

[www.portfolioeffect.com](http://www.portfolioeffect.com)  
High Frequency Portfolio Analytics

User Manual

# **PortfolioEffectEstim MATLAB Toolbox**

High Frequency Price Estimators & Models

Andrey Kostin  
[andrey.kostin@portfolioeffect.com](mailto:andrey.kostin@portfolioeffect.com)

*Released Under GPL-3 License  
by Snowfall Systems, Inc.*

August 20, 2015

# Contents

|   |          |
|---|----------|
| <b>Contents</b>                                     | <b>1</b> |
| <b>1 Package Installation</b>                       | <b>3</b> |
| 1.1 Install Latest JDK/JRE Runtime . . . . .        | 3        |
| 1.2 Configure Java Environment (Optional) . . . . . | 3        |
| 1.3 Install Required Packages (Optional) . . . . .  | 3        |
| <b>2 API Credentials</b>                            | <b>4</b> |
| 2.1 Locate API Credentials . . . . .                | 4        |
| 2.2 Set API Credentials in R . . . . .              | 4        |
| <b>3 Price Variance</b>                             | <b>5</b> |
| 3.1 Integrated Variance . . . . .                   | 5        |
| 3.1.1 Returns . . . . .                             | 5        |
| 3.2 Realized Variance . . . . .                     | 5        |
| 3.2.1 Assumptions . . . . .                         | 5        |
| 3.2.2 Estimator . . . . .                           | 6        |
| 3.2.3 Properties . . . . .                          | 6        |
| 3.3 Two Series Realized Variance . . . . .          | 6        |
| 3.3.1 Assumptions . . . . .                         | 6        |
| 3.3.2 Estimator . . . . .                           | 6        |
| 3.3.3 Properties . . . . .                          | 7        |
| 3.4 Multiple Series Realized Variance . . . . .     | 7        |
| 3.4.1 Assumptions . . . . .                         | 7        |

|          |   |           |
|----------|---|-----------|
| 3.4.2    | Estimator . . . . .                               | 8         |
| 3.4.3    | Properties . . . . .                              | 8         |
| 3.5      | Modulated Realized Variance . . . . .             | 8         |
| 3.5.1    | Assumptions . . . . .                             | 8         |
| 3.5.2    | Estimator . . . . .                               | 8         |
| 3.5.3    | Properties . . . . .                              | 9         |
| 3.6      | Jump Robust Modulated Realized Variance . . . . . | 9         |
| 3.6.1    | Assumptions . . . . .                             | 9         |
| 3.6.2    | Estimator . . . . .                               | 9         |
| 3.6.3    | Properties . . . . .                              | 10        |
| 3.7      | Kernel-based Realized Variance . . . . .          | 10        |
| 3.7.1    | Assumptions . . . . .                             | 10        |
| 3.7.2    | Kernel Types . . . . .                            | 11        |
| 3.7.3    | Estimator . . . . .                               | 11        |
| 3.7.4    | Properties . . . . .                              | 12        |
| 3.8      | IV Estimators Comparison . . . . .                | 13        |
| <b>4</b> | <b>Price Noise Variance</b>                       | <b>14</b> |
| 4.1      | RV Noise Variance . . . . .                       | 14        |
| 4.1.1    | Properties . . . . .                              | 14        |
| 4.2      | Autocovariance Noise Variance . . . . .           | 14        |
| 4.2.1    | Properties . . . . .                              | 15        |
| <b>5</b> | <b>Price Quarticity</b>                           | <b>16</b> |
| 5.1      | Integrated Quarticity . . . . .                   | 16        |
| 5.2      | Realized Quarticity . . . . .                     | 16        |
| 5.2.1    | Assumptions . . . . .                             | 16        |
| 5.2.2    | Estimator . . . . .                               | 16        |
| 5.2.3    | Properties . . . . .                              | 16        |

# 1 Package Installation

PortfolioEffectEstim package for R relies on the rJava package, which assumes that Java runtime is installed and configured on your system. To install Java runtime and to configure your R engine to work with it, follow these steps:

## 1.1 Install Latest JDK/JRE Runtime

Download and install latest Java distribution (JDK or JRE) for your platform from Oracle's website

## 1.2 Configure Java Environment (Optional)

If you are using Windows, installation wizard from the previous step should have done everything for you. If you are on Linux or Mac and you used a tarball file, you will need to manually append the following lines to `/etc/environment` using your favorite text editor:

```
export JAVA_HOME=/path/to/java/folder
export PATH=$PATH:$JAVA_HOME/bin
```

Apply environment changes:

```
source /etc/environment
```

To complete with the set-up of Java environment inside R, run the following line:

```
sudo R CMD javareconf
```

## 1.3 Install Required Packages (Optional)

If you are manually installing PortfolioEffectEstim package (you don't want to use CRAN repositories for some reason), you would need to install all required package dependencies first. Start R from the command line or in your GUI editor and type

```
install.packages(c("PortfolioEffectHFT", "rJava"))
```

You are now ready to install the PortfolioEffectEstim package directly from [www.portfolioeffect.com](http://www.portfolioeffect.com) downloads section.

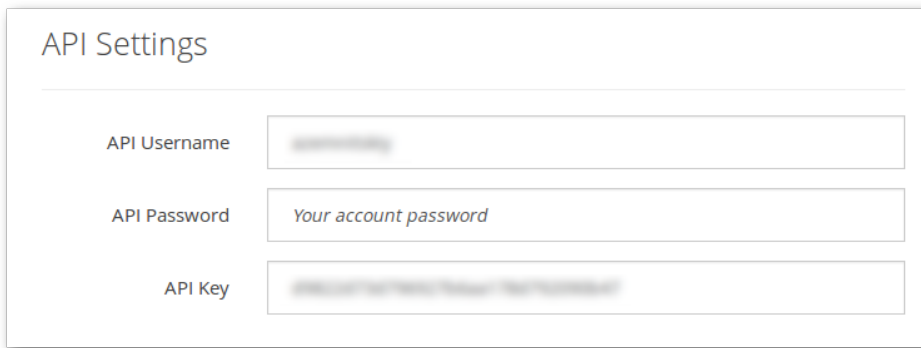
## 2 API Credentials

All portfolio computations are performed on PortfolioEffect cloud servers. To obtain a free non-professional account, you need to follow a quick sign-up process on our website: [www.portfolioeffect.com/registration](http://www.portfolioeffect.com/registration).

Please use a valid sign-up address - it will be used to email your account activation link.

### 2.1 Locate API Credentials

Log in to you account and locate your API credentials on the main page



The screenshot shows a web form titled "API Settings". It contains three input fields, each with a label to its left: "API Username" with a blurred input field, "API Password" with an input field containing the text "Your account password", and "API Key" with a blurred input field.

### 2.2 Set API Credentials in R

Run the following commands to set your account API credentials for the PortfolioEffectEstim R Package installed. You will need to do it only once as your credentials are stored between sessions on your local machine to speed up future logons.

You would need to repeat this procedure if you change your account password or install PortfolioEffectEstim package on another computer.

```
require(PortfolioEffectEstim)
util_setCredentials("API Username", "API Password", "API Key")
```

You are now ready to call PortfolioEffectEstim methods.

# 3 Price Variance

## 3.1 Integrated Variance

Assume that the logarithmic equilibrium price of a financial asset is given by the following diffusion process

$$X_t = \int_0^t \mu(s)ds + \int_0^t \sigma(s)dW(s) \quad (3.1)$$

where

- $W_t$  is a standart Brownian Motion,
- the mean process  $\mu$  is continuous and of finite variation,
- $\sigma(t) > 0$  denotes the cadlag instantaneous volatility.

The object of interest is the integrated variance ( $IV$ ), i.e. the amount of variation at time point  $t$  accumulated over a past time interval  $\Delta$  according to [1, Pigorsch et al.]:

$$IV_t = \int_{t-\Delta}^t \sigma^2(s)ds \quad (3.2)$$

### 3.1.1 Returns

Suppose there exist  $m$  intraday equilibrium returns, the  $i$ th intraday return is then defined as:

$$r_i^{X(m)} = X_{i/m} - X_{(i-1)/m}, \quad i = 1, 2, \dots, m. \quad (3.3)$$

## 3.2 Realized Variance

### 3.2.1 Assumptions

The equilibrium price process.

1. The logarithmic equilibrium price process  $p_t^X$  is a continuous stochastic volatility semimartingale. Specifically,

$$p_t^X = \alpha_t + m_t; \quad (3.4)$$

where  $\alpha_t$  (with  $\alpha_0 = 0$ ) is a predictable drift process of finite variation and  $m_t$  is a continuous local martingale defined as  $\int_0^t \sigma_s dW_s$  with  $W_t : t \geq 0$  denoting standard Brownian motion.

2. The spot volatility process  $\sigma_t$  is cadlag and bounded away from zero.
3. The process  $\int_0^t \sigma_s^4 ds$  is bounded almost surely for all  $t < \infty$ .

### 3.2.2 Estimator

For discretely observed path  $X_{t_i}$ ,  $i = 0, \dots, n$  of the  $X$ , the realized quadratic variation could be estimated consistently using the realized variance measure, defined as [2, Zu and Boswijk, 2014]:

$$RV_t = \sum_{i=1}^n (X_{t_i} - X_{t_{(i-1)}})^2 \xrightarrow{P} IV \quad (3.5)$$

However the realized variance estimator for integrated volatility is not consistent when data is contaminated by market microstructure noise. In particular, when we only observe data with microstructure noise, the realized variance measure will diverge. When sampling frequency increases, realized variance actually estimates the sum of infinite many variances of noises.

### 3.2.3 Properties

- Convergence speed:  $m^{1/2}$  ( $m$  - number of observation)
- Unbiased: no
- Consistent: no
- Jump Robust: no
- Allows for time dependent noise: no
- Allows for endogenous noise: no

## 3.3 Two Series Realized Variance

### 3.3.1 Assumptions

The microstructure noise.

1. The microstructure noise,  $\epsilon_{t,i}$ , has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of  $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$  is  $O(1)$ .

### 3.3.2 Estimator

Two Scale Realized Variance (TSRV) estimates integrated volatility consistently. The idea is to use realized variance type estimators over two time scales to correct the effect of market microstructure noise. Define as:

$$[Y, Y]_t^{avg} = \frac{1}{K} \sum_{i=K}^n (Y_{t_i} - Y_{t_{(i-K)}})^2 \quad (3.6)$$

$$[Y, Y]_t^{all} = \sum_{i=1}^n (Y_{t_i} - Y_{t_{(i-1)}})^2 \quad (3.7)$$

$$\bar{n} = \frac{n - K + 1}{K}, \quad (3.8)$$

the TSRV estimator is defined as [2, Zu and Boswijk, 2008]:

$$TSRV_t = [Y, Y]_t^{(avg)} - \frac{\bar{n}}{n} [Y, Y]_t^{(all)} \xrightarrow{p} IV \quad (3.9)$$

A small sample refinement to the estimator give correction:

$$TSRV_t^{adjust} = \left(1 - \frac{\bar{n}}{n}\right)^{-1} TSRV_t \xrightarrow{p} IV. \quad (3.10)$$

If we have possibly dependent noise we should use an alternative estimator that is also based on the two time scales idea.

To pin down the optimal sampling frequency  $K$  [3, Zhang, Mykland, and Ait-Sahalia, 2005], one can minimize the expected asymptotic variance and to obtain

$$c^* = \left( \frac{16(E\epsilon^2)^2}{TE\eta^2} \right)^{1/3} \quad (3.11)$$

which can be consistently estimated from data in past time periods (before time  $t_0 = 0$ ), using  $E\epsilon^2$  and an estimator of  $\eta^2$ .  $\eta^2$  can be taken to be independent of  $K$  as long as one allocates sampling points to grids regularly. Hence one can choose  $c$ , and so also  $K$ , based on past data.

### 3.3.3 Properties

- Convergence speed:  $m^{1/6}$  ( $m$  - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: no
- Allows for endogenous noise: no

## 3.4 Multiple Series Realized Variance

### 3.4.1 Assumptions

Dependent Noise Structure [4, Podolskij and Vetter, 2009]:

1. The microstructure noise,  $\epsilon_{t,i}$ , has a zero mean, stationary, and strong mixing stochastic process, with the mixing coefficients decaying exponentially. In addition,  $E[(\epsilon_{t,i})^{4+\kappa}]$ , for some  $\kappa > 0$ .
2. The noise is independent of the price process.
3. The variance of  $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$  is  $O(1)$ .



### 3.4.2 Estimator

Under most assumptions, this estimator violates the sufficiency principle, whence we define the average lag  $j$  realized volatility as

$$[Y, Y]_T^{(J)} = \frac{1}{J} \sum_{r=0}^{J-1} [Y, Y]_T^{(J,r)} = \frac{1}{J} \sum_{t=0}^{n-J} (Y_{t+J} - Y_t)^2 \quad (3.12)$$

A generalization of TSRV can be defined for  $1 \leq J < K \leq n$  as

$$MSRV_t = [Y, Y]_T^{(K)} - \frac{\bar{n}_K}{\bar{n}_J} [Y, Y]_T^{(J)} \xrightarrow{p} IV \quad (3.13)$$

thereby combining the two time scales  $J$  and  $K$ . Here  $\bar{n}_K = (n - K + 1)/K$  and similarly for  $\bar{n}_J$ .

### 3.4.3 Properties

- Convergence speed:  $m^{1/4}$  ( $m$  - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: yes
- Allows for endogenous noise: no

## 3.5 Modulated Realized Variance

### 3.5.1 Assumptions

The microstructure noise.

1. The microstructure noise,  $\epsilon_{t,i}$ , has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of  $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$  is  $O(1)$ .

### 3.5.2 Estimator

Modulated Bipower Variation is written as [4, Podolskij and Vetter, 2009]:

$$MBV(Y, r, l)_n = n^{(r+l)/4-1/2} \sum_{m=1}^M |\bar{Y}_m^{(K)}|^r |\bar{Y}_{m+1}^{(K)}|^l, \quad r, l \geq 0; \quad (3.14)$$

$$Y_m^{(K)} = \frac{1}{n/M - K + 1} \sum_{i=(m-1)n/M}^{mn/M-K} (Y_{(i+K)/n} - Y_{i/n}) \quad (3.15)$$

with

$$K = c_1 n^{1/2}, \quad M = \frac{n}{c_2 K} = \frac{n^{1/2}}{c_1 c_2} \quad (3.16)$$

We can choose the constants  $c_1$  and  $c_2$  from specific process:

$$c_1 = 0.25, \quad c_2 = 2. \quad (3.17)$$

Modulated Realized Variance is written as:

$$MRV(Y)_n = \frac{c_1 c_2 MBV(Y, 2, 0)_n - \nu_2 \hat{\omega}^2}{\nu_1} \xrightarrow{p} IV \quad (3.18)$$

where

$$\nu_1 = \frac{c_1(3c_2 - 4 + (2 - c_2)^3 \vee 0)}{3(c_2 - 1)^2} \quad (3.19)$$

$$\nu_2 = \frac{2((c_2 - 1) \wedge 1)}{c_1(c_2 - 1)^2} \quad (3.20)$$

### 3.5.3 Properties

- Convergence speed:  $m^{1/4}$  ( $m$  - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: no
- Allows for for endogenous noise: no

## 3.6 Jump Robust Modulated Realized Variance

### 3.6.1 Assumptions

The microstructure noise.

1. The microstructure noise,  $\epsilon_{t,i}$ , has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of  $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$  is  $O(1)$ .
4. The price is  $Z = Y + J$  where  $Y$  is a noisy diffusion process and  $J$  denotes a finite activity jump process, that is,  $J$  exhibits finitely many jumps on compact intervals.

### 3.6.2 Estimator

We can construct consistent estimates for the integrated volatility, which are robust to noise and finite activity jumps [4, Podolskij and Vetter, 2009].

$$MBV(Y, r, l)_n = \frac{(c_1 c_2 / \mu_1^2)(MBV(Z, 1, 1)_n - \nu_2 \hat{\omega}^2)}{\nu_1} \xrightarrow{p} IV \quad (3.21)$$

where

$$\nu_1 = \frac{c_1(3c_2 - 4 + (2 - c_2)^3 \vee 0)}{3(c_2 - 1)^2} \quad (3.22)$$

$$\nu_2 = \frac{2((c_2 - 1) \wedge 1)}{c_1(c_2 - 1)^2} \quad (3.23)$$

Mixed normal distribution with conditional variance [4, Podolskij and Vetter, 2009]:

$$\beta^2 = \frac{2c_1c_2}{\nu_1^2} \int_0^1 (\nu_1\sigma_u^2 + \nu_2\omega^2)^2 du \quad (3.24)$$

Consistent estimator of  $\beta$  is:

$$\beta_n^2 = \frac{2c_1^2c_2^2}{3\nu_1^2} MBV(Y, 4, 0)_n \quad (3.25)$$

We can choose the constants  $c_1$  and  $c_2$  that minimise the conditional variance. In order to compare our asymptotic variance with the corresponding results of other methods we assume that the volatility process  $\sigma$  is constant. In that case the conditional variance  $\beta^2$  is minimised by

$$c_1 = \sqrt{\frac{18}{(c_2 - 1)(4 - c_2)}} \cdot \frac{\omega}{\sigma}, \quad c_2 = \frac{8}{5} \quad (3.26)$$

We can choose the constants  $c_1$  and  $c_2$  from specific process:

$$c_1 = 0.25, \quad c_2 = 2. \quad (3.27)$$

### 3.6.3 Properties

- Converges to integrated variance
- Convergence speed:  $m^{1/6}$  ( $m$  - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: yes
- Allows for time dependent noise: no
- Allows for for endogenous noise: no

## 3.7 Kernel-based Realized Variance

### 3.7.1 Assumptions

The microstructure noise.

1. The microstructure noise,  $\epsilon_{t,i}$ , has zero mean and is distributed random variable.
2. The variance of  $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$  is  $O(1)$ .

### 3.7.2 Kernel Types

| Flat-Top Kernel               |   |
|-------------------------------|---|
| Bartlett Kernel               | $k(x) = 1 - x$  |
| Epanichnikov Kernel           | $k(x) = 1 - x^2$  |
| Second order Kernel           | $k(x) = 1 - 2x + x^2$   |
| Non-Flat-Top Kernel           |   |
| Cubic Kernel                  | $k(x) = 1 - 3x^2 + 2x^3$  |
| Parzen Kernel                 | $k(x) = \begin{cases} 2(1-x)^3 & x > 0.5 \\ 1 - 6x^2 + 6x^3 & x < 0.5, \end{cases}$ |
| Tukey Hanning Kernel          | $k(x) = \frac{(1 + \sin(\pi/2 - \pi x))}{2}$  |
| Tukey Hanning Modified Kernel | $k(x) = \frac{(1 - \sin(\pi/2 - \pi(1-x)^2))}{2}$                                   |
| Fifth Order Kernel            | $k(x) = 1 - 10x^3 + 15x^4 - 6x^5$   |
| Sixth Order Kernel            | $k(x) = 1 - 15x^4 + 24x^5 - 10x^6$  |
| Seventh Order Kernel          | $k(x) = 1 - 21x^5 + 35x^6 - 15x^7$  |
| Eighth Order Kernel           | $k(x) = 1 - 28x^6 + 48x^7 - 21x^8$  |

### 3.7.3 Estimator

Kernel-based Realized Variance is [5, Barndorff-Nielsen et al., 2006]:

$$\tilde{K}(X_\delta) = \gamma_0(X_\delta) + \sum_{h=1}^H k\left(\frac{h-1}{H}\right) \tilde{\gamma}_h(X_\delta) \quad (3.28)$$

where  $k()$  is kernel function and:

$$\tilde{\gamma}_h(X_\delta) = \gamma_h(X_\delta) + \gamma_{-h}(X_\delta) \quad (3.29)$$

$$\gamma_h(X_\delta) = \sum_{j=1}^n (X_{\delta j} - X_{\delta(j-1)})(X_{\delta(j-h)} - X_{\delta(j-h-1)}) \quad (3.30)$$

with  $h = -H, \dots, -1, 0, 1, \dots, H$  and  $n = \lceil 1/\delta \rceil$ .

Bandwidth for all kernrel is:

$$H = cn^{2/3} \quad (3.31)$$

In this case we have the asymptotic distribution given:

$$n^{1/6} \{ \tilde{K}(X_\delta) - \int_0^t \sigma_u^2 du \} \xrightarrow{L_\S} MN \left[ 0, 4ck_{\bullet}^{0,0} t \int_0^t \sigma_u^4 du + 4\omega^4 c^{-2} \{k'(0)^2 + k'(1)^2\} \right] \quad (3.32)$$

where  $\omega = (1, 1, k(\frac{1}{H}), \dots, k(\frac{H-1}{H}))^T$

$$k_{\bullet}^{0,0} = \int_0^1 k(x)^2 dx, \quad k_{\bullet}^{0,2} = \int_0^1 k(x)k''(x)dx \quad k_{\bullet}^{0,4} = \int_0^1 k(x)k''''(x)dx \quad (3.33)$$

The value of  $c$  which minimises the asymptotic variance is

$$c = d \frac{\omega^{4/3}}{(t \int_0^t \sigma_u^4 du)^{1/3}} \quad (3.34)$$

where

$$d = \left[ \frac{2k^2(0)^2 + k'(1)^2}{k_{\bullet}^{0,0}} \right]^{1/2} \quad (3.35)$$

The lower bound for the asymptotic variance is  $6dk_{\bullet}^{0,0}\omega^{4/3}(t \int_0^t \sigma_u^4 du)^{2/3}$

But for non-flat-top kernel we can review special case.

Bandwidth for non-flat-top kernel is:

$$H = cn^{1/2} \quad (3.36)$$

In this case the asymptotic distribution is given

$$n^{1/4} \{ \tilde{K}(X_\delta) - \int_0^t \sigma_u^2 du \} \xrightarrow{L_\S} MN \left[ 0, 4ck_{\bullet}^{0,0} t \int_0^t \sigma_u^4 du - 8c^{-1}k_{\bullet}^{0,2}\omega^2 \left( \int_0^t \sigma_u^2 du + \frac{\omega^2}{2} \right) + 4\omega^4 c^{-3} \{k'''(0) + k_{\bullet}^{0,4}\} \right] \quad (3.37)$$

For this class of kernels the value of  $\hat{c}$  which minimises the asymptotic variance is

$$c \approx \sqrt{\frac{1}{k_{\bullet}^{0,0}} \left\{ -k_{\bullet}^{0,2} + \sqrt{(k_{\bullet}^{0,2})^2 + 3k_{\bullet}^{0,0}f} \right\}} \quad (3.38)$$

### 3.7.4 Properties

#### Flat-Top Kernel

- Convergence speed:  $m^{1/6}$  ( $m$  - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: no
- Allows for endogenous noise: no

### Non-Flat-Top Kernel

- Convergence speed:  $m^{1/4}$  ( $m$  - number of observation)
- Unbiased: yes
- Consistent: yes
- Jump Robust: no
- Allows for time dependent noise: yes
- Allows for for endogenous noise: yes

## 3.8 IV Estimators Comparison

Figure 3.1: Table Estimators

# 4 Price Noise Variance

## 4.1 RV Noise Variance

Assume that the observed (log) price is contaminated by market microstructure noise  $u$  (or measurement error), i.e.:

$$Y_{i/m} = X_{i/m} + u_{i/m}, \quad i = 1, 2, \dots, m \quad (4.1)$$

where  $X_{i/m}$  is the latent true, or so-called efficient, price that follows the semimartingale. In this case, the observed intraday return is given by:

$$r_i^{Y(m)} = r_i^{X(m)} + \epsilon_i^{(m)}, \quad i = 1, 2, \dots, m, \quad (4.2)$$

i.e. by the efficient intraday return  $r_i^{X(m)} = X_{i/m} - X_{(i-1)/m}$  and the intraday noise increment  $\epsilon_i^{(m)} = u_{i/m} - u_{(i-1)/m}$ .

The noise variance  $E\epsilon^2$  can be estimated consistently by normalized realized variance over the whole interval  $[0, 1]$  for noisy data [3, Zhang et al., 2005]:

$$\hat{\omega}^2 = \frac{1}{2n} \sum_{i=1}^n (Y_{t_i} - Y_{t_{i-1}})^2 \quad (4.3)$$

### 4.1.1 Properties

- Convergence speed:  $m^{1/2}$  ( $m$  - number of observation)
- Unbiased: no
- Consistent: yes
- Jump Robust: yes
- Allows for time dependent noise: no

## 4.2 Autocovariance Noise Variance

The noise variance can be estimated as the negative of the first order autocovariance of observed returns [6, Oomen, 2005]:

$$\hat{\omega}^2 = \max \left\{ 0, -\frac{1}{n-1} \sum_{i=1}^n (Y_{t_i} - Y_{t_{i-1}})(Y_{t_{i-1}} - Y_{t_{i-2}}) \right\} \quad (4.4)$$

### 4.2.1 Properties

- Convergence speed:  $m^{1/2}$  ( $m$  - number of observation)
- Unbiased: no
- Consistent: yes
- Jump Robust: yes
- Allows for time dependent noise: no



# 5 Price Quarticity

## 5.1 Integrated Quarticity

The integrated quarticity is as described in [1, Pigorsch et al.]:

$$IQ = \int_{t-1}^t \sigma^4(s) ds. \quad (5.1)$$

---

## 5.2 Realized Quarticity

### 5.2.1 Assumptions

The microstructure noise.

1. The microstructure noise,  $\epsilon_{t,i}$ , has zero mean and is an independent and identically distributed random variable.
2. The noise is independent of the price process.
3. The variance of  $\nu_{t,i} = \epsilon_{t,i} - \epsilon_{t,i-1}$  is  $O(1)$ .

### 5.2.2 Estimator

The realized fourth-power variation or realized quarticity, defined as [7, Corsi et al., 2005]:

$$RQ_t = \frac{M}{3} \sum_{j=1}^M r_{t,j}^4 \xrightarrow{P} IQ \quad (5.2)$$

where  $M$  is sampling frequency.

### 5.2.3 Properties

- Convergence speed:  $m^{1/n}$  ( $m$  - number of observation)
- Unbiased:
- Consistent:
- Jump Robust:
- Allows for time dependent noise:
- Allows for endogenous noise:

# Contents

- [1] U. Pigorsch, C. Pigorsch, I. Popov, "Volatility estimation based on high-frequency data", In: Duan, JC, Gentle, JE and Hardle E, Handbook of Computational Finance, New York: Springer, 2010
- [2] Y. Zu and Peter Boswijk, "Estimating spot volatility with high-frequency financial data", Journal of Econometrics, 181(2), pp. 117-135, 2014
- [3] L. Zhang, P. A. Mykland, and Y. Ait-Sahalia, "A tale of two time scales: Determining integrated volatility with noisy high-frequency data", Journal of the American Statistical Association, vol. 100, No. 472, pp. 1394-1411, 2005.
- [4] M. Podolskij and M. Vetter, "Estimation of volatility functionals in the simultaneous presence of microstructure noise and jumps", Bernoulli, vol. 15, No. 3, pp. 634-658, 2009.
- [5] O.E.Barndorff-Nielsen, P.Reinhard Hansen, A.Lunde, and N.Shephard, "Designing realised kernels to measure the ex-post variation of equity prices in the presence of noise", Economics Series Working Papers 264, University of Oxford, Department of Economics, 2006.
- [6] R. C. Oomen, "Comment on realized variance and market microstructure noise by Peter R. Hansen and Asger Lunde," pp. 1-15, 2005.
- [7] Corsi, F., U. Kretschmer, S. Mittnik and C. Pigorsch, "The Volatility of Realized Volatility", Econometric Reviews, pp. 46-78, 2008