# Package 'rsvddpd' 

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## Type Package

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Title Robust Singular Value Decomposition using Density Power
Divergence
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Description Computing singular value decomposition with robustness is a challenging task.
This package provides an implementation of computing robust SVD using density power divergence ([arXiv:2109.10680](arXiv:2109.10680)). It combines the idea of robustness and efficiency in estimation based on a tuning parameter. It also provides utility functions to simulate various scenarios to compare performances of different algorithms.
License MIT + file LICENSE
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LinkingTo Rcpp, RcppArmadillo
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## Description

AddOutlier returns a matrix with outliers randomly added to a matrix given certain proportion of contamination

## Usage

AddOutlier (X, proportion, value, seed = NULL, method = "element")

## Arguments

X matrix, to which outliers are added
proportion
numeric, proportion of elements, rows or columns to be contaminated. Must be between 0 and 1 .
value numeric, the outlying value to be used for contamination
seed numeric, a seed to reproduce the randomization behaviour
method character, must be one of the following:

- "element" - For contaminating at random positions of the matrix
- "row" - For contaminating an entire row of the matrix
- "col" - For contaminating an entire column of the matrix


## Value

A matrix with elements / rows / columns contaminated.

## Note

Due to randomization, it is possible that the none of the entries of the matrix become contaminated. In that case, it is recommended to use different seed value.

## Examples

$X=$ matrix $(1: 20$, nrow $=4$, ncol $=5)$
AddOutlier (X, 0.5, 10, seed $=1234$ )

## Description

cv .alpha returns the optimal robustness parameter

## Usage

cv.alpha(X, alphas = 10)

## Arguments

X
matrix, whose singular value decomposition is required
alphas numeric vector, vector of robustness parameters to try.

## Value

A list containing

- The choices of the robust parameters.
- Corresponding cross validation score.
- Best choice of the robustness parameter.


## References

S. Roy, A. Basu and A. Ghosh (2021), A New Robust Scalable Singular Value Decomposition Algorithm for Video Surveillance Background Modelling https://arxiv.org/abs/2109. 10680

rSVDdpd | Robust Singular Value Decomposition using Density Power Diver- |
| :--- |
| gence |

## Description

rSVDdpd returns the singular value decomposition of a matrix with robust singular values in presence of outliers

## Usage

```
rSVDdpd(
    X ,
    alpha,
    nd \(=\) NA,
    tol \(=1 \mathrm{e}-04\),
    eps \(=1 \mathrm{e}-04\),
    maxiter = 100L,
    initu = NULL,
    initv = NULL
)
```


## Arguments

X
matrix, whose singular value decomposition is required
alpha
nd integer, must be lower than $\operatorname{nrow}(X)$ and $n \operatorname{col}(X)$ both. If NA, defaults to $\min (\operatorname{nrow}(X), \operatorname{ncol}(X))$
tol numeric, a tolerance level. If the residual matrix has lower norm than this, then subsequent singular values will be taken as 0 .
eps numeric, a tolerance level for the convergence of singular vectors. If in subsequent iterations the singular vectors do not change its norm beyond this, then the iteration will stop.
maxiter integer, upper limit to the maximum number of iterations.
initu matrix, initializing vectors for left singular values. Must be of dimension $\operatorname{nrow}(X) \times \min (\operatorname{nrow}(X), \operatorname{ncol}(X))$. If NULL, defaults to random initialization.
initv matrix, initializing vectors for right singular values. Must be of dimension $n \operatorname{col}(X) \times \min (\operatorname{nrow}(X), \operatorname{ncol}(X))$. If NULL, defaults to random initialization.

## Details

The usual singular value decomposition is highly prone to error in presence of outliers, since it tries to minimize the $L_{2}$ norm of the errors between the matrix $X$ and its best lower rank approximation. While there is considerable effort to impose robustness using $L_{1}$ norm of the errors instead of $L_{2}$ norm, such estimation lacks efficiency. Application of density power divergence bridges the gap.

$$
D P D(f \mid g)=\int f^{(1+\alpha)}-\left(1+\frac{1}{\alpha}\right) \int f^{\alpha} g+\frac{1}{\alpha} \int g^{(1+\alpha)}
$$

The parameter alpha should be between 0 and 1 , if not, then a warning is shown. Lower alpha means less robustness but more efficiency in estimation, while higher alpha means high robustness but less efficiency in estimation. The recommended value of alpha is 0.3 . The function tries to obtain the best rank one approximation of a matrix by minimizing this density power divergence of the true errors with that of a normal distribution centered at the origin.

## Value

A list containing different components of the decomposition $X=U D V^{\prime}$

- d - The robust singular values, namely the diagonal entries of $D$.
- u - The matrix of left singular vectors $U$. Each column is a singular vector.
- v - The matrix of right singular vectors $V$. Each column is a singular vector.


## References

S. Roy, A. Basu and A. Ghosh (2021), A New Robust Scalable Singular Value Decomposition Algorithm for Video Surveillance Background Modelling https://arxiv.org/abs/2109. 10680

## See Also

svd

## Examples

```
X = matrix(1:20, nrow = 4, ncol = 5)
rSVDdpd(X, alpha = 0.3)
```

simSVD

Simulate SVD and measure performances of various algorithms

## Description

simSVD simulates various models for the errors in the data matrix, and summarize performance of a singular value decomposition algorithm under presence or absence of outlying data introduced through various outlying schemes, using Monte Carlo approach.

## Usage

simSVD(
trueSVD,
svdfun,
$B=100$,
seed $=$ NULL,
dist = "normal", tau $=0.95$, outlier = FALSE, out_method = "element", out_value = 10, out_prop = 0.1, return_details = FALSE,
)

## Arguments

| trueSVD | list, containing three different named components. <br> - d-a vector containing the singular values. <br> - u-a matrix with left singular vectors, each column being a singular vector. <br> - v - a matrix with right singular vectors, each column being a singular vector. |
| :---: | :---: |
| svdfun | function which takes a numeric matrix as first argument and returns singular value decomposition of it as a list, with three components d , u and v as indicated before. |
| B | numeric, denoting the number of Monte Carlo simulation. |
| seed | numeric, a seed value used for reproducibility. |
| dist | character string, denoting the distribution from which errors will be generated. It must be equal to one of the following: normal, cauchy, exp, logis, lognormal |
| tau | numeric, a value between 0 and 1 , see details for more. |
| outlier | logical, if TRUE, simulates the situation by adding outliers. |
| out_method | character, the method to add outliers. Must be one of "element", "row" or "col". See AddOutlier for details. |
| out_value | numeric, the outlying observation. See AddOutlier for details. |
| out_prop | a numeric, between 0 and 1 denoting the proportion of contamination. See AddOutlier for details. |
| return_details | logical, whether to return detailed results for each Monte Carlo simulation. See value for details. |
|  | extra arguments to be passed to svdfun function. |

## Value

Based on whether return_details is TRUE or FALSE, returns a list with two or one components.

- Simulations :
- Lambda - A matrix containing obtained singular values from all Monte Carlo Simulations.
- Left - A matrix containing the dissimilarities between left singular vectors of true SVD and obtained SVD.
- Right - A matrix containing the dissimilarities between right singular vectors of true SVD and obtained SVD.
- Summary:
- Bias - A numeric vector showing biases of the singular vectors obtained by svdfun algorithm.
- MSE - A numeric vector showing MSE of the singular vectors obtained by svdfun algorithm.
- Variance - A numeric vector showing variances of the singular vectors obtained by svdfun algorithm.
- Left - A numeric vector showing average dissimilarities between true and estimated left singular vectors.
- Right - A numeric vector showing average dissimilarities between true and estimated right singular vectors.

If return_details is FALSE, only Summary component of the larger list is returned.

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