# Package 'susieR'

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Type Package

Title Sum of Single Effects Linear Regression

Description Implements methods for variable selection in linear regression based on the ``Sum of Single Effects" (SuSiE) model, as described in Wang et al (2020) <DOI:10.1101/501114> and Zou et al (2021) <DOI:10.1101/2021.11.03.467167>. These methods provide simple summaries, called ``Credible Sets", for accurately quantifying uncertainty in which variables should be selected. The methods are motivated by genetic fine-mapping applications, and are particularly well-suited to settings where variables are highly correlated and detectable effects are sparse. The fitting algorithm, a Bayesian analogue of stepwise selection methods called ``Iterative Bayesian Stepwise Selection" (IBSS), is simple and fast, allowing the SuSiE model be fit to large data sets (thousands of samples and hundreds of thousands of variables).

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coef.susie

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coef.susie

Extract regression coefficients from susie fit

## Description

Extract regression coefficients from susie fit

### Usage

```
## S3 method for class 'susie'
coef(object, ...)
```

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### **Arguments**

object A susie fit.

... Additional arguments passed to the generic coef method.

### Value

A p+1 vector, the first element being an intercept, and the remaining p elements being estimated regression coefficients.

compute\_ss

Compute sufficient statistics for input to susie\_suff\_stat

### Description

This is a synonym for compute\_suff\_stat included for historical reasons (deprecated).

### Usage

```
compute_ss(X, y, standardize = FALSE)
```

### Arguments

X An n by p matrix of covariates.

y An n vector.

standardize Logical flag indicating whether to standardize columns of X to unit variance

prior to computing summary data.

### Value

A list of sufficient statistics (X'X, X'y, y'y and n)

```
data(N2finemapping)
ss = compute_ss(N2finemapping$X, N2finemapping$Y[,1])
```

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compute\_suff\_stat

Compute sufficient statistics for input to susie\_suff\_stat

### **Description**

Computes the sufficient statistics X'X, X'y, y'y and n after centering (and possibly standardizing) the columns of X and centering y to have mean zero. We also store the column means of X and mean of y.

### Usage

```
compute_suff_stat(X, y, standardize = FALSE)
```

### **Arguments**

X An n by p matrix of covariates.

y An n vector.

standardize Logical flag indicating whether to standardize columns of X to unit variance

prior to computing summary data

#### Value

A list of sufficient statistics (XtX, Xty, yty, n) and X\_colmeans, y\_mean.

### **Examples**

```
data(N2finemapping)
ss = compute_suff_stat(N2finemapping$X, N2finemapping$Y[,1])
```

estimate\_s\_rss

Estimate s in susie\_rss Model Using Regularized LD

### **Description**

The estimated s gives information about the consistency between the z scores and LD matrix. A larger s means there is a strong inconsistency between z scores and LD matrix. The "null-mle" method obtains mle of s under  $z|R\ N(0,(1-s)R+sI),\,0< s<1$ . The "null-partialmle" method obtains mle of s under  $U^Tz|R\ N(0,sI)$ , in which U is a matrix containing the of eigenvectors that span the null space of R; that is, the eigenvectors corresponding to zero eigenvalues of R. The estimated s from "null-partialmle" could be greater than 1. The "null-pseudomle" method obtains mle of s under pseudolikelihood  $L(s)=\prod_{i=1}^p p(z_j|z_{-j},s,R),\,0< s<1$ .

### Usage

```
estimate_s_rss(z, R, n, r_tol = 1e-08, method = "null-mle")
```

### **Arguments**

| Z      | A p-vector of z scores.                                                    |
|--------|----------------------------------------------------------------------------|
| R      | A p by p symmetric, positive semidefinite correlation matrix.              |
| n      | The sample size. (Optional, but highly recommended.)                       |
| r_tol  | Tolerance level for eigenvalue check of positive semidefinite matrix of R. |
| method | a string specifies the method to estimate $s$ .                            |

### Value

A number between 0 and 1.

### **Examples**

```
set.seed(1)
n = 500
p = 1000
beta = rep(0,p)
beta[1:4] = 0.01
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
input_ss = compute_suff_stat(X,y,standardize = TRUE)
ss = univariate_regression(X,y)
R = cor(X)
attr(R, "eigen") = eigen(R, symmetric = TRUE)
zhat = with(ss,betahat/sebetahat)
# Estimate s using the unadjusted z-scores.
s0 = estimate_s_rss(zhat,R)
# Estimate s using the adjusted z-scores.
s1 = estimate_s_rss(zhat,R,n)
```

FinemappingConvergence

Simulated Fine-mapping Data with Convergence Problem.

### **Description**

Data simulated using real genotypes from 50,000 individuals and 200 SNPs. Two of the SNPs have non-zero effects on the multivariate response. The response data are generated under a linear regression model. The simulated response and the columns of the genotype matrix are centered.

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### **Format**

FinemappingConvergence is a list with the following elements:

**XtX** Summary statistics computed using the centered and scaled genotype matrix.

**Xty** Summary statistics computed using the centered and scaled genotype data, and the centered simulated response.

yty yty is computed using the centered simulated response.

**n** The sample size (50,000).

true\_coef The coefficients used to simulate the responses.

**z** z-scores from a simple (single-SNP) linear regression.

### See Also

A similar data set with more SNPs is used in the "Refine SuSiE model" vignette.

### **Examples**

data(FinemappingConvergence)

get\_cs\_correlation Get Correlations Between CSs, using Variable with Maximum PIP
From Each CS

### **Description**

This function evaluates the correlation between single effect CSs. It is not part of the SuSiE inference. Rather, it is designed as a diagnostic tool to assess how correlated the reported CS are.

#### Usage

```
get_cs_correlation(model, X = NULL, Xcorr = NULL, max = FALSE)
```

### Arguments

| model | A SuSiE fit, typically an output from susie or one of its variants.                                                                                                                                                                |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| X     | n by p matrix of values of the p variables (covariates) in n samples. When provided, correlation between variables will be computed and used to remove CSs whose minimum correlation among variables is smaller than min_abs_corr. |
| Xcorr | p by p matrix of correlations between variables (covariates). When provided, it will be used to remove CSs whose minimum correlation among variables is smaller than min_abs_corr.                                                 |
| max   | When max = FAFLSE, return a matrix of CS correlations. When max = TRUE, return only the maximum absolute correlation among all pairs of correlations.                                                                              |

#### Value

A matrix of correlations between CSs, or the maximum absolute correlation when max = TRUE.

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| kriging_rss | Compute Distribution of z-scores of Variant j Given Other z-scores, and Detect Possible Allele Switch Issue |
|-------------|-------------------------------------------------------------------------------------------------------------|
|             |                                                                                                             |

### **Description**

Under the null, the rss model with regularized LD matrix is z|R,s N(0,(1-s)R+sI)). We use a mixture of normals to model the conditional distribution of z\_j given other z scores,  $z_j|z_{-j},R,s$   $\sum_{k=1}^K \pi_k N(-\Omega_{j,-j}z_{-j}/\Omega_{j,k})$   $\Omega=((1-s)R+sI)^{-1},\ \sigma_1,...,\sigma_k$  is a grid of fixed positive numbers. We estimate the mixture weights  $\pi$  We detect the possible allele switch issue using likelihood ratio for each variant.

### Usage

```
kriging_rss(
    z,
    R,
    n,
    r_tol = 1e-08,
    s = estimate_s_rss(z, R, n, r_tol, method = "null-mle")
)
```

### **Arguments**

```
    A p-vector of z scores.
    A p by p symmetric, positive semidefinite correlation matrix.
    The sample size. (Optional, but highly recommended.)
    Tolerance level for eigenvalue check of positive semidefinite matrix of R.
    an estimated s from estimate_s_rss
```

### Value

a list containing a ggplot2 plot object and a table. The plot compares observed z score vs the expected value. The possible allele switched variants are labeled as red points ( $\log LR > 2$  and abs(z) > 2). The table summarizes the conditional distribution for each variant and the likelihood ratio test. The table has the following columns: the observed z scores, the conditional expectation, the conditional variance, the standardized differences between the observed z score and expected value, the log likelihood ratio statistics.

```
# See also the vignette, "Diagnostic for fine-mapping with summary # statistics." set.seed(1) n = 500 p = 1000 beta = rep(0,p) beta[1:4] = 0.01
```

N2finemapping

```
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
ss = univariate_regression(X,y)
R = cor(X)
attr(R,"eigen") = eigen(R,symmetric = TRUE)
zhat = with(ss,betahat/sebetahat)
cond_dist = kriging_rss(zhat,R,n = n)
cond_dist$plot
```

N2finemapping

Simulated Fine-mapping Data with Two Effect Variables

### **Description**

This data set contains a genotype matrix for 574 individuals and 1,002 variables. The variables are genotypes after centering and scaling, and therefore retain the correlation structure of the original genotype data. Two of the variables have non-zero effects on the multivariate response. The response data are generated under a multivariate linear regression model. See Wang *et al* (2020) for details.

#### **Format**

N2finemapping is a list with the following elements:

X Centered and scaled genotype data.

**chrom** Chromomsome of the original data, in hg38 coordinates.

**pos** Chromomosomal position of the original data, in hg38 coordinates. The information can be used to compare impact of using other genotype references of the same variables in susie\_rss application.

true\_coef Simulated effect sizes.

residual\_variance Simulated residual covariance matrix.

Y Simulated multivariate response.

allele\_freq Allele frequencies based on the original genotype data.

V Suggested prior covariance matrix for effect sizes of the two non-zero effect variables.

### References

G. Wang, A. Sarkar, P. Carbonetto and M. Stephens (2020). A simple new approach to variable selection in regression, with application to genetic fine-mapping. *Journal of the Royal Statistical Society, Series B* doi: 10.1101/501114.

```
data(N2finemapping)
```

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N3finemapping

Simulated Fine-mapping Data with Three Effect Variables.

### **Description**

The data-set contains a matrix of 574 individuals and 1,001 variables. These variables are real-world genotypes centered and scaled, and therefore retains the correlation structure of variables in the original genotype data. 3 out of the variables have non-zero effects. The response data is generated under a multivariate linear regression model. See Wang *et al* (2020) for more details.

#### **Format**

N3finemapping is a list with the following elements:

**X** N by P variable matrix of centered and scaled genotype data.

**chrom** Chromomsome of the original data, in hg38 coordinate.

**pos** Chromomosomal position of the original data, in hg38 coordinate. The information can be used to compare impact of using other genotype references of the same variables in susie\_rss application.

true\_coef The simulated effect sizes.

**residual\_variance** The simulated residual covariance matrix.

**Y** The simulated response variables.

allele\_freq Allele frequency of the original genotype data.

V Prior covariance matrix for effect size of the three non-zero effect variables.

#### References

G. Wang, A. Sarkar, P. Carbonetto and M. Stephens (2020). A simple new approach to variable selection in regression, with application to genetic fine-mapping. *Journal of the Royal Statistical Society, Series B* doi: 10.1101/501114.

### **Examples**

data(N3finemapping)

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predict.susie

Predict outcomes or extract coefficients from susie fit.

### **Description**

Predict outcomes or extract coefficients from susie fit.

### Usage

```
## S3 method for class 'susie'
predict(object, newx = NULL, type = c("response", "coefficients"), ...)
```

### **Arguments**

object A susie fit.

newx A new value for X at which to do predictions.

type The type of output. For type = "response", predicted or fitted outcomes are

returned; for type = "coefficients", the estimated coefficients are returned.

... Other arguments used by generic predict function. These extra arguments are

not used here.

### Value

For type = "response", predicted or fitted outcomes are returned; for type = "coefficients", the estimated coefficients are returned. If the susie fit has intercept = NA (which is common when using susie\_suff\_stat) then predictions are computed using an intercept of 0, and a warning is emitted.

summary.susie

Summarize Susie Fit.

### Description

summary method for the "susie" class.

### Usage

```
## $3 method for class 'susie'
summary(object, ...)
## $3 method for class 'summary.susie'
print(x, ...)
```

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### **Arguments**

object A susie fit.

... Additional arguments passed to the generic summary or print.summary method.

x A susie summary.

#### Value

summary. susie returns a list containing a data frame of variables and a data frame of credible sets.

SummaryConsistency

Simulated Fine-mapping Data with LD matrix From Reference Panel.

### Description

Data simulated using real genotypes from 10,000 individuals and 200 SNPs. One SNP have non-zero effect on the multivariate response. The response data are generated under a linear regression model. There is also one SNP with flipped allele between summary statistics and the reference panel.

### **Format**

SummaryConsistency is a list with the following elements:

**z** z-scores computed by fitting univariate simple regression variable-by-variable.

**ldref** LD matrix estimated from the reference panel.

flip\_id The index of the SNP with the flipped allele.

**signal\_id** The index of the SNP with the non-zero effect.

### See Also

A similar data set with more samples is used in the "Diagnostic for fine-mapping with summary statistics" vignette.

### **Examples**

data(SummaryConsistency)

Sum of Single Effects (SuSiE) Regression

susie

#### **Description**

Performs a sparse Bayesian multiple linear regression of y on X, using the "Sum of Single Effects" model from Wang et al (2020). In brief, this function fits the regression model  $y = \mu + Xb + e$ , where elements of e are i.i.d. normal with zero mean and variance residual\_variance,  $\mu$  is an intercept term and b is a vector of length p representing the effects to be estimated. The "susie assumption" is that  $b = \sum_{l=1}^L b_l$  where each  $b_l$  is a vector of length p with exactly one nonzero element. The prior on the non-zero element is normal with zero mean and variance var(y) \*scaled\_prior\_variance. The value of L is fixed, and should be chosen to provide a reasonable upper bound on the number of non-zero effects to be detected. Typically, the hyperparameters residual\_variance and scaled\_prior\_variance will be estimated during model fitting, although they can also be fixed as specified by the user. See functions  $susie_get_c$  and other functions of form  $susie_get_*$  to extract the most commonly-used results from a susie fit.

### Usage

```
susie(
  Χ,
  L = min(10, ncol(X)),
  scaled_prior_variance = 0.2,
  residual_variance = NULL,
  prior_weights = NULL,
  null_weight = 0,
  standardize = TRUE,
  intercept = TRUE,
  estimate_residual_variance = TRUE,
  estimate_prior_variance = TRUE,
  estimate_prior_method = c("optim", "EM", "simple"),
  check_null_threshold = 0,
  prior_tol = 1e-09,
  residual_variance_upperbound = Inf,
  s_init = NULL,
  coverage = 0.95,
  min_abs_corr = 0.5,
  compute_univariate_zscore = FALSE,
  na.rm = FALSE,
 max_iter = 100,
  tol = 0.001,
  verbose = FALSE,
  track_fit = FALSE,
  residual_variance_lowerbound = var(drop(y))/10000,
  refine = FALSE,
  n_{purity} = 100
```

```
)
susie_suff_stat(
 XtX,
  Xty,
 yty,
 n,
 X_{colmeans} = NA,
 y_mean = NA,
 maf = NULL,
 maf_thresh = 0,
 L = 10,
  scaled_prior_variance = 0.2,
  residual_variance = NULL,
  estimate_residual_variance = TRUE,
  estimate_prior_variance = TRUE,
  estimate_prior_method = c("optim", "EM", "simple"),
  check_null_threshold = 0,
  prior_tol = 1e-09,
  r_{tol} = 1e-08,
  prior_weights = NULL,
  null_weight = 0,
  standardize = TRUE,
 max_iter = 100,
  s_init = NULL,
  coverage = 0.95,
 min_abs_corr = 0.5,
  tol = 0.001,
  verbose = FALSE,
  track_fit = FALSE,
  check_input = FALSE,
  refine = FALSE,
  check_prior = FALSE,
  n_{purity} = 100
```

### **Arguments**

X An n by p matrix of covariates.

y The observed responses, a vector of length n.

L Maximum number of non-zero effects in the susie regression model. If L is larger than the number of covariates, p, L is set to p.

scaled\_prior\_variance

The prior variance, divided by var(y) (or by (1/(n-1))yty for susie\_suff\_stat); that is, the prior variance of each non-zero element of b is var(y) \* scaled\_prior\_variance. The value provided should be either a scalar or a vector of length L. If estimate\_prior\_variance = TRUE, this provides initial estimates of the prior variances.

residual\_variance

Variance of the residual. If estimate\_residual\_variance = TRUE, this value provides the initial estimate of the residual variance. By default, it is set to var(y) in susie and (1/(n-1))yty in  $susie_suff_stat$ .

prior\_weights

A vector of length p, in which each entry gives the prior probability that corresponding column of X has a nonzero effect on the outcome, y.

null\_weight

Prior probability of no effect (a number between 0 and 1, and cannot be exactly 1).

standardize

If standardize = TRUE, standardize the columns of X to unit variance prior to fitting (or equivalently standardize XtX and Xty to have the same effect). Note that scaled\_prior\_variance specifies the prior on the coefficients of X *after* standardization (if it is performed). If you do not standardize, you may need to think more carefully about specifying scaled\_prior\_variance. Whatever your choice, the coefficients returned by coef are given for X on the original input scale. Any column of X that has zero variance is not standardized.

intercept

If intercept = TRUE, the intercept is fitted; it intercept = FALSE, the intercept is set to zero. Setting intercept = FALSE is generally not recommended.

estimate\_residual\_variance

If estimate\_residual\_variance = TRUE, the residual variance is estimated, using residual\_variance as an initial value. If estimate\_residual\_variance = FALSE, the residual variance is fixed to the value supplied by residual\_variance.

estimate\_prior\_variance

If estimate\_prior\_variance = TRUE, the prior variance is estimated (this is a separate parameter for each of the L effects). If provided, scaled\_prior\_variance is then used as an initial value for the optimization. When estimate\_prior\_variance = FALSE, the prior variance for each of the L effects is determined by the value supplied to scaled\_prior\_variance.

estimate\_prior\_method

The method used for estimating prior variance. When estimate\_prior\_method = "simple" is used, the likelihood at the specified prior variance is compared to the likelihood at a variance of zero, and the setting with the larger likelihood is retained.

check\_null\_threshold

When the prior variance is estimated, compare the estimate with the null, and set the prior variance to zero unless the log-likelihood using the estimate is larger by this threshold amount. For example, if you set check\_null\_threshold = 0.1, this will "nudge" the estimate towards zero when the difference in log-likelihoods is small. A note of caution that setting this to a value greater than zero may lead the IBSS fitting procedure to occasionally decrease the ELBO.

When the prior variance is estimated, compare the estimated value to prior\_tol at the end of the computation, and exclude a single effect from PIP computation if the estimated prior variance is smaller than this tolerance value.

residual\_variance\_upperbound

Upper limit on the estimated residual variance. It is only relevant when estimate\_residual\_variance = TRUE.

s\_init A previous susie fit with which to initialize.

prior\_tol

coverage A number between 0 and 1 specifying the "coverage" of the estimated confi-

dence sets.

min\_abs\_corr Minimum absolute correlation allowed in a credible set. The default, 0.5, cor-

responds to a squared correlation of 0.25, which is a commonly used threshold

for genotype data in genetic studies.

compute\_univariate\_zscore

If compute\_univariate\_zscore = TRUE, the univariate regression z-scores are

outputted for each variable.

na.rm Drop any missing values in y from both X and y.

max\_iter Maximum number of IBSS iterations to perform.

tol A small, non-negative number specifying the convergence tolerance for the IBSS

fitting procedure. The fitting procedure will halt when the difference in the variational lower bound, or "ELBO" (the objective function to be maximized), is

less than tol.

verbose If verbose = TRUE, the algorithm's progress, and a summary of the optimization

settings, are printed to the console.

track\_fit If track\_fit = TRUE, trace is also returned containing detailed information

about the estimates at each iteration of the IBSS fitting procedure.

residual\_variance\_lowerbound

Lower limit on the estimated residual variance. It is only relevant when estimate\_residual\_variance

= TRUE.

refine If refine = TRUE, then an additional iterative refinement procedure is used, after

the IBSS algorithm, to check and escape from local optima (see details).

n\_purity Passed as argument n\_purity to susie\_get\_cs.

XtX A p by p matrix X'X in which the columns of X are centered to have mean zero.

Xty A p-vector X'y in which y and the columns of X are centered to have mean zero.

yty A scalar y'y in which y is centered to have mean zero.

n The sample size.

X\_colmeans A p-vector of column means of X. If both X\_colmeans and y\_mean are provided,

the intercept is estimated; otherwise, the intercept is NA.

y\_mean A scalar containing the mean of y. If both X\_colmeans and y\_mean are provided,

the intercept is estimated; otherwise, the intercept is NA.

maf Minor allele frequency; to be used along with maf\_thresh to filter input sum-

mary statistics.

maf\_thresh Variants having a minor allele frequency smaller than this threshold are not used.

r\_tol Tolerance level for eigenvalue check of positive semidefinite matrix of R.

check\_input If check\_input = TRUE, susie\_suff\_stat performs additional checks on XtX

and Xty. The checks are: (1) check that XtX is positive semidefinite; (2) check

that Xty is in the space spanned by the non-zero eigenvectors of XtX.

check\_prior If check\_prior = TRUE, it checks if the estimated prior variance becomes un-

reasonably large (comparing with  $10 * max(abs(z))^2$ ).

#### **Details**

The function susie implements the IBSS algorithm from Wang et al (2020). The option refine = TRUE implements an additional step to help reduce problems caused by convergence of the IBSS algorithm to poor local optima (which is rare in our experience, but can provide misleading results when it occurs). The refinement step incurs additional computational expense that increases with the number of CSs found in the initial run.

The function  $susie\_suff\_stat$  implements essentially the same algorithms, but using sufficient statistics. (The statistics are sufficient for the regression coefficients b, but not for the intercept  $\mu$ ; see below for how the intercept is treated.) If the sufficient statistics are computed correctly then the results from  $susie\_suff\_stat$  should be the same as (or very similar to) susie, although runtimes will differ as discussed below. The sufficient statistics are the sample size n, and then the n by n matrix n and the p-vector n and the sum of squared n values n and the vector n and the vector n to have mean n; these can be computed using n compute\_suff\_stat.

The handling of the intercept term in  $susie\_suff\_stat$  needs some additional explanation. Computing the summary data after centering X and y effectively ensures that the resulting posterior quantities for b allow for an intercept in the model; however, the actual value of the intercept cannot be estimated from these centered data. To estimate the intercept term the user must also provide the column means of X and the mean of y (X\_colmeans and y\_mean). If these are not provided, they are treated as NA, which results in the intercept being NA. If for some reason you prefer to have the intercept be 0 instead of NA then set X\_colmeans = 0, y\_mean = 0.

For completeness, we note that if  $susie\_suff\_stat$  is run on X'X, X'y, y'y computed without centering X and y, and with  $X\_colmeans = 0$ ,  $y\_mean = 0$ , this is equivalent to susie applied to X, y with intercept = FALSE (although results may differ due to different initializations of residual\\_variance and  $scaled\_prior\_variance$ ). However, this usage is not recommended for for most situations.

The computational complexity of susie is O(npL) per iteration, whereas susie\_suff\_stat is  $O(p^2L)$  per iteration (not including the cost of computing the sufficient statistics, which is dominated by the  $O(np^2)$  cost of computing X'X). Because of the cost of computing X'X, susie will usually be faster. However, if n >> p, and/or if X'X is already computed, then susie\_suff\_stat may be faster.

### Value

A "susie" object with some or all of the following elements:

alpha An L by p matrix of posterior inclusion probabilites.

mu An L by p matrix of posterior means, conditional on inclusion.

mu2 An L by p matrix of posterior second moments, conditional on inclusion.

Xr A vector of length n, equal to X %\*% colSums(alpha \* mu).

lbf log-Bayes Factor for each single effect.

lbf\_variable log-Bayes Factor for each variable and single effect.

intercept Intercept (fixed or estimated).

sigma2 Residual variance (fixed or estimated).

V Prior variance of the non-zero elements of b, equal to scaled\_prior\_variance

\* var(y).

| elbo                                  | The value of the variational lower bound, or "ELBO" (objective function to be maximized), achieved at each iteration of the IBSS fitting procedure. |
|---------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| fitted                                | Vector of length n containing the fitted values of the outcome.                                                                                     |
| sets                                  | Credible sets estimated from model fit; see <pre>susie_get_cs</pre> for details.                                                                    |
| pip                                   | A vector of length p giving the (marginal) posterior inclusion probabilities for all p covariates.                                                  |
| z                                     | A vector of univariate z-scores.                                                                                                                    |
| niter                                 | Number of IBSS iterations that were performed.                                                                                                      |
| converged                             | TRUE or FALSE indicating whether the IBSS converged to a solution within the chosen tolerance level.                                                |
| susie_suff_stat returns also outputs: |                                                                                                                                                     |
| XtXr                                  | A p-vector of t(X) times the fitted values, X %*% colSums(alpha*mu).                                                                                |

#### References

G. Wang, A. Sarkar, P. Carbonetto and M. Stephens (2020). A simple new approach to variable selection in regression, with application to genetic fine-mapping. *Journal of the Royal Statistical Society, Series B* **82**, 1273-1300 doi: 10.1101/501114.

Y. Zou, P. Carbonetto, G. Wang and M. Stephens (2021). Fine-mapping from summary data with the "Sum of Single Effects" model. *bioRxiv* doi: 10.1101/2021.11.03.467167.

#### See Also

susie\_get\_cs and other susie\_get\_\* functions for extracting results; susie\_trendfilter for applying the SuSiE model to non-parametric regression, particularly changepoint problems, and susie\_rss for applying the SuSiE model when one only has access to limited summary statistics related to X and y (typically in genetic applications).

```
# susie example
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p), nrow = n, ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res1 = susie(X,y,L = 10)
susie_get_cs(res1) # extract credible sets from fit
plot(beta, coef(res1)[-1])
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")
plot(y,predict(res1))
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")
# susie_suff_stat example
```

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susie\_auto

Attempt at Automating SuSiE for Hard Problems

### **Description**

susie\_auto is an attempt to automate reliable running of susie even on hard problems. It implements a three-stage strategy for each L: first, fit susie with very small residual error; next, estimate residual error; finally, estimate the prior variance. If the last step estimates some prior variances to be zero, stop. Otherwise, double L, and repeat. Initial runs are performed with relaxed tolerance; the final run is performed using the default susie tolerance.

### Usage

```
susie_auto(
   X,
   y,
   L_init = 1,
   L_max = 512,
   verbose = FALSE,
   init_tol = 1,
   standardize = TRUE,
   intercept = TRUE,
   max_iter = 100,
   tol = 0.01,
   ...
)
```

### **Arguments**

| Χ        | An n by p matrix of covariates.                                                                                      |
|----------|----------------------------------------------------------------------------------------------------------------------|
| у        | The observed responses, a vector of length n.                                                                        |
| L_init   | The initial value of L.                                                                                              |
| L_max    | The largest value of L to consider.                                                                                  |
| verbose  | If verbose = TRUE, the algorithm's progress, and a summary of the optimization settings, are printed to the console. |
| init_tol | The tolerance to passed to susie during early runs (set large to shorten the initial runs).                          |

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standardize If standardize = TRUE, standardize the columns of X to unit variance prior to fitting. Note that scaled\_prior\_variance specifies the prior on the coefficients of X after standardization (if it is performed). If you do not standardize, you may need to think more carefully about specifying scaled\_prior\_variance. Whatever your choice, the coefficients returned by coef are given for X on the original input scale. Any column of X that has zero variance is not standardized. intercept If intercept = TRUE, the intercept is fitted; it intercept = FALSE, the intercept is set to zero. Setting intercept = FALSE is generally not recommended. max\_iter Maximum number of IBSS iterations to perform. tol A small, non-negative number specifying the convergence tolerance for the IBSS fitting procedure. The fitting procedure will halt when the difference in the variational lower bound, or "ELBO" (the objective function to be maximized), is less than tol.

Additional arguments passed to susie.

#### Value

See susie for a description of return values.

#### See Also

susie

### **Examples**

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res = susie_auto(X,y)
plot(beta,coef(res)[-1])
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")
plot(y,predict(res))
abline(a = 0,b = 1,col = "skyblue",lty = "dashed")
```

susie\_get\_objective

Inferences From Fitted SuSiE Model

### **Description**

These functions access basic properties or draw inferences from a fitted susie model.

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### Usage

```
susie_get_objective(res, last_only = TRUE, warning_tol = 1e-06)
susie_get_posterior_mean(res, prior_tol = 1e-09)
susie_get_posterior_sd(res, prior_tol = 1e-09)
susie_get_niter(res)
susie_get_prior_variance(res)
susie_get_residual_variance(res)
susie_get_lfsr(res)
susie_get_posterior_samples(susie_fit, num_samples)
susie_get_cs(
 res,
 X = NULL
 Xcorr = NULL,
 coverage = 0.95,
 min_abs_corr = 0.5,
 dedup = TRUE,
 squared = FALSE,
 check_symmetric = TRUE,
 n_{purity} = 100,
 use_rfast
)
susie_get_pip(res, prune_by_cs = FALSE, prior_tol = 1e-09)
```

### Arguments

| res         | A susie fit, typically an output from susie or one of its variants. For susie_get_pip and susie_get_cs, this may instead be the posterior inclusion probability matrix, alpha.                                                     |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| last_only   | If last_only = FALSE, return the ELBO from all iterations; otherwise return the ELBO from the last iteration only.                                                                                                                 |
| warning_tol | Warn if ELBO is decreasing by this tolerance level.                                                                                                                                                                                |
| prior_tol   | Filter out effects having estimated prior variance smaller than this threshold.                                                                                                                                                    |
| susie_fit   | A susie fit, an output from susie.                                                                                                                                                                                                 |
| num_samples | The number of draws from the posterior distribution.                                                                                                                                                                               |
| X           | n by p matrix of values of the p variables (covariates) in n samples. When provided, correlation between variables will be computed and used to remove CSs whose minimum correlation among variables is smaller than min_abs_corr. |

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Xcorr p by p matrix of correlations between variables (covariates). When provided,

it will be used to remove CSs whose minimum correlation among variables is

smaller than min\_abs\_corr.

coverage A number between 0 and 1 specifying desired coverage of each CS.

min\_abs\_corr A "purity" threshold for the CS. Any CS that contains a pair of variables with

correlation less than this threshold will be filtered out and not reported.

dedup = TRUE, remove duplicate CSs.

squared If squared = TRUE, report min, mean and median of squared correlation instead

of the absolute correlation.

check\_symmetric

If check\_symmetric = TRUE, perform a check for symmetry of matrix Xcorr

when Xcorr is provided (not NULL).

n\_purity The maximum number of credible set (CS) variables used in calculating the

correlation ("purity") statistics. When the number of variables included in the CS is greater than this number, the CS variables are randomly subsampled.

use\_rfast Use the Rfast package for the purity calculations. By default use\_rfast = TRUE

if the Rfast package is installed.

prune\_by\_cs Whether or not to ignore single effects not in a reported CS when calculating

PIP.

#### Value

cs

susie\_get\_objective returns the evidence lower bound (ELBO) achieved by the fitted susie model and, optionally, at each iteration of the IBSS fitting procedure.

susie\_get\_residual\_variance returns the (estimated or fixed) residual variance parameter.

susie\_get\_prior\_variance returns the (estimated or fixed) prior variance parameters.

susie\_get\_posterior\_mean returns the posterior mean for the regression coefficients of the fitted susie model.

susie\_get\_posterior\_sd returns the posterior standard deviation for coefficients of the fitted susie model.

susie\_get\_niter returns the number of model fitting iterations performed.

susie\_get\_pip returns a vector containing the posterior inclusion probabilities (PIPs) for all variables.

susie\_get\_lfsr returns a vector containing the average lfsr across variables for each single-effect, weighted by the posterior inclusion probability (alpha).

susie\_get\_posterior\_samples returns a list containing the effect sizes samples and causal status with two components: b, an num\_variables x num\_samples matrix of effect sizes; gamma, an num\_variables x num\_samples matrix of causal status random draws.

susie\_get\_cs returns credible sets (CSs) from a susie fit, as well as summaries of correlation among the variables included in each CS. If desired, one can filter out CSs that do not meet a specified "purity" threshold; to do this, either X or Xcorr must be supplied. It returns a list with the following elements:

A list in which each list element is a vector containing the indices of the variables in the CS.

susie\_init\_coef

coverage The nominal coverage specified for each CS.

purity If X or Xcorr iis provided), the purity of each CS.

cs\_index If X or Xcorr is provided) the index (number between 1 and L) of each reported CS in the supplied susie fit.

### **Examples**

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p), nrow = n, ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
s = susie(X,y,L = 10)
susie_get_objective(s)
susie_get_objective(s, last_only=FALSE)
susie_get_residual_variance(s)
susie_get_prior_variance(s)
susie_get_posterior_mean(s)
susie_get_posterior_sd(s)
susie_get_niter(s)
susie_get_pip(s)
susie_get_lfsr(s)
```

susie\_init\_coef

*Initialize a susie object using regression coefficients* 

### **Description**

Initialize a susie object using regression coefficients

### Usage

```
susie_init_coef(coef_index, coef_value, p)
```

### **Arguments**

coef\_index An L-vector containing the the indices of the nonzero coefficients.

coef\_value An L-vector containing initial coefficient estimates.

p A scalar giving the number of variables.

### Value

A list with elements alpha, mu and mu2 to be used by susie.

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### **Examples**

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[sample(1:1000,4)] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
# Initialize susie to ground-truth coefficients.
s = susie_init_coef(which(beta != 0),beta[beta != 0],length(beta))
res = susie(X,y,L = 10,s_init=s)
```

susie\_plot

SuSiE Plots.

### Description

susie\_plot produces a per-variable summary of the SuSiE credible sets. susie\_plot\_iteration produces a diagnostic plot for the susie model fitting. For susie\_plot\_iteration, several plots will be created if track\_fit = TRUE when calling susie.

### Usage

```
susie_plot(
  model,
  y,
  add_bar = FALSE,
  pos = NULL,
  b = NULL,
  max_cs = 400,
  add_legend = NULL,
  ...
)
susie_plot_iteration(model, L, file_prefix, pos = NULL)
```

### **Arguments**

model

A SuSiE fit, typically an output from susie or one of its variants. For suse\_plot, the susie fit must have model\$z, model\$PIP, and may include model\$sets. model may also be a vector of z-scores or PIPs.

У

A string indicating what to plot: either "z\_original" for z-scores, "z" for z-score derived p-values on (base-10) log-scale, "PIP" for posterior inclusion probabilities, "log10PIP" for posterior inclusion probabilities on the (base-10) log-scale. For any other setting, the data are plotted as is.

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If add\_bar = TRUE, add horizontal bar to signals in credible interval. add\_bar Indices of variables to plot. If pos = NULL all variables are plotted. pos For simulated data, set b = TRUE to highlight "true" effects (highlights in red). b The largest credible set to display, either based on purity (set max\_cs between 0 max\_cs and 1), or based on size (set  $\max_{c} > 1$ ). add\_legend If add\_legend = TRUE, add a legend to annotate the size and purity of each CS discovered. It can also be specified as location where legends should be added, e.g., add\_legend = "bottomright" (default location is "topright"). Additional arguments passed to plot. . . . An integer specifying the number of credible sets to plot. L Prefix to path of output plot file. If not specified, the plot, or plots, will be saved file\_prefix to a temporary directory generated using tempdir.

#### Value

Invisibly returns NULL.

#### See Also

```
susie_plot_changepoint
```

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[sample(1:1000,4)] = 1
X = matrix(rnorm(n*p), nrow = n, ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res = susie(X,y,L = 10)
susie_plot(res,"PIP")
susie_plot(res,"PIP",add_bar = TRUE)
susie_plot(res,"PIP",add_legend = TRUE)
susie_plot(res,"PIP", pos=1:500, add_legend = TRUE)
# Plot selected regions with adjusted x-axis position label
res$genomic_position = 1000 + (1:length(res$pip))
susie_plot(res,"PIP",add_legend = TRUE,
           pos = list(attr = "genomic_position", start = 1000, end = 1500))
# True effects are shown in red.
susie_plot(res,"PIP",b = beta,add_legend = TRUE)
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[sample(1:1000,4)] = 1
X = matrix(rnorm(n*p), nrow = n, ncol = p)
```

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```
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res = susie(X,y,L = 10)
susie_plot_iteration(res, L=10)
```

```
susie_plot_changepoint
```

Plot changepoint data and susie fit using ggplot2

### **Description**

Plots original data, y, overlaid with line showing susie fitted value and shaded rectangles showing credible sets for changepoint locations.

### Usage

```
susie_plot_changepoint(
    s,
    y,
    line_col = "blue",
    line_size = 1.5,
    cs_col = "red"
)
```

### **Arguments**

| S         | A susie fit generated by susie_trendfilter(y, order = $\emptyset$ ).                       |
|-----------|--------------------------------------------------------------------------------------------|
| У         | An $n$ -vector of observations that are ordered in time or space (assumed equally-spaced). |
| line_col  | Color for the line showing fitted values.                                                  |
| line_size | Size of the lines showing fitted values                                                    |
| cs_col    | Color of the shaded rectangles showing credible sets.                                      |

### Value

A ggplot2 plot object.

```
set.seed(1)
mu = c(rep(0,50),rep(1,50),rep(3,50),rep(-2,50),rep(0,300))
y = mu + rnorm(500)
# Here we use a less sensitive tolerance so that the example takes
# less time; in practice you will likely want to use a more stringent
# setting such as tol = 0.001.
s = susie\_trendfilter(y,tol = 0.1)
```

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```
# Produces ggplot with credible sets for changepoints.
susie_plot_changepoint(s,y)
```

susie\_rss

Sum of Single Effects (SuSiE) Regression using Summary Statistics

### **Description**

susie\_rss performs variable selection under a sparse Bayesian multiple linear regression of Y on X using the z-scores from standard univariate regression of Y on each column of X, an estimate, R, of the correlation matrix for the columns of X, and optionally, but strongly recommended, the sample size n. See "Details" for other ways to call susie\_rss

### Usage

```
susie_rss(
   z,
   R,
   n,
   bhat,
   shat,
   var_y,
   z_ld_weight = 0,
   estimate_residual_variance = FALSE,
   prior_variance = 50,
   check_prior = TRUE,
   ...
)
```

### **Arguments**

| Z           | p-vector of z-scores.                                                                                                                                                                                                                                         |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| R           | p x p correlation matrix.                                                                                                                                                                                                                                     |
| n           | The sample size.                                                                                                                                                                                                                                              |
| bhat        | Alternative summary data giving the estimated effects (a vector of length p). This, together with shat, may be provided instead of z.                                                                                                                         |
| shat        | Alternative summary data giving the standard errors of the estimated effects (a vector of length p). This, together with bhat, may be provided instead of z.                                                                                                  |
| var_y       | The sample variance of y, defined as $y'y/(n-1)$ . When the sample variance is not provided, the coefficients (returned from coef) are computed on the "standardized" X, y scale.                                                                             |
| z_ld_weight | This parameter is included for backwards compatibility with previous versions of the function, but it is no longer recommended to set this to a non-zero value. When $z_1d_weight > 0$ , the matrix R is adjusted to be $cov2cor((1-w)*R + cov2cor((1-w)*R))$ |

w\*tcrossprod(z)), where w = z\_ld\_weight.

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estimate\_residual\_variance

The default is FALSE, the residual variance is fixed to 1 or variance of y. If the in-sample LD matrix is provided, we recommend setting estimate\_residual\_variance = TRUE.

prior\_variance The prior variance(s) for the non-zero noncentrality parameterss  $b_l$ . It is either a scalar, or a vector of length L. When the susie\_suff\_stat option estimate\_prior\_variance is set to TRUE (which is highly recommended) this simply provides an initial value for the prior variance. The default value of 50 is simply intended to be a large initial value. Note this setting is only relevant when n is unknown. If n is known, the relevant option is scaled\_prior\_variance in susie\_suff\_stat.

check\_prior

When check\_prior = TRUE, it checks if the estimated prior variance becomes unreasonably large (comparing with  $100 * max(abs(z))^2$ ).

Other parameters to be passed to susie\_suff\_stat.

#### **Details**

In some applications, particularly genetic applications, it is desired to fit a regression model (Y =Xb + E say, which we refer to as "the original regression model" or ORM) without access to the actual values of Y and X, but given only some summary statistics. susie\_rss assumes availability of z-scores from standard univariate regression of Y on each column of X, and an estimate, R, of the correlation matrix for the columns of X (in genetic applications R is sometimes called the "LD" matrix").

With the inputs z, R and sample size n, susie\_rss computes PVE-adjusted z-scores z\_tilde, and calls susie\_suff\_stat with XtX = (n-1)R, Xty =  $\sqrt{n-1}z_tilde$ , yty = n-1, n = n. The output effect estimates are on the scale of b in the ORM with standardized X and y. When the LD matrix R and the z-scores z are computed using the same matrix X, the results from susie\_rss are same as, or very similar to, susie with standardized X and y.

Alternatively, if the user provides n, bhat (the univariate OLS estimates from regressing y on each column of X), shat (the standard errors from these OLS regressions), the in-sample correlation matrix R = cov2cor(crossprod(X)), and the variance of y, the results from susie\_rss are same as susie with X and y. The effect estimates are on the same scale as the coefficients b in the ORM with X and y.

In rare cases in which the sample size, n, is unknown, susie\_rss calls susie\_suff\_stat with XtX = R and Xty = z, and with residual\_variance = 1. The underlying assumption of performing the analysis in this way is that the sample size is large (i.e., infinity), and/or the effects are small. More formally, this combines the log-likelihood for the noncentrality parameters,  $b = \sqrt{n}b$ ,

$$L(\tilde{b}; z, R) = -(\tilde{b}'R\tilde{b} - 2z'\tilde{b})/2,$$

with the "susie prior" on b; see susie and Wang et al (2020) for details. In this case, the effect estimates returned by susie\_rss are on the noncentrality parameter scale.

The estimate\_residual\_variance setting is FALSE by default, which is recommended when the LD matrix is estimated from a reference panel. When the LD matrix R and the summary statistics z (or bhat, shat) are computed using the same matrix X, we recommend setting estimate\_residual\_variance = TRUE.

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#### Value

A "susie" object with the following elements:

alpha An L by p matrix of posterior inclusion probabilites.

mu An L by p matrix of posterior means, conditional on inclusion.

mu2 An L by p matrix of posterior second moments, conditional on inclusion.

lbf log-Bayes Factor for each single effect.

lbf\_variable log-Bayes Factor for each variable and single effect.

V Prior variance of the non-zero elements of b.

elbo The value of the variational lower bound, or "ELBO" (objective function to be

maximized), achieved at each iteration of the IBSS fitting procedure.

sets Credible sets estimated from model fit; see susie\_get\_cs for details.

pip A vector of length p giving the (marginal) posterior inclusion probabilities for

all p covariates.

niter Number of IBSS iterations that were performed.

converged TRUE or FALSE indicating whether the IBSS converged to a solution within the

chosen tolerance level.

#### References

G. Wang, A. Sarkar, P. Carbonetto and M. Stephens (2020). A simple new approach to variable selection in regression, with application to genetic fine-mapping. *Journal of the Royal Statistical Society, Series B* **82**, 1273-1300 doi: 10.1101/501114.

Y. Zou, P. Carbonetto, G. Wang and M. Stephens (2021). Fine-mapping from summary data with the "Sum of Single Effects" model. *bioRxiv* doi: 10.1101/2021.11.03.467167.

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p), nrow = n, ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
input_ss = compute_suff_stat(X,y,standardize = TRUE)
   = univariate_regression(X,y)
    = with(input_ss,cov2cor(XtX))
zhat = with(ss,betahat/sebetahat)
res = susie_rss(zhat,R, n=n)
# Toy example illustrating behaviour susie_rss when the z-scores
# are mostly consistent with a non-invertible correlation matrix.
# Here the CS should contain both variables, and two PIPs should
# be nearly the same.
```

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```
z = c(6,6.01)
R = matrix(1,2,2)
fit = susie_rss(z,R)
print(fit$sets$cs)
print(fit$pip)

# In this second toy example, the only difference is that one
# z-score is much larger than the other. Here we expect that the
# second PIP will be much larger than the first.
z = c(6,7)
R = matrix(1,2,2)
fit = susie_rss(z,R)
print(fit$sets$cs)
print(fit$pip)
```

susie\_trendfilter

Apply susie to trend filtering (especially changepoint problems), a type of non-parametric regression.

### **Description**

Fits the non-parametric Gaussian regression model y=mu+e, where the mean mu is modelled as mu=Xb, X is a matrix with columns containing an appropriate basis, and b is vector with a (sparse) SuSiE prior. In particular, when order = 0, the jth column of X is a vector with the first j elements equal to zero, and the remaining elements equal to 1, so that  $b_j$  corresponds to the change in the mean of y between indices j and j+1. For background on trend filtering, see Tibshirani (2014). See also the "Trend filtering" vignette, vignette("trend\_filtering").

### Usage

```
susie_trendfilter(y, order = 0, standardize = FALSE, use_mad = TRUE, ...)
```

### **Arguments**

y An n-vector of obs

An n-vector of observations ordered in time or space (assumed to be equally spaced).

order An integer specifying the order of trend filtering. The default, order = 0, cor-

responds to "changepoint" problems (i.e., piecewise constant mu). Although order > 0 is implemented, we do not recommend its use; in practice, we have found problems with convergence of the algorithm to poor local optima, pro-

ducing unreliable inferences.

standardize Logical indicating whether to standardize the X variables ("basis functions");

standardize = FALSE is recommended as these basis functions already have a

natural scale.

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use\_mad

Logical indicating whether to use the "median absolute deviation" (MAD) method to the estimate residual variance. If use\_mad = TRUE, susie is run twice, first by fixing the residual variance to the MAD value, then a second time, initialized to the first fit, but with residual variance estimated the usual way (by maximizing the ELBO). We have found this strategy typically improves reliability of the results by reducing a tendency to converge to poor local optima of the ELBO.

... Other arguments passed to susie.

#### **Details**

This implementation exploits the special structure of X, which means that the matrix-vector product  $X^T y$  is fast to compute; in particular, the computation time is O(n) rather than  $O(n^2)$  if X were formed explicitly. For implementation details, see the "Implementation of SuSiE trend filtering" vignette by running vignette ("trendfiltering\_derivations").

### Value

A "susie" fit; see susie for details.

#### References

R. J. Tibshirani (2014). Adaptive piecewise polynomial estimation via trend filtering. *Annals of Statistics* **42**, 285-323.

### **Examples**

```
set.seed(1)
mu = c(rep(0,50),rep(1,50),rep(3,50),rep(-2,50),rep(0,200))
y = mu + rnorm(400)
s = susie_trendfilter(y)
plot(y)
lines(mu,col = 1,lwd = 3)
lines(predict(s),col = 2,lwd = 2)
# Calculate credible sets (indices of y that occur just before # changepoints).
susie_get_cs(s)
# Plot with credible sets for changepoints.
susie_plot_changepoint(s,y)
```

univariate\_regression Perform Univariate Linear Regression Separately for Columns of X

### **Description**

This function performs the univariate linear regression  $y \sim x$  separately for each column x of X. Each regression is implemented using .lm.fit(). The estimated effect size and stardard error for each variable are outputted.

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### Usage

```
univariate_regression(
   X,
   y,
   Z = NULL,
   center = TRUE,
   scale = FALSE,
   return_residuals = FALSE
)
```

### **Arguments**

|                  | Χ      | n by p matrix of regressors.                                                                                                                                                                              |
|------------------|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                  | У      | n-vector of response variables.                                                                                                                                                                           |
|                  | Z      | Optional n by k matrix of covariates to be included in all regresions. If $Z$ is not NULL, the linear effects of covariates are removed from y first, and the resulting residuals are used in place of y. |
|                  | center | If center = TRUE, center $X$ , $y$ and $Z$ .                                                                                                                                                              |
|                  | scale  | If $scale = TRUE$ , $scale X$ , y and Z.                                                                                                                                                                  |
| return_residuals |        |                                                                                                                                                                                                           |
|                  |        | TTT -1                                                                                                                                                                                                    |

Whether or not to output the residuals if Z is not NULL.

### Value

A list with two vectors containing the least-squares estimates of the coefficients (betahat) and their standard errors (sebetahat). Optionally, and only when a matrix of covariates Z is provided, a third vector residuals containing the residuals is returned.

```
set.seed(1)
n = 1000
p = 1000
beta = rep(0,p)
beta[1:4] = 1
X = matrix(rnorm(n*p),nrow = n,ncol = p)
X = scale(X,center = TRUE,scale = TRUE)
y = drop(X %*% beta + rnorm(n))
res = univariate_regression(X,y)
plot(res$betahat/res$sebetahat)
```

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