distribution. |
| :--- | :--- |
| Q | a numeric vector storing the second distribution. |
| testNA | a logical value indicating whether or not distributions shall be checked for NA <br> values. |

## Author(s)

Hajk-Georg Drost

## Examples

```
euclidean(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

```
fidelity Fidelity distance (lowlevel function)
```


## Description

The lowlevel function for computing the fidelity distance.

## Usage

fidelity(P, Q, testNA)

## Arguments

| P | a numeric vector storing the first distribution. |
| :--- | :--- |
| Q | a numeric vector storing the second distribution. |
| testNA | a logical value indicating whether or not distributions shall be checked for NA <br> values. |

## Author(s)

Hajk-Georg Drost

## Examples

```
fidelity(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

    getDistMethods Get method names for distance
    
## Description

This function returns the names of the methods that can be applied to compute distances between probability density functions using the distance function.

## Usage

getDistMethods()

## Author(s)

Hajk-Georg Drost

## Examples

```
getDistMethods()
```

    gJSD Generalized Jensen-Shannon Divergence
    
## Description

This function computes the Generalized Jensen-Shannon Divergence of a probability matrix.

## Usage

```
gJSD(x, unit = "log2", weights = NULL, est.prob = NULL)
```


## Arguments

x
unit a character string specifying the logarithm unit that shall be used to compute distances that depend on log computations.
weights a numeric vector specifying the weights for each distribution in $x$. Default: weights $=$ NULL; in this case all distributions are weighted equally (= uniform distribution of weights). In case users wish to specify non-uniform weights for e.g. 3 distributions, they can specify the argument weights $=c(0.5,0.25,0.25)$. This notation denotes that vec 1 is weighted by 0.5 , vec 2 is weighted by 0.25 , and vec 3 is weighted by 0.25 as well.
est.prob method to estimate probabilities from input count vectors such as non-probability vectors. Default: est. prob $=$ NULL. Options are:

- est. prob = "empirical": The relative frequencies of each vector are computed internally. For example an input matrix rbind ( $1: 10,11: 20$ ) will be transformed to a probability vector rbind(1:10/sum(1:10), 11:20/ $\operatorname{sum}(11: 20))$


## Details

Function to compute the Generalized Jensen-Shannon Divergence
$J S D_{\pi_{1}, \ldots, \pi_{n}}\left(P_{1}, \ldots, P_{n}\right)=H\left(\sum_{i=1}^{n} \pi_{i} * P_{i}\right)-\sum_{i=1}^{n} \pi_{i} * H\left(P_{i}\right)$
where $\pi_{1}, \ldots, \pi_{n}$ denote the weights selected for the probability vectors $P_{-} 1, \ldots, P_{-} n$ and $H\left(P_{-} i\right)$ denotes the Shannon Entropy of probability vector P_i.

## Value

The Jensen-Shannon divergence between all possible combinations of comparisons.

## Author(s)

Hajk-Georg Drost

## See Also

KL, H, JSD, CE, JE

## Examples

```
# define input probability matrix
Prob <- rbind(1:10/sum(1:10), 20:29/sum(20:29), 30:39/sum(30:39))
# compute the Generalized JSD comparing the PS probability matrix
gJSD(Prob)
# Generalized Jensen-Shannon Divergence between three vectors using different log bases
gJSD(Prob, unit = "log2") # Default
gJSD(Prob, unit = "log")
gJSD(Prob, unit = "log10")
```

\# Jensen-Shannon Divergence Divergence between count vectors P.count and Q.count
P.count <- 1:10
Q.count <- 20:29
R.count <- 30:39
x.count <- rbind(P.count, Q.count, R.count)
$\operatorname{gJSD}(x$. count, est.prob $=$ "empirical")
gower Gower distance (lowlevel function)

## Description

The lowlevel function for computing the gower distance.

## Usage

gower(P, Q, testNA)

## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

$\operatorname{gower}(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 /$ sum(20:29), testNA $=$ FALSE $)$

H
Shannon's Entropy $H(X)$

## Description

Compute the Shannon's Entropy $H(X)=-\sum P(X) * \log 2(P(X))$ based on a given probability vector $P(X)$.

## Usage

$H(x$, unit $=" \log 2 ")$

## Arguments

x
a numeric probability vector $P(X)$ for which Shannon's Entropy $H(X)$ shall be computed.
unit a character string specifying the logarithm unit that shall be used to compute distances that depend on log computations.

## Details

This function might be useful to fastly compute Shannon's Entropy for any given probability vector.

## Value

a numeric value representing Shannon's Entropy in bit.

## Author(s)

Hajk-Georg Drost

## References

Shannon, Claude E. 1948. "A Mathematical Theory of Communication". Bell System Technical Journal 27 (3): 379-423.

## See Also

JE, CE, KL, JSD, gJSD

## Examples

```
H(1:10/sum(1:10))
```

harmonic_mean_dist Harmonic mean distance (lowlevel function)

## Description

The lowlevel function for computing the harmonic_mean_dist distance.

## Usage

harmonic_mean_dist(P, Q, testNA)

## Arguments

P
a numeric vector storing the first distribution.
Q
testNA
a numeric vector storing the second distribution.
a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

harmonic_mean_dist $(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 / \operatorname{sum}(20: 29)$, testNA $=$ FALSE $)$
hellinger Hellinger distance (lowlevel function)

## Description

The lowlevel function for computing the hellinger distance.

## Usage

hellinger ( $\mathrm{P}, \mathrm{Q}$, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
hellinger(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```


## Description

The lowlevel function for computing the inner_product distance.

## Usage

inner_product(P, Q, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
inner_product(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

```
intersection_dist Intersection distance (lowlevel function)
```


## Description

The lowlevel function for computing the intersection_dist distance.

## Usage

intersection_dist(P, Q, testNA)

## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
    intersection_dist(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

    jaccard Jaccard distance (lowlevel function)
    
## Description

The lowlevel function for computing the jaccard distance.

## Usage

jaccard(P, Q, testNA)

## Arguments

P
Q
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

$\operatorname{jaccard}(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 / \operatorname{sum}(20: 29), \operatorname{testNA}=F A L S E)$

JE Shannon's Joint-Entropy $H(X, Y)$

## Description

This funciton computes Shannon's Joint-Entropy $H(X, Y)=-\sum \sum P(X, Y) * \log 2(P(X, Y))$ based on a given joint-probability vector $P(X, Y)$.

## Usage

JE(x, unit = "log2")

## Arguments

x
a numeric joint-probability vector $P(X, Y)$ for which Shannon's Joint-Entropy $H(X, Y)$ shall be computed.
unit a character string specifying the logarithm unit that shall be used to compute distances that depend on log computations.

## Value

a numeric value representing Shannon's Joint-Entropy in bit.

## Author(s)

Hajk-Georg Drost

## References

Shannon, Claude E. 1948. "A Mathematical Theory of Communication". Bell System Technical Journal 27 (3): 379-423.

## See Also

H, CE, KL, JSD, gJSD, distance

## Examples

$$
\operatorname{JE}(1: 100 / \operatorname{sum}(1: 100))
$$

jeffreys Jeffreys distance (lowlevel function)

## Description

The lowlevel function for computing the jeffreys distance.

## Usage

jeffreys(P, Q, testNA, unit, epsilon)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.
unit type of log function. Option are

- unit = "log"
- unit = "log2"
- unit = " $\log 10 "$
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.


## Author(s)

Hajk-Georg Drost

## Examples

```
jeffreys(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE,
    unit = "log2", epsilon = 0.00001)
```

    jensen_difference Jensen difference (lowlevel function)
    
## Description

The lowlevel function for computing the jensen_difference distance.

## Usage

jensen_difference(P, Q, testNA, unit)

## Arguments

P
Q
testNA
unit
a numeric vector storing the first distribution.
a numeric vector storing the second distribution
a logical value indicating whether or not distributions shall be checked for NA values. type of log function. Option are

- unit = "log"
- unit $=" \log 2 "$
- unit $=" \log 10 "$


## Author(s)

Hajk-Georg Drost

## Examples

jensen_difference $(P=1: 10 /$ sum (1:10), $Q=20: 29 /$ sum(20:29), testNA $=$ FALSE, unit $=" \log 2 ")$
jensen_shannon Jensen-Shannon distance (lowlevel function)

## Description

The lowlevel function for computing the jensen_shannon distance.

## Usage

jensen_shannon(P, Q, testNA, unit)

## Arguments

P
Q
testNA
unit type of $\log$ function. Option are

- unit $=" \log "$
- unit = "log2"
- unit $=" \log 10 "$


## Author(s)

Hajk-Georg Drost

## Examples

jensen_shannon $(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 / \operatorname{sum}(20: 29)$, testNA $=$ FALSE, unit $=" \log 2 ")$

## Description

This function computes a distance matrix or distance value based on the Jensen-Shannon Divergence with equal weights.

## Usage

JSD(x, test.na = TRUE, unit = "log2", est.prob = NULL)

## Arguments

x
test.na
unit
est.prob

a numeric data.frame or matrix (storing probability vectors) or a numeric data.frame or matrix storing counts (if est. prob = TRUE). See distance for details.
a boolean value specifying whether input vectors shall be tested for NA values.
a character string specifying the logarithm unit that shall be used to compute distances that depend on log computations.
method to estimate probabilities from input count vectors such as non-probability vectors. Default: est. prob = NULL. Options are:

- est.prob = "empirical": The relative frequencies of each vector are computed internally. For example an input matrix rbind ( $1: 10,11: 20$ ) will be transformed to a probability vector rbind(1:10/sum(1:10), 11:20/ $\operatorname{sum}(11: 20))$


## Details

Function to compute the Jensen-Shannon Divergence $\operatorname{JSD}(\mathrm{P} \| \mathrm{Q})$ between two probability distributions P and Q with equal weights $\pi_{1}=\pi_{2}=1 / 2$.
The Jensen-Shannon Divergence $\operatorname{JSD}(\mathrm{P} \| \mathrm{Q})$ between two probability distributions P and Q is defined as:

$$
J S D(P \| Q)=0.5 *(K L(P \| R)+K L(Q \| R))
$$

where $R=0.5 *(P+Q)$ denotes the mid-point of the probability vectors P and Q , and $\mathrm{KL}(\mathrm{P} \| \mathrm{R})$, $\mathrm{KL}(\mathrm{Q} \| \mathrm{R})$ denote the Kullback-Leibler Divergence of P and R , as well as Q and R .

## General properties of the Jensen-Shannon Divergence:

- 1) JSD is non-negative.
- 2) $\operatorname{JSD}$ is a symmetric measure $\operatorname{JSD}(\mathrm{P} \| \mathrm{Q})=\operatorname{JSD}(\mathrm{Q} \| \mathrm{P})$.
- 3) $\mathrm{JSD}=0$, if and only if $\mathrm{P}=\mathrm{Q}$.


## Value

a distance value or matrix based on JSD computations.

## Author(s)

Hajk-Georg Drost

## References

Lin J. 1991. "Divergence Measures Based on the Shannon Entropy". IEEE Transactions on Information Theory. (33) 1: 145-151.

Endres M. and Schindelin J. E. 2003. "A new metric for probability distributions". IEEE Trans. on Info. Thy. (49) 3: 1858-1860.

## See Also

KL, H, CE, gJSD, distance

## Examples

```
# Jensen-Shannon Divergence between P and Q
P <- 1:10/sum(1:10)
Q <- 20:29/sum(20:29)
x <- rbind(P,Q)
JSD (x)
# Jensen-Shannon Divergence between P and Q using different log bases
JSD(x, unit = "log2") # Default
JSD(x, unit = "log")
JSD(x, unit = "log10")
# Jensen-Shannon Divergence Divergence between count vectors P.count and Q.count
P.count <- 1:10
Q.count <- 20:29
x.count <- rbind(P.count,Q.count)
JSD(x.count, est.prob = "empirical")
# Example: Distance Matrix using JSD-Distance
Prob <- rbind(1:10/sum(1:10), 20:29/sum(20:29), 30:39/sum(30:39))
# compute the KL matrix of a given probability matrix
JSDMatrix <- JSD(Prob)
# plot a heatmap of the corresponding JSD matrix
heatmap(JSDMatrix)
```

KL Kullback-Leibler Divergence

## Description

This function computes the Kullback-Leibler divergence of two probability distributions P and Q .

## Usage

KL(x, test.na = TRUE, unit = "log2", est.prob = NULL, epsilon = 1e-05)

## Arguments

x
test.na
unit
est.prob method to estimate probabilities from a count vector. Default: est.prob=NULL.
epsilon a small value to address cases in the KL computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon $=$ 0.01 ). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.

## Details

$$
K L(P \| Q)=\sum P(P) * \log 2(P(P) / P(Q))=H(P, Q)-H(P)
$$

where $H(P, Q)$ denotes the joint entropy of the probability distributions $P$ and $Q$ and $H(P)$ denotes the entropy of probability distribution $P$. In case $P=Q$ then $K L(P, Q)=0$ and in case $P!=Q$ then $K L(P, Q)>0$.
The KL divergence is a non-symmetric measure of the directed divergence between two probability distributions P and Q . It only fulfills the positivity property of a distance metric.
Because of the relation $\mathrm{KL}(\mathrm{P} \| \mathrm{Q})=\mathrm{H}(\mathrm{P}, \mathrm{Q})-\mathrm{H}(\mathrm{P})$, the Kullback-Leibler divergence of two probability distributions P and Q is also named Cross Entropy of two probability distributions P and Q.

## Value

The Kullback-Leibler divergence of probability vectors.

## Author(s)

Hajk-Georg Drost

## References

Cover Thomas M. and Thomas Joy A. 2006. Elements of Information Theory. John Wiley \& Sons.

## See Also

H, CE, JSD, gJSD, distance

## Examples

```
# Kulback-Leibler Divergence between P and Q
P<- 1:10/sum(1:10)
Q <- 20:29/sum(20:29)
x <- rbind(P,Q)
KL(x)
# Kulback-Leibler Divergence between P and Q using different log bases
KL(x, unit = "log2") # Default
KL(x, unit = "log")
KL(x, unit = "log10")
# Kulback-Leibler Divergence between count vectors P.count and Q.count
P.count <- 1:10
Q.count <- 20:29
x.count <- rbind(P.count,Q.count)
KL(x, est.prob = "empirical")
# Example: Distance Matrix using KL-Distance
Prob <- rbind(1:10/sum(1:10), 20:29/sum(20:29), 30:39/sum(30:39))
# compute the KL matrix of a given probability matrix
KLMatrix <- KL(Prob)
# plot a heatmap of the corresponding KL matrix
heatmap(KLMatrix)
```

```
kulczynski_d Kulczynski_d distance (lowlevel function)
```


## Description

The lowlevel function for computing the kulczynski_d distance.

## Usage

kulczynski_d(P, Q, testNA, epsilon)

## Arguments

P
a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.

## Author(s)

Hajk-Georg Drost

## Examples

```
kulczynski_d(P = 1:10/sum(1:10), Q = 20:29/sum(20:29),
    testNA = FALSE, epsilon = 0.00001)
```

kullback_leibler_distance
kullback-Leibler distance (lowlevel function)

## Description

The lowlevel function for computing the kullback_leibler_distance distance.

## Usage

kullback_leibler_distance(P, Q, testNA, unit, epsilon)

## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.
unit type of $\log$ function. Option are

- unit = "log"
- unit $=" \log 2 "$
- unit = "log10"
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon=0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.


## Author(s)

Hajk-Georg Drost

## Examples

```
kullback_leibler_distance(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE,
    unit = "log2", epsilon = 0.00001)
```


## Description

The lowlevel function for computing the kumar_hassebrook distance.

## Usage

kumar_hassebrook(P, Q, testNA)

## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

kumar_hassebrook( $P=1: 10 /$ sum(1:10) $, ~ Q=20: 29 /$ sum(20:29), testNA $=$ FALSE)
kumar_johnson Kumar-Johnson distance (lowlevel function)

## Description

The lowlevel function for computing the kumar_johnson distance.

## Usage

kumar_johnson(P, Q, testNA, epsilon)

## Arguments

P
Q
testNA
epsilon
a numeric vector storing the first distribution.
a numeric vector storing the second distribution.
a logical value indicating whether or not distributions shall be checked for NA values.
epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $x / 0$ or $0 / 0$ cases.

## Author(s)

Hajk-Georg Drost

## Examples

kumar_johnson( $P=1: 10 /$ sum (1:10), $Q=20: 29 /$ sum(20:29),
testNA $=$ FALSE, epsilon $=0.00001$ )

## k_divergence K-Divergence (lowlevel function)

## Description

The lowlevel function for computing the k_divergence distance.

## Usage

k_divergence(P, Q, testNA, unit)

## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.
unit type of log function. Option are

- unit = "log"
- unit = "log2"
- unit = "log10"


## Author(s)

Hajk-Georg Drost

## Examples

k_divergence ( $P=1: 10 /$ sum (1:10), $Q=20: 29 /$ sum(20:29), testNA $=$ FALSE, unit $=" \log 2 ")$
lin.cor Linear Correlation

## Description

This function computed the linear correlation between two vectors or a correlation matrix for an input matrix.
The following methods to compute linear correlations are implemented in this function:

## Usage

lin.cor (x, y = NULL, method = "pearson", test.na = FALSE)

## Arguments

$x \quad$ a numeric vector, matrix, or data.frame.
$y \quad$ a numeric vector that should be correlated with $x$.
method the method to compute the linear correlation between $x$ and $y$.
test.na a boolean value indicating whether input data should be checked for NA values.

## Details

- method = "pearson" : Pearson's correlation coefficient (centred).
- method = "pearson2" : Pearson's uncentred correlation coefficient.
- method = "sq_pearson" . Squared Pearson's correlation coefficient.
- method = "kendall" : Kendall's correlation coefficient.
- method = "spearman" : Spearman's correlation coefficient.

Further Details:

- Pearson's correlation coefficient (centred) :


## Author(s)

Hajk-Georg Drost

## Description

The lowlevel function for computing the lorentzian distance.

## Usage

lorentzian(P, Q, testNA, unit)

## Arguments

P
Q
testNA
unit
a numeric vector storing the first distribution.
a numeric vector storing the second distribution.
a logical value indicating whether or not distributions shall be checked for NA values.
type of log function. Option are

- unit = "log"
- unit $=" \log 2 "$
- unit = " $\log 10 "$


## Author(s)

Hajk-Georg Drost

## Examples

lorentzian( $P=1: 10 /$ sum(1:10), $Q=20: 29 / s u m(20: 29)$, testNA $=$ FALSE, unit $=$ "log2")
manhattan Manhattan distance (lowlevel function)

## Description

The lowlevel function for computing the manhattan distance.

## Usage

manhattan(P, Q, testNA)

## Arguments

P
a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA
a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

manhattan $(P=1: 10 /$ sum(1:10) $, ~ Q=20: 29 / s u m(20: 29)$, testNA $=F A L S E)$

```
matusita Matusita distance (lowlevel function)
```


## Description

The lowlevel function for computing the matusita distance.

## Usage

matusita(P, Q, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

matusita( $P=1: 10 /$ sum(1:10), $Q=20: 29 /$ sum(20:29), testNA $=$ FALSE $)$

## Description

Compute Shannon's Mutual Information based on the identity $I(X, Y)=H(X)+H(Y)-$ $H(X, Y)$ based on a given joint-probability vector $P(X, Y)$ and probability vectors $P(X)$ and $P(Y)$.

## Usage

$M I(x, y, x y, u n i t=" \log 2 ")$

## Arguments

x
$\mathrm{y} \quad$ a numeric probability vector $P(Y)$.
xy a numeric joint-probability vector $P(X, Y)$.
unit a character string specifying the logarithm unit that shall be used to compute distances that depend on log computations.

## Details

This function might be useful to fastly compute Shannon's Mutual Information for any given jointprobability vector and probability vectors.

## Value

Shannon's Mutual Information in bit.

## Author(s)

Hajk-Georg Drost

## References

Shannon, Claude E. 1948. "A Mathematical Theory of Communication". Bell System Technical Journal 27 (3): 379-423.

## See Also

H, JE, CE

## Examples

```
MI( x = 1:10/sum(1:10), y = 20:29/sum(20:29), xy = 1:10/sum(1:10) )
```

minkowski Minkowski distance (lowlevel function)

## Description

The lowlevel function for computing the minkowski distance.

## Usage

minkowski(P, Q, n, testNA)

## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
$\mathrm{n} \quad$ index for the minkowski exponent.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

minkowski $(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 / \operatorname{sum}(20: 29), n=2$, testNA $=$ FALSE $)$
motyka $\quad$ Motyka distance (lowlevel function)

## Description

The lowlevel function for computing the motyka distance.

## Usage

motyka(P, Q, testNA)

## Arguments

| $P$ | a numeric vector storing the first distribution. |
| :--- | :--- |
| $Q$ | a numeric vector storing the second distribution. |
| testNA | a logical value indicating whether or not distributions shall be checked for NA <br> values. |

## Author(s)

Hajk-Georg Drost

## Examples

```
motyka(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

```
neyman_chi_sq Neyman chi-squared distance (lowlevel function)
```


## Description

The lowlevel function for computing the neyman_chi_sq distance.

## Usage

neyman_chi_sq(P, Q, testNA, epsilon)

## Arguments

P a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.

## Author(s)

Hajk-Georg Drost

## Examples

```
neyman_chi_sq(P = 1:10/sum(1:10), Q = 20:29/sum(20:29),
    testNA = FALSE, epsilon = 0.00001)
```

```
pearson_chi_sq Pearson chi-squared distance (lowlevel function)
```


## Description

The lowlevel function for computing the pearson_chi_sq distance.

## Usage

pearson_chi_sq(P, Q, testNA, epsilon)

## Arguments

P a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.

## Author(s)

Hajk-Georg Drost

## Examples

```
pearson_chi_sq(P = 1:10/sum(1:10), Q = 20:29/sum(20:29),
    testNA = FALSE, epsilon = 0.00001)
```


## Description

The lowlevel function for computing the prob_symm_chi_sq distance.

## Usage

prob_symm_chi_sq(P, Q, testNA)

## Arguments

P a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
prob_symm_chi_sq(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

```
ruzicka Ruzicka distance (lowlevel function)
```


## Description

The lowlevel function for computing the ruzicka distance.

## Usage

ruzicka(P, Q, testNA)

## Arguments

P a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
ruzicka( \(P=1: 10 /\) sum(1:10), \(Q=20: 29 /\) sum(20:29), testNA \(=F A L S E)\)
```

```
soergel Soergel distance (lowlevel function)
```


## Description

The lowlevel function for computing the soergel distance.

## Usage

soergel( $\mathrm{P}, \mathrm{Q}$, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

soergel $(P=1: 10 /$ sum $(1: 10), Q=20: 29 /$ sum(20:29), testNA $=$ FALSE $)$

| sorensen $\quad$ Sorensen distance (lowlevel function) |
| :--- |

## Description

The lowlevel function for computing the sorensen distance.

## Usage

```
sorensen(P, Q, testNA)
```


## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

sorensen( $P$ = 1:10/sum(1:10), $Q=20: 29 /$ sum(20:29), testNA $=$ FALSE)

```
squared_chi_sq Squared chi-squared distance (lowlevel function)
```


## Description

The lowlevel function for computing the squared_chi_sq distance.

## Usage

squared_chi_sq(P, Q, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

squared_chi_sq( $P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 / \operatorname{sum}(20: 29)$, testNA $=$ FALSE $)$
squared_chord Squared chord distance (lowlevel function)

## Description

The lowlevel function for computing the squared_chord distance.

## Usage

squared_chord(P, Q, testNA)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
squared_chord(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

```
squared_euclidean Squared euclidean distance (lowlevel function)
```


## Description

The lowlevel function for computing the squared_euclidean distance.

## Usage

squared_euclidean(P, Q, testNA)

## Arguments

P a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

```
squared_euclidean(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```

```
taneja Taneja difference (lowlevel function)
```


## Description

The lowlevel function for computing the taneja distance.

## Usage

taneja(P, Q, testNA, unit, epsilon)

## Arguments

P
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.
unit type of $\log$ function. Option are

- unit = "log"
- unit $=" \log 2 "$
- unit = " $\log 10 "$
epsilon epsilon a small value to address cases in the distance computation where division by zero occurs. In these cases, x/0 or $0 / 0$ will be replaced by epsilon. The default is epsilon $=0.00001$. However, we recommend to choose a custom epsilon value depending on the size of the input vectors, the expected similarity between compared probability density functions and whether or not many 0 values are present within the compared vectors. As a rough rule of thumb we suggest that when dealing with very large input vectors which are very similar and contain many 0 values, the epsilon value should be set even smaller (e.g. epsilon $=0.000000001$ ), whereas when vector sizes are small or distributions very divergent then higher epsilon values may also be appropriate (e.g. epsilon = 0.01). Addressing this epsilon issue is important to avoid cases where distance metrics return negative values which are not defined and only occur due to the technical issues of computing $\mathrm{x} / 0$ or $0 / 0$ cases.


## Author(s)

Hajk-Georg Drost

## Examples

```
taneja(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE,
    unit = "log2", epsilon = 0.00001)
```

    tanimoto Tanimoto distance (lowlevel function)
    
## Description

The lowlevel function for computing the tanimoto distance.

## Usage

tanimoto( $\mathrm{P}, \mathrm{Q}$, testNA)

## Arguments

$P \quad$ a numeric vector storing the first distribution.
Q a numeric vector storing the second distribution.
testNA a logical value indicating whether or not distributions shall be checked for NA values.

## Author(s)

Hajk-Georg Drost

## Examples

tanimoto $(P=1: 10 / \operatorname{sum}(1: 10), Q=20: 29 / \operatorname{sum}(20: 29)$, testNA $=$ FALSE $)$

| Topsoe Topsoe distance (lowlevel function) |
| :--- |

## Description

The lowlevel function for computing the topsoe distance.

## Usage

topsoe(P, Q, testNA, unit)

## Arguments

P
Q
testNA
unit
a numeric vector storing the first distribution.
a numeric vector storing the second distribution.
a logical value indicating whether or not distributions shall be checked for NA values.

- unit = "log"
- unit = " $\log 2 "$
- unit $=" \log 10 "$


## Author(s)

Hajk-Georg Drost

## Examples

```
topsoe(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE, unit = "log2")
```

```
wave_hedges Wave hedges distance (lowlevel function)
```


## Description

The lowlevel function for computing the wave_hedges distance.

## Usage

wave_hedges(P, Q, testNA)

## Arguments

| P | a numeric vector storing the first distribution. |
| :--- | :--- |
| Q | a numeric vector storing the second distribution. |
| testNA | a logical value indicating whether or not distributions shall be checked for NA <br> values. |

## Author(s)

Hajk-Georg Drost

## Examples

```
wave_hedges(P = 1:10/sum(1:10), Q = 20:29/sum(20:29), testNA = FALSE)
```


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