

Package ‘rbart’

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Type Package

Title Bayesian Trees for Conditional Mean and Variance

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Description A model of the form $Y = f(x) + s(x)$ Z is fit where functions f and s are modeled with ensembles of trees and Z is standard normal.

This model is developed in the paper 'Heteroscedastic BART Via Multiplicative Regression Trees' (Pratola, Chipman, George, and McCulloch, 2019, [arXiv:1709.07542v2](https://arxiv.org/abs/1709.07542v2)).

BART refers to Bayesian Additive Regression Trees. See the R-package 'BART'.

The predictor vector x may be high dimensional.

A Markov Chain Monte Carlo (MCMC) algorithm provides Bayesian posterior uncertainty for both f and s .

The MCMC uses the recent innovations in Efficient Metropolis--Hastings proposal mechanisms for Bayesian regression tree models (Pratola, 2015, Bayesian Analysis, [doi:10.1214/16-BA999](https://doi.org/10.1214/16-BA999)).

License GPL (>= 2)

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Suggests knitr, rmarkdown, MASS, nnet

LinkingTo Rcpp

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Author Robert McCulloch [aut, cre, cph],
Matthew Pratola [aut, cph],
Hugh Chipman [aut, cph]

Maintainer Robert McCulloch <robert.e.mcculloch@gmail.com>

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hbartqqplot

Predictive qqplot for heterbart

Description

Given results from predict.rbart, gets draws from the predictive distribution at each x and then computes the empirical inverse cdf to get draws which would be uniform if the predictive were the true distribution. Then draws a qqplot against the uniform. In large enough samples, if the model is correct, the qqplot should look like a straight line with intercept 0 and slope 1. In small samples, we expect the predictive to be more spread out than the true distribution, even if the model is correct.

Usage

```
hbartqqplot(y,rbmod,nunif=10000,linecolor="red",linewd=3,...)
```

Arguments

<code>y</code>	<code>y</code> values corresponding to <code>x</code> values given to <code>predict.rbart</code> .
<code>rbmod</code>	Output list from <code>predict.rbart</code> .
<code>nunif</code>	Number of $\text{uniform}(0,1)$ draws used in constructing the qqplot.
<code>linecolor,linewd</code>	Line color and width for $(0,1)$ line.
<code>...</code>	Arguments passed on to <code>stats::qqplot</code> .

Value

quantiles of `y` in draws from the predictive (conditional on each `x` value).

Examples

```
#####
## please see vignette and/or www.rob-mcculloch.org for more realistic examples
#####

## get simulated data
data(simdat)

##get rbart run on the simulated data
data(rbartonsimd)

## Predictive quantile-quantile plot
temp = hbartqqplot(simdat$yp,rbartonsimd,xlab="predictive quantile",ylab="uniform",
cex.axis=1.4,cex.lab=1.2)
```

plotFunctionDraws

Plot matrix of function draws evaluated on a set of x

Description

Given draws of a function f_d , $d = 1, 2, \dots, D$ and a set of x vectors x_j , $j = 1, 2, \dots, J$, we have a $D \times J$ matrix of evaluations whose (d, j) element is $f_d(x_j)$, the d^{th} draw of the function evaluated at the $j^{th} x$. This function plots the draws by plotting estimates of $f(x_j)$ versus intervals for $f(x_j)$. The estimates are the mean of the j^{th} column and the intervals are two quantiles of the j^{th} column (e.g 5% and 95%).

Usage

```
plotFunctionDraws(fd,complevel=mean(fd),probs=c(.025,.975),
  xlab="posterior mean of function",ylab="posterior intervals",
  intervalcol="green",linecol="red",
  pts=NA,ptscol="blue", ptspch=1, ptscex=1, ...)
```

Arguments

<code>fd</code>	$D \times J$ matrix whose (d, j) element is the d^{th} function draw evaluated at the $j^{th} x$.
<code>complevel</code>	A horizontal line is drawn a complevel to compare the intervals to.
<code>probs</code>	The two quantiles used to construct the intervals.
<code>xlab</code>	Label for x axis.
<code>ylab</code>	Label for y axis.
<code>intervalcol</code>	Color to draw the intervals with.
<code>linecol</code>	Color to draw the comparizon horizontal line with.
<code>pts</code>	Add (x_j, pts_j) to the plot. For example pts could be fitted values from and alternative model such as the linear model.

ptscol Color to draw the points pts with.
 ptspch plot character to plot the points pts with.
 ptscex cex to plot the points pts with.
 ... Arguments passed on to call to graphics::plot.

Value

NULL

Examples

```

#####
## please see vignette and/or www.rob-mcculloch.org for more realistic examples
#####

## get simulated data
data(simdat)

##get rbart run on the simulated data
data(rbartonsimd)

## plot function (f and s) draws
shat = sqrt(mean((simdat$yp-rbartonsimd$mmean)^2)) #overall estimate of sigma
lmfit = lm(y~x,data.frame(x=simdat$x,y=simdat$y))
yhatlm = predict(lmfit,data.frame(x=simdat$xp)) #fits from a linear model

#Now we use plotFunctionDraws to look at mdraws (left panel) and sdraws (right panel).

## in the mean inference, you can see that the linear model seem unlikely
## in the variance inference, you can see that the posteriors of s(x) are far from a constant value
par(mfrow=c(1,2))

## look at mean inference
plotFunctionDraws(rbartonsimd$mdraws,complevel=mean(simdat$y), probs=c(.05,.95),
  xlab=expression(hat(f)(x)), pts=yhatlm, ptscol="black",
  cex.lab=1.2, cex.axis=1.4, main="intervals for f(x)")

##look at the standard deviation inference
plotFunctionDraws(rbartonsimd$sdraws, complevel=shat, xlab=expression(hat(s)(x)),
  intervalcol="magenta", linecol="blue",
  cex.lab=1.2, cex.axis=1.4, main="intervals for s(x)")

```

Description

The function `predict.rbart()` is the main function for drawing posterior predictive realizations at new inputs using a fitted model stored in a `rbart` object returned from `rbart()`.

Usage

```
## S3 method for class 'rbart'
predict(
  object,
  x.test=object$x.train,
  tc=1,
  fmean=mean(object$y.train),
  q.lower=0.025,
  q.upper=0.975,...)
```

Arguments

<code>object</code>	Object of type <code>rbart</code> from a previous call to <code>rbart()</code>
<code>x.test</code>	New input settings in the form of an <code>npred</code> x <code>p</code> matrix at which to construct predictions. Defaults to the training inputs.
<code>tc</code>	Number of OpenMP threads to use for parallel computing.
<code>fmean</code>	Mean-centering vector for the training data. Defaults to the value used by <code>rbart()</code> when fitting the model. Usually should be left to the default.
<code>q.lower</code>	Lower quantile to return.
<code>q.upper</code>	Upper quantile to return.
<code>...</code>	not used.

Details

`predict.rbart()` is the main function for calculating posterior predictions and uncertainties once a model has been fit by `rbart()`.

Returns an object of type `rbart` with the following entries.

Value

<code>mdraws</code>	Posterior realizations of the mean function, $f(\mathbf{x})$ stored in an <code>ndpost</code> x <code>npred</code> matrix, where <code>ndpost</code> is the number of kept MCMC draws in the <code>rbart</code> run.
<code>sdraws</code>	Posterior realizations of the standard deviation function, $s(\mathbf{x})$ stored in an <code>ndpost</code> x <code>npred</code> matrix, where <code>ndpost</code> is the number of kept MCMC draws in the <code>rbart</code> run.
<code>mmean</code>	Posterior predictive mean of $f(\mathbf{x})$.
<code>smean</code>	Posterior predictive mean of the standard deviation, $s(\mathbf{x})$.
<code>msd</code>	Posterior standard deviation of the mean, $f(\mathbf{x})$.
<code>ssd</code>	Posterior standard deviation of the standard deviation, $s(\mathbf{x})$.
<code>m.5</code>	Posterior median of the mean function realizations, $f(\mathbf{x})$.
<code>m.lower</code>	Posterior <code>q.lower</code> quantile of the mean function realizations.
<code>m.upper</code>	Posterior <code>q.upper</code> quantile of the mean function realizations.
<code>s.5</code>	Posterior median of the standard deviation function realizations, $s(\mathbf{x})$.

<code>s.lower</code>	Posterior q.lower quantile of the standard deviation function realizations.
<code>s.upper</code>	Posterior q.upper quantile of the standard deviation function realizations.
<code>q.lower</code>	Lower quantile used in constructing the above.
<code>q.upper</code>	Upper quantile used in constructing the above.

Author(s)

Matthew T. Pratola <mpratola@stat.osu.edu> [aut, cre, cph], Robert E. McCulloch <robert.e.mcculloch@gmail.com> [aut, cre, cph], Hugh Chipman <chugh(chipman@gmail.com)> [aut, cph] Maintainer: Matthew T. Pratola <mpratola@stat.osu.edu>, Robert E. McCulloch <robert.e.mcculloch@gmail.com>

References

- Chipman, Hugh A., George, Edward I., and McCulloch, Robert E. (1998) Bayesian CART model search. *Journal of the American Statistical Association*, **93**, 935–948.
- Chipman, Hugh A., George, Edward I., and McCulloch, Robert E. (2010) BART: Bayesian additive regression trees. *The Annals of Applied Statistics*, **4**, 266–298.
- Pratola, Matthew T. (2016) Efficient Metropolis Hastings proposal mechanisms for Bayesian regression tree models. *Bayesian analysis*, **11**, 885–911.
- Pratola, Matthew T., Chipman, Hugh A., George, Edward I. and McCulloch, Robert E. (2017) Heteroscedastic BART Using Multiplicative Regression Trees. *arXiv preprint*, **arXiv:1709.07542**, 1–20.

See Also

[rbart](#)

Examples

```
#####
## This is just a stub (runs fast) example for testing.
## For more realistic examples, please see:
##   (i) the vignette at www.rob-mcculloch.org
##   (ii) the example simulated data (see ?simdat)
##       and the longer run in ?rbartonsimd,
##       where a saved run of rbart is run on simdat is plotted.
#####

##simulate data
set.seed(99)

# train data
n=500 #train data sample size
p=1 #just one x
x = matrix(sort(runif(n*p)),ncol=p) #iid uniform x values
fx = 4*(x[,1]^2) #quadratic function f
sx = .2*exp(2*x[,1]) # exponential function s
y = fx + sx*rnorm(n) # y = f(x) + s(x) Z
```

```

#test data (the p added to the variable names is for predict)
np=500 #test data sample size
xp = matrix(sort(runif(np*p)),ncol=p)
fxp = 4*(xp[,1]^2)
sxp = .2*exp(2*xp[,1])
yp = fxp + sxp*rnorm(np)

##run rbart MCMC
# The number of interations is kept small to make example run,
##!!!! REAL APPLICATIONS MAY NEED LONGER RUNS !!!!
# nskip: burn in draws,
# ndpost:kept draws,
# nadapt: initial draws to tune MCMC,
# numcut: number of cutpoints used for each x
# k: bigger k gives smoother f (default is 2)
set.seed(19)
res = rbart(x,y,nskip=10,ndpost=20,nadapt=0,numcut=1000,k=5) #again, this is way too short a run!!!
## now predict to get inference
resp = predict(res,x.test=xp)

##check out of sample fit
cat("out of sample cor(f,fhat) is ",cor(fxp,resp$mmean),"\\n")
cat("out of sample cor(s,sht) is ",cor(sxp,resp$smean),"\\n")

##plot estimated vs. true
##plot the data
plot(xp,yp,cex.axis=1.5,cex.lab=1.5)
lines(xp,fxp,col="blue")
lines(xp,fx+2*sxp,col="blue",lty=2)
lines(xp,fxp-2*sxp,col="blue",lty=2)

## add the fit
lines(xp,resp$mmean) #estimate of f
lines(xp,resp$mmean+2*resp$smean) #estimate of sd
lines(xp,resp$mmean-2*resp$smean) #estimate of sd

```

rbart

Fitting Bayesian Regression Tree models supported by rbart.

Description

The function `rbart()` is the main function for fitting Bayesian Regression Tree models, including single-tree models, Bayesian Additive Regression Tree (BART) models and Heteroscedastic BART models. `rbart()` maintains some degree of backwards compatibility with `BayesTree::bart()`, while offering many new options.

Usage

```
rbart(x.train, y.train, x.test=matrix(0.0,0,0), ntree=200, ntreesh=40, ndpost=1000,
      nskip=100, k=2, power=2.0, base=.95, tc=1, sigmav=rep(1,length(y.train)),
```

```
fmean=mean(y.train), overallsd = sd(y.train), overallnu=10,
chv = cor(x.train,method="spearman"), pbd=.7, pb=.5, stepwpert=.1, probchv=.1,
minnumbot=5, printevery=100, numcut=100, xicuts=NULL, nadapt=1000, adaptevery=100,
summarystats=FALSE)
```

Arguments

x.train	nxp matrix of predictor variables for the training data.
y.train	nx1 vector of the observed response for the training data.
x.test	mxp matrix of predictor variables for the test set. Deprecated.
ntree	Number of trees used for the mean model.
ntreeh	Number of trees used for the variance model.
ndpost	Number of iterations to run the MCMC algorithm after burn-in.
nskip	Number of MCMC iterations treated as burn-in and discarded.
k	Prior hyperparameter for the mean model.
power	Power parameter in the tree depth penalizing prior.
base	Base parameter in the tree depth penalizing prior.
tc	Number of OpenMP threads to use.
sigmav	Initialization of square-root of variance parameter.
fmean	Overall mean of the data for pre-centering the data before running the model.
overallsd	A rudimentary estimate of the process standard deviation. Used in calibrating the variance prior.
overallnu	Shape parameter for the variance prior.
chv	Predictor correlation matrix used as a pre-conditioner for MCMC change-of-variable proposals.
pbd	Probability of performing a birth/death proposal, otherwise perform a rotate proposal.
pb	Probability of performing a birth proposal given that we choose to perform a birth/death proposal.
stepwpert	Initial width of proposal distribution for peturbing cutpoints.
probchv	Probability of performing a change-of-variable proposal. Otherwise, only do a perturb proposal.
minnumbot	Minimum number of observations required in bottom (terminal) nodes.
printevery	Outputs MCMC algorithm status every printevery iterations.
numcut	Number of cutpoints to use for each predictor variable.
xicuts	More detailed construction of cutpoints can be specified using makecuts() and passed as an argument here.
nadapt	Number of MCMC iterations allowed for adaptive MCMC. These are also discarded.
adaptevery	Adapt MCMC proposal distributions every adaptevery iterations until the algorithm has run for nadapt iterations.
summarystats	Return detailed summary statistics about the fitting procedure.

Details

`rbart()` is the main model fitting function for continuous response data. The most general form of the model allowed is $Y(\mathbf{x}) = f(\mathbf{x}) + s(\mathbf{x})Z$ where Z is $N(0, 1)$ and $f(\mathbf{x}) = \sum_{j=1}^m g(\mathbf{x}; T_j, M_j)$ and $s(\mathbf{x}) = \prod_{j=1}^{m'} h(\mathbf{x}; T'_j, M'_j)$, where the $g(\cdot; T_j, M_j)$ represent additive tree components used for modeling the mean and $h(\cdot; T'_j, M'_j)$ represent multiplicative tree components used for modeling the variance.

The most common models to fit are a homoscedastic single-tree model, a homoscedastic BART model and a heteroscedastic BART model.

For a BART model, set `pbd=c(0.7, 0.0)` and `ntreeh=1`. This forces a scalar (homoscedastic) variance term.

For a single-tree model, set `pbd=(0.7, 0.0)`, `ntreeh=1` and `ntree=1`. This forces the mean component to be modeled using only one tree.

The heteroscedastic BART model is the default.

Value

`res` Fitted model object of S3 class `rbart`.

Author(s)

Matthew T. Pratola <mpratola@stat.osu.edu> [aut, cre, cph], Robert E. McCulloch <robert.e.mcculloch@gmail.com> [aut, cre, cph], Hugh Chipman <[chugh\(chipman@gmail.com](mailto:chugh(chipman@gmail.com))> [aut, cph] Maintainer: Matthew T. Pratola <mpratola@stat.osu.edu>, Robert E. McCulloch <robert.e.mcculloch@gmail.com>

References

- Chipman, Hugh A., George, Edward I., and McCulloch, Robert E. (1998) Bayesian CART model search. *Journal of the American Statistical Association*, **93**, 935–948.
- Chipman, Hugh A., George, Edward I., and McCulloch, Robert E. (2010) BART: Bayesian additive regression trees. *The Annals of Applied Statistics*, **4**, 266–298.
- Pratola, Matthew T. (2016) Efficient Metropolis Hastings proposal mechanisms for Bayesian regression tree models. *Bayesian analysis*, **11**, 885–911.
- Pratola, Matthew T., Chipman, Hugh A., George, Edward I. and McCulloch, Robert E. (2017) Heteroscedastic BART Using Multiplicative Regression Trees. *arXiv preprint*, **arXiv:1709.07542**, 1–20.

See Also

[predict.rbart](#)

Examples

```
#####
## This is just a stub (runs fast) example for testing.
## For more realistic examples, please see:
##   (i) the vignette at www.rob-mcculloch.org
##   (ii) the example simulated data (see ?simdat)
```

```

##      and the longer run in ?rbartonsimd,
##      where a saved run of rbart is run on simdat is plotted.
#####
##simulate data
set.seed(99)

# train data
n=500 #train data sample size
p=1 #just one x
x = matrix(sort(runif(n*p)),ncol=p) #iid uniform x values
fx = 4*(x[,1]^2) #quadratic function f
sx = .2*exp(2*x[,1]) # exponential function s
y = fx + sx*rnorm(n) # y = f(x) + s(x) Z

#test data (the p added to the variable names is for predict)
np=500 #test data sample size
xp = matrix(sort(runif(np*p)),ncol=p)
fxp = 4*(xp[,1]^2)
sxp = .2*exp(2*xp[,1])
yp = fxp + sxp*rnorm(np)

##run rbart MCMC
# The number of interations is kept small to make example run,
##!!!! REAL APPLICATIONS MAY NEED LONGER RUNS !!!!
#   nskip: burn in draws,
#   ndpost:kept draws,
#   nadapt: initial draws to tune MCMC,
#   numcut: number of cutpoints used for each x
#   k: bigger k gives smoother f (default is 2)
set.seed(19)
res = rbart(x,y,nskip=10,ndpost=20,nadapt=0,numcut=1000,k=5) #again, this is way too short a run!!!
## now predict to get inference
resp = predict(res,x.test=xp)

##check out of sample fit
cat("out of sample cor(f,fhat) is ",cor(fxp,resp$mmean),"\n")
cat("out of sample cor(s,shat) is ",cor(sxp,resp$smean),"\n")

##plot estimated vs. true
##plot the data
plot(xp,yp,cex.axis=1.5,cex.lab=1.5)
lines(xp,fxp,col="blue")
lines(xp,fx+2*sxp,col="blue",lty=2)
lines(xp,fxp-2*sxp,col="blue",lty=2)

## add the fit
lines(xp,resp$mmean) #estimate of f
lines(xp,resp$mmean+2*resp$smean) #estimate of sd
lines(xp,resp$mmean-2*resp$smean) #estimate of sd

```

<code>rbartModelMatrix</code>	<i>Model Matrix for BART</i>
-------------------------------	------------------------------

Description

`rbartModelMatrix` takes a data frame of explanatory variables (`x`) and turns it into a numeric matrix suitable for BART. This is used when some of the `x` variables are factors. The returned matrix will first have columns for all the numeric variables in the data frame and then columns for all the factors expanded into binary dummy variables. Note that if a factor has `k` levels, then BART wants `k` dummies (not `k-1` as in linear regression). So for example, a factor with two levels named `xf`, will result in two columns named `xf1` and `xf2`.

Usage

```
rbartModelMatrix(xdf)
```

Arguments

<code>xdf</code>	Data frame to be turned into a matrix for BART.
------------------	---

Value

Matrix from of explanatory variables in the data frame `xdf`.

Examples

```
set.seed(99)
xdf = data.frame(x1=1:15,x2=as.factor(c(rep(1,5),rep(2,5),rep(3,5))),
                 x3=runif(15),x4=as.factor(c(rep(1,5),rep(2,10))))
print(head(xdf))

xm = rbartModelMatrix(xdf)
print(head(xm))
```

<code>rbartonsimd</code>	<i>rbart run on simulated data</i>
--------------------------	------------------------------------

Description

`predict.rbart` results for simulated data.

Usage

```
data("rbartonsimd")
```

Format

`rbartonsimd` returned list from a call to `predict.rbart` on the simulated data.

Details

The data `rbartonsimd` is the results of an `rbart` run on the simulated data in `simdat`.

The code for the `rbart` run is:

```
## load data
data(simdat)
attach(simdat) #some people think this is a bad idea

## run rbart
set.seed(99)
res = rbart(x,y,nskip=500,ndpost=100,nadapt=500,adaptevery=50)
rbartonsimd = predict(res,xp) #get prediction for test x in xp
```

Examples

```
## load simulated data and rbart run.
data(rbartonsimd)
data(simdat)

## plot data and x vs f(x), x vs f(x) +/- 2s(x), x in test data, true and estimated
## data
plot(simdat$xp,simdat$yp)
## true
lines(simdat$xp,simdat$fp,col="blue",lty=2,lwd=2)
lines(simdat$xp,simdat$fp+2*simdat$sp,col="blue",lty=2,lwd=2)
lines(simdat$xp,simdat$fp-2*simdat$sp,col="blue",lty=2,lwd=2)
##estimated
mhat = rbartonsimd$mmean; shat = rbartonsimd$smean
lines(simdat$xp,mhat,col="red",lty=1,lwd=2)
lines(simdat$xp,mhat + 2*shat,col="red",lty=1,lwd=2)
lines(simdat$xp,mhat - 2*shat,col="red",lty=1,lwd=2)

## note that you can get "nicer" looking fits by
## (i) running rbart longer (e.g. ndpost=500),
## (ii) using numcut=1000,k=5 in rbart.
```

Description

Simulated data with nonlinear mean and heteroskedasticity.

Usage

```
data("simdat")
```

Format

- x simulated train x values
- y simulated train y values
- xp simulated test xp values
- yp simulated test yp values
- fx true f evaluated on train x
- sx true s evaluated on train x
- fxp true f evaluated on test xp
- sxp true s evaluated on test xp

Details

The simulated data in simdat was generated using the code:

```
##simulate data
set.seed(99)

# train data
n=500 #train data sample size
p=1 #just one x
x = matrix(sort(runif(n*p)),ncol=p) #iid uniform x values
fx = 4*(x[,1]^2) #quadratic function f
sx = .2*exp(2*x[,1]) # exponential function s
y = fx + sx*rnorm(n) # y = f(x) + s(x) Z

#test data (the p added to the variable names is for predict)
np=1000 #test data sample size
xp = matrix(sort(runif(np*p)),ncol=p)
fpx = 4*(xp[,1]^2)
sxp = .2*exp(2*xp[,1])
yp = fpx + sxp*rnorm(np)
```

Examples

```
data(simdat)

## plot x vs y with f(x) and f(x) +/- 2s(x) for train and test simulated data
##train
plot(simdat$x,simdat$y,xlab="x",ylab="y")
##test
points(simdat$xp,simdat$yp,col="red",pch=2)
lines(simdat$xp,simdat$fpx,col="blue",lwd=2)
lines(simdat$xp,simdat$fpx+2*simdat$sxp,col="blue",lwd=2,lty=2)
```

```
lines(simdat$xp,simdat$fxp-2*simdat$sxp,col="blue",lwd=2,lty=2)
legend("topleft",legend=c("train","test"),pch=c(1,2),col=c("black","red"))
```

ucarprice

Used Car Prices

Description

Prices of used cars and variables describing the cars.

Usage

```
data("ucarprice")
```

Format

A data frame with 1,000 observations on the following 7 variables.

price numeric: price of a car in dollars
trim factor: trim of the car, for example, how the interior is styled
isOneOwner factor: has the car had just one owner, t=yes, f=no
mileage numeric: number of miles the car has been driven
year numeric: model year of the car
color factor: color of the car
displacement factor: displacement of the car engine

Examples

```
data(ucarprice)
```

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